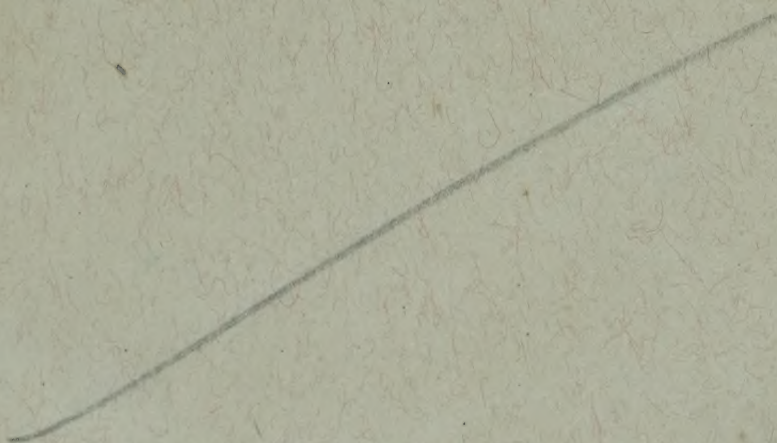


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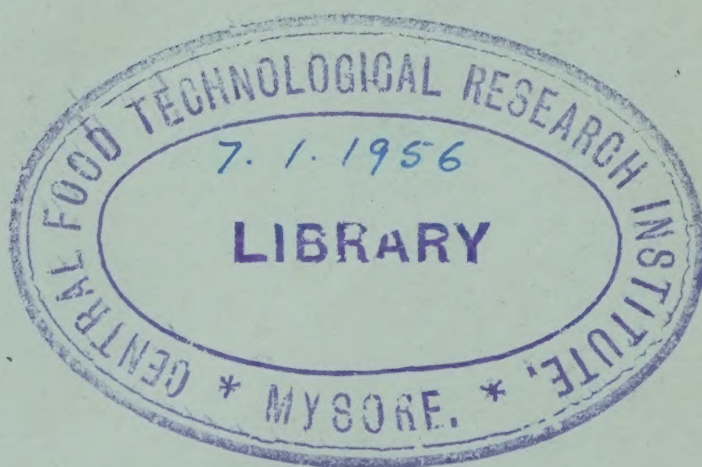
FLUID FLOW, HEAT TRANSFER
AND MASS TRANSFER

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CONTENTS

	<i>page</i>
PREFACE	vii
1. UNITS AND DIMENSIONS	1

SECTION A—Flow of Fluids

INTRODUCTION	11
2. ENERGY RELATIONSHIPS	13
Internal energy—compressible and incompressible fluids—energy of a fluid in motion—fluid head and pressure	
3. FRICTION IN PIPES AND CHANNELS	28
Calculation of pressure drop in a pipe—roughness of pipe surface—types of flow—significance of Reynolds Number—streamline flow through pipes, between plates, and through annuli—turbulent flow—curved pipes—sudden enlargement and contraction—pipe fittings—tube banks—separation—flow of compressible fluids in pipes (isothermal, non-isothermal and adiabatic)—maximum flow—power requirements for pumping—converging-diverging nozzles—flow in open channels—uniform and non-uniform flow—specific energy—critical depth—hydraulic jump	
4. <u>FLOW MEASUREMENT</u>	74
Measurement of pressure—measurement of flow—pitot tube—orifice meter—nozzle—venturi meter—rotameter—notches—hot wire anemometer—quantity meters	
5. <u>PUMPS FOR CHEMICAL WORKS</u>	104
Piston pump—ram pump—diaphragm pump—glandless pump—gear pump and other rotary pumps—centrifugal pump—virtual head—metering pumps—air-operated elevators—air lift pump—gas pumps and compressors—thermodynamics of gas compression	

SECTION B—Heat Transfer

6. HEAT TRANSFER	151
Conduction—resistances in series—logarithmic mean temperature difference—unsteady conduction—convection—forced convection inside and outside tubes—tube bundles—natural convection—radiation—gas radiation—heat transfer from condensing vapours—heat transfer to boiling liquids—heat transfer in reaction vessels—shell and tube heat exchangers—determination of film coefficients—finned tubes—plate heat exchangers—thermal insulation	

SECTION C—Momentum, Heat and Mass Transfer

7. MASS TRANSFER	233
Fick's law—diffusion in gas phase—equimolecular counterdiffusion—diffusion through stationary gas—Maxwell's law—diffusion in liquids—mass transfer by natural convection and forced convection—unsteady mass transfer—applications	

8. MOMENTUM, HEAT AND MASS TRANSFER . . . 245

Transfer by molecular diffusion—eddy transfer—quantitative relations between heat, mass, and momentum transfer—use of “j-factors”

9. THE BOUNDARY LAYER . . . 259

Momentum equation—streamline and turbulent boundary layer—laminar sub-layer—flow at entry to pipe—fully developed flow—universal velocity profile—effect of surface roughness—boundary layer for heat transfer—streamline flow over plane surface—streamline flow through heated tube—boundary layer for mass transfer—Reynolds’ analogy between heat transfer and skin friction—Taylor-Prandtl modification—use of universal velocity profile

SECTION D—Humidification and Water Cooling

10. HUMIDIFICATION AND WATER COOLING . . . 305

Wet bulb temperature—adiabatic saturation temperature—humidity charts (temperature and enthalpy)—determination of humidity—humidification and dehumidification—water cooling—design of water cooling tower—design of humidifying tower—systems other than air-water

APPENDIX

TABLES OF PHYSICAL PROPERTIES . . . 336

STEAM TABLES . . . 353

PROBLEMS . . . 361

INDEX . . . 373

NOTE TO THE SECOND IMPRESSION

IN view of the short time which has elapsed since this book was first issued, it has not been thought desirable to make any major revisions for the second impression. The opportunity has been taken to incorporate a number of corrections and we should like to thank the readers who have pointed out errors—notably Messrs. G. H. ANDERSON and D. TRAIN. We shall be very pleased to hear of any further shortcomings so that suitable amendments can be made when a revised edition is prepared.

PREFACE

THE idea of treating the various processes of the chemical industry as a series of unit operations was first brought out as a basis for a new technology by WALKER, LEWIS and MCADAMS in their book in 1923. Before this, the engineering of chemical plants had been regarded as individual to an industry and there was little common ground between one industry and another. Since the early 1920's chemical engineering as a separate subject has been introduced into the universities of both America and England and has expanded considerably in recent years so that there are now a number of university courses in both countries. During the past twenty years the subject matter has been extensively increased by various researches described in a number of technical journals to which frequent reference is made in the present work.

Despite the increased attention given to the subject there are few general books, although there have been a number of specialised books on certain sections such as distillation, heat transfer, etc. It is the purpose of the present work to present to the student an account of the fundamentals of the subject. The physical basis of the mechanism of many of the chemical engineering operations forms a major feature of chemical engineering technology. Before tackling the individual operations it is important to stress the general mechanisms which are found in so many of the operations. We have therefore divided the subject matter into two volumes, the first of which contains an account of these fundamentals—diffusion, fluid flow and heat transfer. In Volume 2 we shall show how these theoretical foundations are applied in the design of individual units such as distillation columns, filters, crystallisers, evaporators, etc.

Volume 1 is divided into four sections, fluid flow, heat transfer, mass transfer and humidification. Since the chemical engineer must handle fluids of all kinds, including compressible gases at high pressures, we believe that it is a good plan to consider the problem from a thermodynamic aspect and to derive general equations for flow which can be used in a wide range of circumstances. We have paid special attention to showing how the boundary layer is developed over plane surfaces and in pipes, since it is so important in controlling heat and mass transfer. At the same time we have included a chapter on pumping since chemical engineering is an essentially practical subject, and the normal engineering texts do not cover the problem as experienced in the chemical and petroleum industries.

The chapter on heat transfer contains an account of the generally accepted techniques for calculation of film transfer coefficients for a wide range of conditions, and includes a section on the general construction of tubular exchangers which form a major feature of many works. The possibilities of the newer plate type units are indicated.

In section three, the chapter on mass transfer introduces the mechanism of diffusion and this is followed by an account of the common relationships between

heat, mass and momentum transfer and the elementary boundary layer theory. The final section includes the practical problem of humidification where both heat and mass transfer are taking place simultaneously.

It will be seen that in all chapters there are sections in small print. In a subject such as this, which ranges from very theoretical and idealised systems to the practical problems with empirical or experimentally determined relations, there is much to be said for omitting the more theoretical features in a first reading, and in fact this is frequently done in the more practical courses. For this reason the more difficult theoretical sections have been put in small print and the whole of Chapter 9 may be omitted by those who are more concerned with the practical utility of the subject.

In many of the derivations we have given the mathematical analysis in more detail than is customary. It is our experience that the mathematical treatment should be given in full and that the student should then apply similar analysis to a variety of problems.

We have introduced into each chapter a number of worked examples which we believe are essential to a proper understanding of the methods of treatment given in the text. It is very desirable for a student to understand a worked example before tackling fresh practical problems himself. Chemical engineering problems require a numerical answer and it is essential to become familiar with the different techniques so that the answer is obtained by systematic methods rather than by intuition.

In preparing this text we have been guided by courses of lectures which we have given over a period of years and have presented an account of the subject with the major emphasis on the theoretical side. With a subject that has grown so rapidly, and which extends from the physical sciences to practical techniques, the choice of material must be a matter of personal selection. It is, however, more important to give the principles than the practice, which is best acquired in the factory. We hope that the text may also prove useful to those in industry who, whilst perhaps successfully employing empirical relationships, feel that they would like to find the extent to which the fundamentals are of help.

We should like to take this opportunity of thanking a number of friends who have helped by their criticism and suggestions, amongst whom we are particularly indebted to Mr. F. E. WARNER, to Dr. M. GUTER, to Dr. D. J. RASBASH and to Dr. L. L. KATAN. We are also indebted to a number of companies who have kindly permitted us to use illustrations of their equipment. We have given a number of references to technical journals and we are grateful to the publishers for permission to use illustrations from their works. In particular we would thank the Institution of Chemical Engineers, the American Institute of Chemical Engineers, the American Chemical Society, the Oxford University Press and the McGraw-Hill Book Company.

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CHAPTER 1

Units and Dimensions

At an early stage, the student of chemical engineering will discover that the data which he uses are expressed in a great variety of different units, so that he must express his quantities in a common system before proceeding with his calculations. Most of the physical properties which are determined in the laboratory will be expressed in the C.G.S. system, referred to later, whereas the dimensions of the full-scale plant, its throughput and operating characteristics, will be expressed either in some form of general engineering units or in special units which have their origin in the history of the particular industry. This inconsistency is quite unavoidable and is a reflection of the fact that chemical engineering has in many cases developed as a synthesis of scientific knowledge and practical experience. Familiarity with the various systems of units and an ability readily to convert from one to another are therefore essential and it therefore seems desirable to give a brief summary of the most common systems of units and the method of conversion.

Since the physical properties of a system are interconnected by a series of mechanical and physical laws, it is possible to regard certain quantities as basic and other quantities as derived. The choice of basic units varies from one system to another but it is usual to take length and time as fundamental. We will denote these quantities by L and T . The dimensions of velocity, which is a rate of increase of distance with time, can then be written as LT^{-1} and those of acceleration, which is a rate of increase of velocity, are LT^{-2} . An area has the dimensions L^2 and a volume has the dimensions L^3 .

The volume of a body does not completely define the amount of material which it contains and therefore it is usual to define a third basic quantity. This third quantity is usually taken either as the gravitational force F acting on the body, i.e. its weight, or as the amount of matter in the body, i.e. its mass M . Force and mass are related by the lemma that the net force acting on a body in a given direction is equal to the product of its mass and its acceleration in that direction. Thus F and M are connected by the dimensional equation

$$F = MLT^{-2} \text{ or } M = FL^{-1}T^2$$

In scientific work it is usual to take mass as the third fundamental quantity, but in engineering force is more often used. Three common systems of units will now be discussed.

The Centimetre-gram-second (C.G.S.) System

In this system the basic units are as follows:

Length	. 1 centimetre	. L
Mass	. 1 gram	. M
Time	. 1 second	. T

The unit of force is that force which will give a mass of 1 gram an acceleration of 1 centimetre per second per second and is known as the dyne (MLT^{-2}).

$$\text{Thus} \quad 1 \text{ dyne} = 1 (\text{gram})(\text{centimetre})(\text{second})^{-2}$$

The Foot-pound-second (F.P.S.) System

The following basic units are used in this system:

Length	. 1 foot	. L
Mass	. 1 pound	. M
Time	. 1 second	. T

The unit of force, the poundal, is that force which will impart an acceleration of 1 foot per second per second to a mass of 1 pound, i.e.

$$1 \text{ poundal} = 1 (\text{pound mass})(\text{foot})(\text{second})^{-2}$$

In some cases, particularly in heat transfer, it is convenient to use the hour as the unit of time.

The British Engineering System (Gravitational Units)

This system again uses the foot and second as the units of length and time but employs the pound weight as the third fundamental unit. The pound weight is defined as the force exerted by the earth's gravitational field on a mass of 1 pound. We thus have the following fundamental units:

Length	. 1 foot	. L
Force	. 1 pound weight	. F
Time	. 1 second	. T

The unit of mass in this system is known as the slug and is the mass which is given an acceleration of 1 foot per second per second by the application of a force of 1 pound weight, i.e.

$$1 \text{ slug} = 1 (\text{pound force})(\text{foot})^{-1}(\text{second})^2$$

Comparison of the Two English Systems

In the F.P.S. system, the unit of force is defined as that required to give an acceleration of 1 foot per second per second to a mass of 1 pound. In the engineering system, the basic unit of force is taken as the attraction of the earth for 1 pound mass, i.e. the force required to give an acceleration of g (32.2 feet per second per second) to a mass of 1 pound. Thus the unit of force in the engineering system is 32.2 times greater than that in the F.P.S. system and the unit of mass is correspondingly greater, i.e.

$$1 \text{ pound weight} = 32.2 \text{ poundals}$$

$$1 \text{ slug} = 32.2 \text{ pounds mass}$$

Misunderstanding often arises from the fact that the unit of mass in the one system has the same name as the unit of force in the other, viz. the pound. In order to avoid confusion the unit of mass will in future be referred to simply as the pound and the unit of force as the pound weight. It will be noted that the units of 1 pound mass and 1 pound weight belong to two different sets of units

and therefore these two quantities must not be used together in any dimensionally consistent equation.

In the present work we shall use mass, length, and time as the fundamental units. Our units are therefore independent of the acceleration due to gravity, and “ g ” will be involved only where work is done in the earth’s gravitational field. If gravitational units are used, the units of force and mass are related to the acceleration due to gravity, with the result that “ g ” has to be introduced explicitly in those terms representing energy not connected with work in the earth’s gravitational field; further no explicit “ g ” appears where work is done in the gravitational field, since it is involved implicitly in the units of force or mass. This latter system is therefore regarded as more confusing.

Derived Units

Properties of a system, involving motion, forces, and mechanical work can be expressed in terms of the three fundamental units of mass, length, and time. We have already shown, for instance, that force has dimensions MLT^{-2} . We now find that the dimensions of pressure, which is defined as a force per unit area, are $\frac{\text{MLT}^{-2}}{\text{L}^2} = \text{ML}^{-1}\text{T}^{-2}$. Again viscosity is a force per unit area and unit velocity gradient and therefore has the dimensions $\frac{\text{MLT}^{-2}}{\text{L}^2 \cdot \frac{\text{LT}^{-1}}{\text{L}}}$, i.e. $\text{ML}^{-1}\text{T}^{-1}$; and density is a mass, per unit volume, and therefore has dimensions ML^{-3} . Kinematic viscosity is viscosity divided by density, and therefore has dimensions of $\frac{\text{ML}^{-1}\text{T}^{-1}}{\text{ML}^{-3}}$ or L^2T^{-1} . Energy is given by the product of force and distance and has dimensions ML^2T^{-2} . In this way the dimensions of each of the quantities involved in a problem can be calculated. A list is given in Table 1.1.

Thermal Units

As heat is a form of energy, it must necessarily have the dimensions ML^2T^{-2} but it is not always convenient to express it in this manner. The heat content of a body is usually expressed as the product of its mass, temperature above an arbitrary datum level, and specific heat. Energy is then expressed in thermal units which differ from the mechanical units by a factor known as the mechanical equivalent of heat. Provided consistent units are employed for heat, mass, and temperature, the specific heat of a material has a fixed numerical value and is therefore conveniently regarded as dimensionless. Denoting temperature by θ , we have that thermal energy has the dimensions $\text{M}\theta$. The temperature scale could be defined so that the numerical values of mechanical and thermal energy were identical but such a scale would be unfamiliar and we shall therefore use the Kelvin and Rankine scales and regard temperature as a fourth variable. Using this notation, we can then define our mechanical equivalent of heat J by the relation

$$J \times \text{thermal energy} = \text{mechanical energy}$$

The dimensions of J are therefore $\frac{ML^2T^{-2}}{M\theta}$, i.e. $L^2T^{-2}\theta^{-1}$. Since J is a factor connecting two quantities of essentially the same dimensions, it must itself be dimensionless, so that θ has the dimensions of L^2T^{-2} , or velocity squared. As already mentioned, however, we shall regard temperature as a fourth fundamental dimension.

Quantities involving energy can therefore be expressed in terms of either thermal or mechanical energy. Thus, for example, dividing a small quantity of heat added to a system under reversible conditions by the temperature at which it is added, gives the small change in the entropy which can be said to have dimensions of either $ML^2T^{-2}\theta^{-1}$ or M . Thus entropy per unit mass has dimensions of $L^2T^{-2}\theta^{-1}$ in mechanical energy units and is dimensionless in thermal energy units; and similarly for other thermal quantities which are tabulated in Table 1.1.

In some cases mechanical energy and thermal energy may appear in the same equation and both must then be expressed in the same units. In these circumstances we shall normally use mechanical energy units so that the specific heat of the material, instead of being truly dimensionless, is given the dimensions of mechanical energy

$$\frac{\text{thermal energy}}{\text{mechanical energy}} = \frac{ML^2T^{-2}}{M\theta} = L^2T^{-2}\theta^{-1}, \text{ i.e. the same as those of the mechanical equivalent of heat.}$$

Electrical Units and Magnetic Units

Two systems of units are in use, one depending on the definition of the magnetic pole strength and the other on the definition of a unit electrical charge. The two sets of units differ by a factor equal to the product of the magnetic permeability and the electrical susceptibility; this has the dimensions of velocity and is numerically equal to the velocity of light in air. In the present work we shall only occasionally be concerned with these units and shall always use the electromagnetic system. The dimensions of the more important quantities are given in Table 1.1.

Conversion of Units

Conversion of units from one system to another is simply carried out if the quantities are expressed in terms of the fundamental units of mass, length, time, temperature. The conversion factors for the British and Metric Systems are then:

<i>Mass</i>	. 1 pound = 453.6 grams = $\frac{1}{32.2}$ slugs
<i>Length</i>	. 1 foot = 30.48 centimetres
<i>Time</i>	. 1 second = $\frac{1}{3600}$ hour
<i>Temperature</i>	. 1 degree Fahrenheit = $\frac{1}{1.8}$ degree Centigrade
<i>Force</i>	. 1 gram weight = 981 dynes
	1 pound weight = 32.2 poundals
	1 pound weight = 453.6 grams weight
	1 poundal = 13,800 dynes

TABLE 1.1. Units

Quantity	Absolute C.G.S.	Absolute F.P.S.	Gravitational F.P.S.	Dimensions in M, L, T, θ
Mass	gram	pound	slug	M
Length	centimetre	foot	foot	L
Time	second	second	second	T
Force	dyne	poundal	pound weight	MLT^{-2}
Energy	erg (= 10^{-7} joules)	foot-poundal	foot-pound	ML^2T^{-2}
Pressure	dynes/square centimetre	poundals/square foot	pound weight/square foot	$ML^{-1}T^{-2}$
Power	ergs/second	foot-poundals/second	foot-pounds/second	ML^2T^{-3}
Entropy per unit mass	ergs/gram-°C	foot-poundals/pound-°C	foot-pounds/pound-°C	$L^2T^{-2}\theta^{-1}$

Heat units

Quantity	C.G.S.	Absolute F.P.S. (°C)	Absolute F.P.S. (°F)	Dimensions
Temperature	degree centigrade	degree centigrade	degree Fahrenheit	θ
Thermal energy or heat	gram-calorie	pound-calorie	British thermal unit (B.Th.U.)	$M\theta$
Entropy per unit mass	(dimensionless)	(dimensionless)	(dimensionless)	—
Mechanical equivalent of heat, J	4.18×10^7 ergs/gram-°C	4.5×10^4 foot-pounds/pound-°C	2.5×10^4 foot-pounds/pound-°F	$L^2T^{-2}\theta^{-1}$

Electrical units (electromagnetic) (e.m.)

	Dimensions	Practical units
Potential	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{+\frac{1}{2}}$	volt = 10^8 e.m. units
Current	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$	ampere = 10^{-1} e.m. units
Resistance	$KT^{-1}\mu$	ohm = 10^9 e.m. units
Magnetic permeability	μ	10^7 e.m. units
Power	ML^2T^{-3}	watt = 10^7 e.m. units = 1 joule/sec.

If we now wish to convert a viscosity from poises (grams per centimetre-second) to a viscosity in pounds per foot second, we proceed as follows:

$$\begin{aligned}
 1 \text{ gram/cm-sec.} &= \frac{1 \text{ gram}}{1 \text{ cm} \times 1 \text{ sec.}} \\
 &= \frac{\frac{1}{453.6} \text{ lb}}{\frac{1}{30.48} \text{ ft} \times 1 \text{ sec.}} \\
 &= \frac{30.48}{453.6} = 0.0672 \text{ lb/ft-sec.} \\
 &= 242 \text{ lb/ft-hr}
 \end{aligned}$$

Viscosities are commonly expressed in centipoises; one centipoise is thus 2.42 lb/ft-hr.

Again we can express our dimensions of viscosity in terms of force rather than of mass, since

$$\mathbf{F} = \mathbf{MLT}^{-2}$$

$$\mathbf{M} = \mathbf{FL}^{-1}\mathbf{T}^2$$

$$\text{Viscosity} = \mathbf{ML}^{-1}\mathbf{T}^{-1} = \mathbf{FL}^{-2}\mathbf{T}$$

Thus we can convert our viscosity from poises into pounds weight—sec./ft² (for example) as follows:

$$1 \text{ lb wt.-sec./ft}^2 = 1 \text{ slug/ft-sec.}$$

$$\begin{aligned}
 1 \text{ gram/cm-sec.} &= \frac{1 \text{ gram}}{1 \text{ cm} \times 1 \text{ sec.}} \\
 &= \frac{\frac{1}{453.6} \times \frac{1}{32.2} \text{ slugs}}{\frac{1}{30.48} \text{ ft} \times 1 \text{ sec.}} \\
 &= \frac{30.48}{453.6 \times 32.2} = 0.0021 \text{ slugs/ft-sec. or lb wt-sec./ft}^2 \\
 &= 7.5 \text{ slugs/ft-hr.}
 \end{aligned}$$

The conversion of electrical units into mechanical energy units is illustrated by the conversion of 1 kilowatt into horse power:

$$\begin{aligned}
 1 \text{ kW} &= 10^3 \times 10^7 \text{ ergs/sec.} \\
 &= 10^{10} \times \frac{1}{453.6} \times \frac{1}{(30.48)^2} \times \frac{1}{32.2} \\
 &= 737 \text{ ft-lb/sec.} \\
 &= \frac{737}{550} = 1.34 \text{ h.p.}
 \end{aligned}$$

i.e.

$$1 \text{ h.p.} = 0.746 \text{ kW}$$

In later chapters, the method of dimensional analysis will be employed in order to arrange the variables involved in a given process in the form of dimensionless groups. The method depends on the fact that any equation must be dimensionally consistent, for if not, the actual value of any property would depend on the units employed. The method is discussed fully by BRIDGMAN and an illustration of the method is given in Chapter 6 in connection with heat transfer by convection.

REFERENCE

BRIDGMAN, P. W.: *Dimensional Analysis*. Yale University Press (1931).

SYMBOLS

g	Acceleration due to gravity	LT^{-2}
J	Mechanical equivalent of heat	$L^2T^{-2}\theta^{-1}$
F	Force	
L	Length	
M	Mass	
T	Time	
μ	Magnetic permeability	
θ	Temperature	

Section A

Flow of Fluids

Introduction

THE chemical engineer is interested in many aspects of the problems involved in the flow of fluids. In the first place, in common with many other engineers, he is concerned with the transport of fluids from one location to another through pipes or open ducts; this requires determination of the pressure drops in the system, and hence of the power required for pumping, selection of the most suitable type of pump, and measurement of the flow rates. In many cases, the fluid contains solid particles in suspension and it is necessary to determine the effect of these particles on the flow characteristics of the fluid or, alternatively, the drag force exerted by the fluid on the particles. In some cases, such as filtration, the particles are in the form of a fairly stable bed and the fluid has to pass through the tortuous channels formed by the pore spaces. In other cases the shape of the boundary surfaces must be so arranged that a particular flow pattern is obtained: for example, when solids are maintained in suspension in a liquid by means of agitation, the desired effect can be obtained with the minimum expenditure of energy if the most suitable flow pattern is produced in the fluid. Further, in those processes where heat transfer or mass transfer to a flowing fluid occurs, the nature of the flow may have a profound effect on the transfer coefficient for the process.

Fluid flow processes may be classified in a number of different ways. Thus the proximity and extent of boundary surfaces will affect the shear stresses and the velocity distribution within the fluid. Where the fluid is a comparatively large distance from a boundary there will be an almost complete absence of shear stresses within the fluid and no appreciable velocity gradient at right angles to the direction of flow. The behaviour of the fluid will approximate to that of an ideal fluid of zero viscosity. On the other hand, velocity gradients and shear stresses near boundary surfaces may be large. In these circumstances pressure gradients in the direction of flow will exist and there will be a close relation between the pressure gradient and the transfer coefficients for heat and mass at the surface. It will be necessary to examine both the effect of the surface on the flow properties of the fluid and the drag force exerted by the fluid on the surface.

The nature of the fluid has an important bearing on the type of flow which will occur. The fluid may be substantially incompressible and the density independent of the pressure; for most practical purposes liquids can be considered incompressible. On the other hand, gases are highly compressible and the density may vary considerably from one end of the system to the other. Further, the behaviour of the fluid when subjected to shear stresses is important. In the so-called Newtonian fluids the rate of shear within the fluid is directly proportional to the shear stress applied; that is to say, the viscosity is independent of the rate of shear. In other fluids, particularly those containing solid

particles in suspension and emulsions, the relation between shear stress and rate of shear is not linear and properties such as thixotropy are exhibited. In addition the presence of suspended solid particles may present special problems in connection with the design of valves and pumps. If the fluid is corrosive, special materials or special methods of construction may have to be employed.

In this section, the flow of Newtonian fluids through simple ducts will be considered. The flow of suspensions and flow through granular beds will be treated in Volume 2.

CHAPTER 2

Energy Relationships

IN the absence of magnetic, electrical and other similar fields, the properties of a given quantity of a particular fluid are determined by its pressure and temperature; other properties can then be evaluated by using the equation of state of the fluid. In order to understand fully the processes which are taking place in a fluid it is necessary to consider the nature of these two variables.

Temperature is a statistical quantity which is related to the energy of the individual molecules. In a gas, attractive forces between the molecules are generally small and the individual molecules are in a state of random motion: the temperature is proportional to the mean kinetic energy of the molecules which will all be moving at different speeds and in a random direction. In a liquid the molecules are free to move relative to one another but are subject to the action of the large intermolecular forces. The liquid state is difficult to define in terms of molecular motion but many of the expressions which are obtained for the properties of a gaseous system are applicable qualitatively to liquids. In solids the vibrational energy of the molecules, which are not free to move relative to one another, gives rise to temperature effects.

The temperature difference is a measure of the driving force available for the transfer of heat. Heat will flow in a direction of decreasing temperature until the system attains a uniform temperature; heat transfer in the opposite direction can be effected only by the performance of external work on the system. This is a statement of the Second Law of Thermodynamics.

The fact that a system always tends spontaneously to attain a uniform temperature suggests that a random distribution of molecular velocities is a stable one; this would be expected from probability. However, it is conceivable that at some instant there may be a preponderance of fast moving molecules at one end of the system and a preponderance of slow moving ones at the other. Because the number of molecules involved is very large, this behaviour is contrary to normal experience and the possibility will therefore be disregarded.

The pressure exerted by a fluid on any surface which is immersed in it results from the impingement of molecules of the fluid upon it. In a gas, the pressure is directly related to the rate of change of momentum of the molecules which collide with the surface.

Consider an ideal gas (one in which intermolecular forces are negligible and in which the volumes of the molecules are small compared with the space between them) in which there are N molecules per unit volume each of mass m . These molecules will have a wide range of velocities. Suppose that N_a of these molecules have a velocity c_a , irrespective of direction. Of these N_a molecules, N_1 have a velocity u_1 in any given

direction, N_2 molecules have a velocity u_2 in that direction, and so on. Imagine a surface of unit area immersed in the fluid at right angles to this direction.

When each of the N_1 molecules of velocity component u_1 hits the surface and rebounds, there will be a net change in momentum $2mu_1$ at right angles to the surface. This assumes that the molecules are perfectly elastic bodies. If the molecules were not perfectly elastic they would rebound from the surface with a lower velocity and therefore a certain amount of their energy would be converted into heat energy. Since heat energy and kinetic energy of the molecules are one and the same thing, this heat would appear again as an equivalent amount of kinetic energy.

Now in a time interval dt the molecules in a volume of $u_1 dt$ will strike the surface. As equal numbers will be moving away from and towards the surface, the numerical concentration of molecules of velocity u_1 moving towards the surface is $\frac{1}{2}N_1$.

Thus the change of momentum of molecules of velocity u_1 at the surface in time dt is $2mu_1 \times u_1 dt \times \frac{1}{2}N_1$.

The rate of change of momentum is therefore $mN_1u_1^2$.

Thus the total rate of change of momentum due to all molecules of velocity c_a is $m\sum N_1u_1^2$.

Now the mean square of u_1 , u_2 , etc., is given by

$$u_a^2 = \frac{N_1u_1^2 + N_2u_2^2 + \dots}{N_1 + N_2 + \dots} = \frac{\sum N_1u_1^2}{N_a}$$

Since the direction of motion of the molecules is random, the mean square velocity in each of the other two principal directions will also be equal to u_a^2 . Then combining the components of the velocities in the three directions, we have

$$\begin{aligned} (u_a^2)_x + (u_a^2)_y + (u_a^2)_z &= c_a^2 \\ \text{i.e.} \quad u_a^2 &= \frac{1}{3}c_a^2 \end{aligned}$$

The rate of change of momentum of all molecules of velocity c_a is therefore $\frac{1}{3}mN_ac_a^2$; and the total rate of change of momentum at the surface is $\frac{1}{3}m\sum N_ac_a^2 = \frac{1}{3}mNc^2$, where c^2 is the mean square of the velocities of all the molecules.

Now the pressure on the surface is equal to the rate of change of momentum per unit area,

$$\therefore P = \frac{1}{3}mNc^2 = \frac{1}{3}\rho c^2 \quad \dots(2.1)$$

where ρ is the density of the gas.

Hence the kinetic energy of the molecules in unit volume of gas

$$= \frac{1}{2}\rho c^2 = \frac{3}{2}P$$

This method introduces small errors because the molecules are assumed to approach the surface at their root mean square velocity.

Now the equation of state of an ideal gas is

$$PV = nRT \quad \dots(2.2)$$

where V is the volume of the gas,

n is the number of moles of gas (mass divided by molecular weight),

T is its absolute temperature, and

R is the universal gas constant.

This equation is obtained by extrapolation of the data for real gases to conditions of very low pressures: the distances between the molecules are then large, so that the

attractive forces between the molecules are small and the volume of the molecules is small compared with the space between them.

If we consider unit mass of gas,

$$P \cdot \frac{1}{\rho} = \frac{1}{M} RT$$

where M is the molecular weight,

$$\text{i.e.} \quad T = \frac{1}{3} \frac{M}{R} \cdot c^2 \quad \dots (2.3)$$

Thus at a given temperature, T , the mean kinetic energies of the molecules of all gases are the same, since the molecular weight is directly proportional to the weight of the individual molecules. Thus the molecules of gases of high molecular weight are travelling with correspondingly low velocities.

From equation 2.1, we have that the root mean square velocity of the molecules is equal to $\sqrt{\frac{3P}{\rho}}$ and is therefore proportional to the reciprocal of the square root of the density of the gas. This is a statement of Graham's Law of Diffusion, since the rate of diffusion of a gas is proportional to the mean velocity of the molecules.

INTERNAL ENERGY

When a fluid flows from one location to another, energy will, in general, be converted from one form to another. The energy which is attributable to the physical state of the fluid is known as internal energy; it is arbitrarily taken as zero at some reference state, such as the absolute zero of temperature or the melting point of ice at atmospheric pressure. A change in the physical state of a fluid will, in general, cause an alteration in the internal energy. An elementary reversible change results from an infinitesimal change in one of the intensive factors acting on the system; the change proceeds at an infinitesimal rate and a small change in the extensive factor in the opposite direction would have caused the process to take place in the reverse direction. Truly reversible changes never occur in practice but they provide a useful standard with which actual processes can be compared. In an irreversible process changes are caused by a finite difference in the intensive factor and take place at a finite rate. In general the process will be accompanied by the conversion of electrical or mechanical energy into heat, or the reduction of the temperature difference between different parts of the system. In fluid flow, we shall normally be concerned with irreversible processes.

For a stationary material the change in the internal energy is equal to the difference between the net amount of heat added to the system and the net amount of work done by the system on its surroundings. For an infinitesimal change,

$$dU = \delta q - \delta W \quad \dots (2.4)$$

where dU is the small change in the internal energy,

δq is the small amount of heat added, and

δW is the net amount of work done on the surroundings.

In this expression, consistent units must be used, either heat units or mechanical energy units. dU is a small change in the internal energy which is a property of the system; it is therefore a perfect differential. On the other hand, δq and δW are small quantities of heat and work; they are not properties of the system and their values depend on the manner in which the change is effected; they are, therefore, not perfect differentials. For a reversible process, however, both δq and δW can be expressed in terms of properties of the system. For convenience, we shall refer to systems of unit mass and disregard the effect on the surroundings.

We can define a property called entropy by the relation

$$dS = \frac{\delta q}{T} \quad \dots(2.5)$$

where dS is the small change in entropy resulting from the addition of a small quantity of heat δq , at a temperature T , under reversible conditions. From the definition of the thermodynamic scale of temperature, $\oint \frac{\delta q}{T} = 0$ for a reversible cyclic process*; and the net change in the entropy is also zero. Thus, for a particular condition of the system, the entropy has a definite value and must be a property of the system; dS is, therefore, a perfect differential.

For an irreversible process,

$$\frac{\delta q}{T} < dS = \frac{\delta q}{T} + \frac{\delta F}{T} \text{ (say)} \quad \dots(2.6)$$

δF is then a measure of the degree of irreversibility of the process; it represents the amount of mechanical energy converted into heat or the conversion of heat energy at one temperature to heat energy at another temperature. For a finite process,

$$\int_1^2 TdS = \Sigma \delta q + \Sigma \delta F = q + F \text{ (say)} \quad \dots(2.7)$$

When a process is isentropic, $q = -F$; a reversible process is isentropic when $q = 0$; i.e. a reversible adiabatic process is isentropic.

The increase in the entropy of an irreversible process can be illustrated in the following manner. Consider the spontaneous transfer of a quantity of heat δq from one part of a system at a temperature T_1 to another part at a temperature T_2 . The net change in the entropy of the system as a whole is then,

$$dS = \frac{\delta q}{T_2} - \frac{\delta q}{T_1}$$

T_1 must be greater than T_2 and dS is therefore positive. If the process had been carried out reversibly, there would have been an infinitesimal difference between T_1 and T_2 and the change in entropy would have been zero.

* This is discussed fully in *Chemical Engineering Thermodynamics*, by DODGE⁽⁴⁾.

Now the change in the internal energy can be expressed in terms of properties of the system itself. For a reversible process,

$$\delta q = TdS \text{ (from 2.5)}$$

and

$$\delta W = Pd v$$

if the only work done is that resulting from a change in volume, $d v$.

Thus, from equation (2.4),

$$dU = TdS - Pd v \quad \dots (2.8)$$

Since this relation is in terms of properties of the system, it must also apply to a system in motion and to irreversible changes, where the only work done is the result of change of volume.

Thus, in an irreversible process, for a stationary system,

$$\text{from 2.4 and 2.5,} \quad dU = \delta q - \delta W = TdS - Pd v$$

and from 2.6

$$\delta q + \delta F = TdS$$

$$\therefore \quad \delta W = Pd v - \delta F \quad \dots (2.9)$$

i.e. the useful work performed by the system is less than $Pd v$ by an amount δF , which represents the amount of mechanical energy converted into heat energy.

The relation between the internal energy and the temperature of a fluid will now be considered. In a system consisting of unit mass of material and where the only work done is that resulting from volume change, the change in internal energy after a reversible change is given by,

$$\begin{aligned} dU &= \delta q - Pd v && \text{(from 2.4)} \\ &= \delta q && \text{(for a process carried out at constant volume)} \\ &= C_v dT && \dots (2.10) \end{aligned}$$

where C_v is the specific heat at constant volume.

As this relation is in terms of properties of the system, it must be applicable to all changes at constant volume.

In an irreversible process,

$$\begin{aligned} dU &= \delta q - (Pd v - \delta F) && \text{(from 2.4 and 2.9)} \\ &= \delta q + \delta F && \text{(under conditions of constant volume)} \end{aligned} \quad \dots (2.11)$$

This quantity δF thus represents the mechanical energy which has been converted into heat and which is therefore available for increasing the temperature.

$$\text{Thus} \quad \delta q + \delta F = C_v dT = dU$$

For changes that take place under conditions of constant pressure, it is more satisfactory to consider variations in the enthalpy H . The enthalpy is defined by the relation,

$$H = U + P v \quad \dots (2.12)$$

$$\begin{aligned}\text{Thus } dH &= dU + Pdv + vdP \\ &= \delta q - Pdv + \delta F + Pdv + vdP \quad (\text{from 2.11})\end{aligned}$$

for an irreversible process. (For a reversible process $\delta F = 0$.)

$$= \delta q + \delta F + vdP \quad \dots(2.13)$$

$$= \delta q + \delta F \quad (\text{at constant pressure})$$

$$= C_p dT \quad \dots(2.14)$$

where C_p is the specific heat at constant pressure.

No assumptions have been made concerning the properties of the system and, therefore, the following relations apply to all fluids.

$$\text{From equation (2.10)} \quad \left(\frac{\partial U}{\partial T}\right)_v = C_v \quad \dots(2.15)$$

$$\text{From equation (2.14)} \quad \left(\frac{\partial H}{\partial T}\right)_P = C_p \quad \dots(2.16)$$

TYPES OF FLUID

The properties of liquids, ideal gases, and non-ideal gases will now be discussed.

The Incompressible Fluid (Liquid)

By definition, v is independent of P , so that $\left(\frac{\partial v}{\partial P}\right)_T = 0$. The internal energy will be a function of temperature but not a function of pressure.

The Ideal Gas

An ideal gas is defined as a gas whose properties obey the law

$$P.V = nRT \quad \dots(2.2)$$

where V is the volume occupied by n moles of the gas and R is the universal gas constant.

This law is closely obeyed by real gases under conditions where the actual volume of the molecules is small compared with the total volume, and where the molecules exert only a very small attractive force on one another. These conditions are met at very low pressures when the distance apart of the individual molecules is large. The value of R is then the same for all gases.

When the only external force on a gas is the fluid pressure, the equation of state is given by the equation,

$$f(P, V, T, n) = 0$$

Any property can be expressed in terms of any three other properties. Consider the dependence of the internal energy on temperature and volume.

$$U = f(T, V, n)$$

For unit mass of gas,

$$U = f(T, v)$$

where v is the volume per unit mass and $Pv = \frac{1}{M} \mathbf{R}T$,(2.2a)

so that
$$dU = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial v}\right)_T dv$$
(2.17)

Now $Tds = dU + Pd v$ (from 2.8)

$\therefore Tds = \left(\frac{\partial U}{\partial T}\right)_v dT + \left\{P + \left(\frac{\partial U}{\partial v}\right)_T\right\} dv$

and
$$dS = \left(\frac{\partial U}{\partial T}\right)_v \frac{dT}{T} + \frac{1}{T} \left\{P + \left(\frac{\partial U}{\partial v}\right)_T\right\} dv$$

Thus
$$\left(\frac{\partial S}{\partial T}\right)_v = \frac{1}{T} \left(\frac{\partial U}{\partial T}\right)_v$$
(2.17a)

and
$$\left(\frac{\partial S}{\partial v}\right)_T = \frac{1}{T} \left\{P + \left(\frac{\partial U}{\partial v}\right)_T\right\}$$
(2.17b)

Then differentiating equation (2.17a) by v and equation (2.17b) by T and equating,

$$\frac{1}{T} \cdot \frac{\partial^2 U}{\partial T \partial v} = \frac{1}{T} \left\{ \left(\frac{\partial P}{\partial T}\right)_v + \frac{\partial^2 U}{\partial v \cdot \partial T} \right\} - \frac{1}{T^2} \left\{ P + \left(\frac{\partial U}{\partial v}\right)_T \right\}$$

i.e.
$$\left(\frac{\partial U}{\partial v}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_v - P$$
(2.18)

This relation applies to any fluid. For the particular case of an ideal gas, since $Pv = \frac{1}{M} \mathbf{R}T$, (2.2a)

$$T \cdot \left(\frac{\partial P}{\partial T}\right)_v = T \cdot \frac{\mathbf{R}}{Mv} = P$$

so that
$$\left(\frac{\partial U}{\partial v}\right)_T = 0$$
(2.19)

and
$$\left(\frac{\partial U}{\partial P}\right)_T = \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial P}\right)_T = 0$$
(2.20)

Thus the internal energy of an ideal gas is a function of temperature only. The variation of internal energy and enthalpy with temperature will now be calculated.

$$dU = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial v}\right)_T dv$$
(2.17)

$$= C_v dT \quad (\text{from 2.15 and 2.19})$$
(2.21)

Thus

$$\frac{dU}{dT} = C_v$$

In general, this relation applies only to changes at constant volume. For the particular case of the ideal gas, however, it applies under all circumstances.

Again, since $H = f(T, P)$,

$$\begin{aligned} dH &= \left(\frac{\partial H}{\partial T}\right)_P dT + \left(\frac{\partial H}{\partial P}\right)_T dP \\ &= C_p dT + \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial(Pv)}{\partial P}\right)_T dP \\ &\quad \text{(from 2.14 and 2.12)} \\ &= C_p dT \end{aligned} \quad \dots(2.22)$$

since $\left(\frac{\partial U}{\partial P}\right)_T = 0$ and $\left(\frac{\partial(Pv)}{\partial P}\right)_T = 0$ for an ideal gas

Thus $\frac{dH}{dT} = C_p$ under all conditions for an ideal gas.

The Non-ideal Gas

For a non-ideal gas, equation 2.2 is modified by including a compressibility factor C which is a function of temperature and pressure,

$$P.V = C.n.R.T \quad \dots(2.23)$$

At very low pressures, deviations from the ideal gas law are caused mainly by the attractive forces between the molecules and the compressibility factor has a value less than unity. At higher pressures, deviations are caused mainly by the fact that the volume of the molecules themselves, which can be regarded as incompressible, becomes significant compared with the total volume of the gas. Under these circumstances the compressibility factor has a value greater than unity.

Many equations have been given to denote the approximate relation between the properties of a non-ideal gas. Of these the simplest, and probably the most commonly used, is Van der Waals' equation—

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT \quad \dots(2.24)$$

where b is a quantity which is a function of the incompressible volume of the molecules themselves, and a/V^2 is a function of the attractive forces between the molecules. It is seen that as P approaches zero and V approaches infinity, this equation reduces to the equation of state for the ideal gas.

A useful estimate of the properties of a non-ideal gas can be obtained by the use of "reduced co-ordinates."

The "reduced temperature" T_R , the "reduced pressure" P_R , and the

“reduced volume” V_R are defined as the ratio of the actual temperature, pressure, and volume of the gas to the corresponding values of these properties at the critical state. It is found that, at a given value of T_R and P_R , nearly all gases have the same molar volume, compressibility factor and other thermodynamic properties. This empirical relationship applies to within about 2% for most gases. The most important exception to the rule is ammonia.

It has already been shown that the change of internal energy of unit mass of fluid with volume at constant temperature is given by the relation

$$\left(\frac{\partial U}{\partial v}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_v - P \quad \dots(2.18)$$

For a non-ideal gas

$$T \left(\frac{\partial P}{\partial T}\right)_v \neq P$$

and therefore $\left(\frac{\partial U}{\partial v}\right)_T$ and $\left(\frac{\partial U}{\partial P}\right)_T$ are not equal to zero.

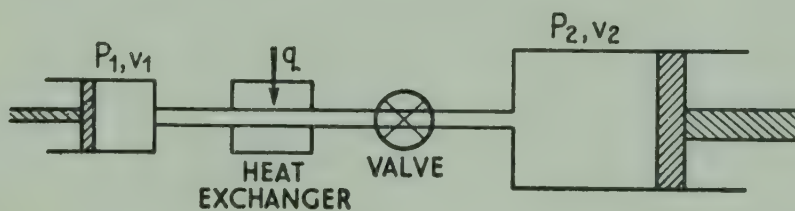


Fig. 2.1. Joule-Thomson effect

Thus the internal energy of the non-ideal gas is a function of pressure as well as temperature. As the gas is expanded, the molecules are separated from each other against the action of the attractive forces between them. Energy is therefore stored in the gas; this is released when the gas is compressed and the molecules are allowed to approach one another again.

A characteristic of the non-ideal gas is that it has a finite Joule-Thomson effect. This relates to the amount of heat which must be added during an expansion of a gas from a pressure P_1 to a pressure P_2 in order to maintain isothermal conditions. Imagine a gas flowing from a cylinder, fitted with piston, at a pressure P_1 to a second cylinder at a pressure P_2 . The process takes place sufficiently slowly for the kinetic energy of the gas and the losses due to fluid friction to be negligible (Fig. 2.1).

The net work done by unit mass of gas on the surroundings in expanding from P_1 to P_2 is given by

$$W = P_2 v_2 - P_1 v_1$$

A quantity of heat (q , say) is added during the expansion so as to maintain isothermal conditions. The change in the internal energy is therefore given by

$$\Delta U = q - W \quad (\text{from 2.4})$$

$$\therefore q = \Delta U - P_2 v_2 + P_1 v_1$$

For an ideal gas, under isothermal conditions, $\Delta U = 0$ and $P_2 v_2 = P_1 v_1$.

Thus $q = 0$ and the ideal gas is said to have a zero Joule-Thomson effect. A non-ideal gas has a Joule-Thomson effect which may be either positive or negative.

ENERGY OF A FLUID IN MOTION

The total energy of a fluid in motion is made up of a number of components. We shall refer to unit mass of fluid and neglect changes in magnetic and electrical energy, etc.

(1) *Internal Energy* U . This has already been discussed.

(2) *Pressure Energy*. This represents the work which must be done in order to introduce the fluid, without change in volume, into the system. It is therefore given by the product Pv , where P is the pressure of the system and v is the volume of unit mass of fluid.

(3) *Potential Energy*. The potential energy of the fluid, due to its position in the earth's gravitational field, is equal to the work which must be done on it in order to raise it to that position from some arbitrarily chosen datum level at which the potential energy is taken as zero. Thus, if the fluid is situated at a height z above the datum level, the potential energy is $z \cdot g$, where g is the acceleration due to gravity which is taken as constant unless otherwise stated.

(4) *Kinetic Energy*. The fluid possesses kinetic energy by virtue of its motion with reference to some arbitrarily fixed body (normally taken as the earth). If the fluid is moving with a uniform velocity u , the kinetic energy is $\frac{1}{2}u^2$.

The total energy of unit mass of fluid is, therefore,

$$U + Pv + gz + \frac{1}{2}u^2$$

If the fluid flows from section 1 to section 2 (where the values of the various quantities are denoted by suffixes 1 and 2 respectively), and q is the net heat absorbed from the surroundings and W is the net work done by the fluid on the surroundings, we have

$$U_2 + P_2v_2 + gz_2 + \frac{1}{2}u_2^2 = U_1 + P_1v_1 + gz_1 + \frac{1}{2}u_1^2 + q - W \quad \dots (2.25a)$$

$$\text{i.e.} \quad q - W = \Delta U + \Delta(Pv) + g\Delta Z + \frac{1}{2}\Delta u^2 \quad \dots (2.25b)$$

where Δ denotes a finite change in the quantities.

$$\text{Thus} \quad q - W = \Delta H + g\Delta z + \frac{1}{2}\Delta u^2 \quad \dots (2.25c)$$

For a small change in the system,

$$\delta q - \delta W = dH + g \cdot dz + u \cdot du \quad \dots (2.26)$$

For many purposes it is convenient to eliminate H , by using equation 2.13.

$$dH = \delta q + \delta F + v dP \quad \dots (2.13)$$

Thus
$$u \cdot du + g \cdot dz + v \cdot dP + \delta W + \delta F = 0 \quad \dots(2.27)$$

Integrating the above equation for flow from section 1 to section 2 and summing the terms δW and δF ,

$$\frac{1}{2}\Delta u^2 + g \cdot \Delta z + \int_1^2 v \cdot dP + W + F = 0 \quad \dots(2.28)$$

Equations 2.25 to 2.28 are quite general and apply therefore to any type of fluid.

With incompressible fluids the energy F is either lost to the surroundings or causes a very small rise in temperature. If the fluid is compressible, however, the rise in temperature may result in an increase in the pressure energy and part of it may be available for doing useful work.

If the fluid is flowing through a channel or pipe a frictional drag arises in the region of the boundaries and gives rise to a velocity distribution across any section perpendicular to the direction of flow. For the unidirectional flow of fluid, it is convenient to define a mean velocity of flow as the ratio of the volumetric flow rate to the cross-sectional area. Then if no material enters or leaves the system between two sections, the mass rate of flow must remain constant, i.e.

$$G = u_1 A_1 \rho_1 = u_2 A_2 \rho_2 \quad \dots(2.29)$$

$$= \frac{u_1 A_1}{v_1} = \frac{u_2 A_2}{v_2} \quad \dots(2.30)$$

where A denotes the cross-sectional area, ρ the density, and subscripts 1 and 2 refer to the two sections.

The actual variation of velocity over the section will depend on the conditions of flow. It will be shown that for flow in a circular pipe the mean velocity is between 0.5 and 0.82 times the axial velocity.

The kinetic energy of a small element of fluid is proportional to the square of its velocity whereas the mean velocity in a duct is defined as a mean linear velocity. A correction factor α must therefore be introduced into the expression for the kinetic energy of the fluid to take account of the velocity distribution.

Thus the kinetic energy per unit mass can be written as $\frac{u^2}{2\alpha}$.

For flow in a circular pipe, α will be shown to be equal to $\frac{1}{2}$ for streamline flow and approximately equal to 1 for turbulent flow.

Equation 2.27, therefore, becomes

$$d \left(\frac{u^2}{2\alpha} \right) + g \cdot dz + v \cdot dP + \delta W + \delta F = 0 \quad \dots(2.31)$$

equation 2.28 becomes

$$\Delta \frac{u^2}{2\alpha} + g \Delta z + \int_1^2 v \cdot dP + W + F = 0 \quad \dots(2.32)$$

and equation 2.25c becomes

$$\Delta \frac{u^2}{2\alpha} + g\Delta z + \Delta H = q - W \quad \dots(2.32a)$$

Before equation 2.32 can be applied to any particular flow problem, the term $\int_1^2 v \cdot dP$ must be evaluated.

Incompressible Fluids

Here v is independent of pressure so that

$$\int_1^2 v \cdot dP = (P_2 - P_1) \cdot v \quad \dots(2.33)$$

We can, therefore, write equation 2.32

$$\frac{u_1^2}{2\alpha_1} + g \cdot z_1 + P_1 v = \frac{u_2^2}{2\alpha_2} + g \cdot z_2 + P_2 v + W + F \quad \dots(2.34)$$

or
$$\Delta \frac{u^2}{2\alpha} + g\Delta z + v\Delta P + W + F = 0 \quad \dots(2.34a)$$

In a frictionless system in which the fluid does not work on the surroundings and α_1 and α_2 are taken as unity (turbulent flow),

$$\frac{u_1^2}{2} + gz_1 + P_1 v = \frac{u_2^2}{2} + gz_2 + P_2 v \quad \dots(2.35)$$

This equation is known as Bernoulli's equation.

Compressible Fluids

The term $\int_1^2 v dP$ will now be evaluated for the ideal gas under various conditions. In most cases the results so obtained can be applied to the non-ideal gas without introducing an error greater than is involved in estimating the other factors concerned in the process. The only common exception to this occurs in the flow of gases at very high pressures, when it is necessary to employ one of the approximate equations for the state of a non-ideal gas, in place of the equation for the ideal gas. Alternatively, one may use equation 2.25c and work in terms of changes in enthalpy.

The relation between the pressure and the volume of an ideal gas depends on the rate of transfer of heat to the surroundings and the degree of irreversibility of the process. The following conditions will be considered.

- (1) An isothermal process.
- (2) An isentropic process.
- (3) A reversible process which is neither isothermal nor adiabatic.
- (4) An irreversible process which is not isothermal.

Isothermal Process

For an isothermal process $Pv = \frac{1}{M} \mathbf{R}T = P_1 v_1$ where the subscript 1 denotes the values at the beginning of the operation, and M is the molecular weight.

$$\text{Thus} \quad \int_1^2 v \cdot dP = P_1 v_1 \int_1^2 P^{-1} dP = P_1 v_1 \ln \frac{P_2}{P_1} \quad \dots (2.36)$$

Isentropic Process

For unit mass of any fluid, the enthalpy can be expressed as a function of the pressure and volume, so that,

$$H = f(P, v)$$

$$\begin{aligned} dH &= \left(\frac{\partial H}{\partial P} \right)_v dP + \left(\frac{\partial H}{\partial v} \right)_P dv \\ &= \left(\frac{\partial H}{\partial T} \right)_v \left(\frac{\partial T}{\partial P} \right)_v dP + \left(\frac{\partial H}{\partial T} \right)_P \left(\frac{\partial T}{\partial v} \right)_P dv \end{aligned}$$

$$\text{Now} \quad \left(\frac{\partial H}{\partial T} \right)_P = C_p \quad \dots (2.16)$$

$$\begin{aligned} \text{and} \quad \left(\frac{\partial H}{\partial T} \right)_v &= \left(\frac{\partial U}{\partial T} \right)_v + \left(\frac{\partial Pv}{\partial T} \right)_v \\ &= C_v + v \cdot \left(\frac{\partial P}{\partial T} \right)_v \quad (\text{from 2.15}) \end{aligned}$$

$$\begin{aligned} \text{Further} \quad dH &= dU + P \cdot dv + v \cdot dP \quad (\text{from 2.12}) \\ &= TdS - P \cdot dv + P \cdot dv + v \cdot dP \quad (\text{from 2.8}) \\ &= TdS + v \cdot dP \quad \dots (2.37) \end{aligned}$$

$$= v \cdot dP \quad (\text{for an isentropic process}) \quad \dots (2.38)$$

Thus for an isentropic process,

$$v \cdot dP = \left\{ C_v + v \left(\frac{\partial P}{\partial T} \right)_v \right\} \left(\frac{\partial T}{\partial P} \right)_v dP + C_p \cdot \left(\frac{\partial T}{\partial v} \right)_P dv$$

$$\text{i.e.} \quad \left(\frac{\partial T}{\partial P} \right)_v dP + \frac{C_p}{C_v} \left(\frac{\partial T}{\partial v} \right)_P \cdot dv = 0$$

From the equation of state for an ideal gas,

$$\left(\frac{\partial T}{\partial P} \right)_v = \frac{T}{P} \text{ and } \left(\frac{\partial T}{\partial v} \right)_P = \frac{T}{v}$$

$$\therefore \quad \frac{dP}{P} + \gamma \frac{dv}{v} = 0 \quad (\text{where } \gamma = C_p/C_v)$$

Integration gives

$$\ln P + \gamma \ln v = \text{constant}$$

i.e.

$$P \cdot v^\gamma = \text{constant} \quad \dots (2.39)$$

This relation holds only approximately, even for an ideal gas, since γ has been taken as a constant in the integration, although it does vary somewhat with pressure.

Now that the relation between P and v has been established, the value of the term $\int_1^2 v \cdot dP$ can be calculated.

$$\begin{aligned} \int_1^2 v \cdot dP &= \int_1^2 \left\{ \frac{P_1 v_1^\gamma}{P} \right\}^{\frac{1}{\gamma}} \cdot dP \\ &= P_1^{\frac{1}{\gamma}} v_1 \int_1^2 P^{-\frac{1}{\gamma}} dP \\ &= P_1^{\frac{1}{\gamma}} v_1 \frac{1}{1 - \frac{1}{\gamma}} \{P_2^{1-\frac{1}{\gamma}} - P_1^{1-\frac{1}{\gamma}}\} \\ &= \frac{\gamma}{\gamma - 1} P_1 v_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \quad \dots (2.40) \end{aligned}$$

$$\begin{aligned} &= \frac{\gamma}{\gamma - 1} \left\{ P_1 \cdot \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \cdot \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \cdot v_2 - P_1 v_1 \right\} \\ &= \frac{\gamma}{\gamma - 1} \{P_2 v_2 - P_1 v_1\} \quad \dots (2.40a) \end{aligned}$$

Further, from equations 2.38 and 2.22,

$$\int_1^2 v dP = \int_1^2 dH = C_p \Delta T \quad \dots (2.40b)$$

(taking C_p as constant).

The above relations apply for an ideal gas to a reversible adiabatic process which, as already shown, is isentropic.

Reversible Process—neither Isothermal nor Adiabatic

In general the conditions under which a change in state of a gas takes place is neither isothermal nor adiabatic and the relation between pressure and volume is approximately of the form $P \cdot v^k = \text{constant}$ for a reversible process, where k is a numerical quantity whose value depends on the heat transfer between the gas and its surroundings. k usually lies between 1 and γ but may, under certain circumstances, lie outside these limits; it will have the same value for a reversible compression as for a reversible expansion under similar conditions. Under these conditions therefore, equation 2.40 becomes

$$\int_1^2 v \cdot dP = \frac{k}{k - 1} P_1 v_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \quad \dots (2.41)$$

Irreversible Process

For an irreversible process, it may not be possible to express the relation between pressure and volume as a continuous mathematical function, but by choosing a suitable value for the constant, k , an equation of the form $P \cdot v^k = \text{constant}$ may be used over a limited range of conditions. The above equation can then be used for the evaluation of $\int_1^2 v \cdot dP$. It should be noted that, for an irreversible process, k will have different values for compression and expansion under otherwise similar conditions. Thus, for the irreversible adiabatic compression of a gas, k will be greater than γ : for the corresponding expansion k will be less than γ . This means that more energy has to be put into an irreversible compression than will be received back when the gas expands to its original condition.

FLUID HEAD AND PRESSURE

The energy balance equation for unit mass of fluid is

$$\Delta \frac{u^2}{2\alpha} + g \cdot \Delta z + \int_1^2 v \cdot dP + W + F = 0 \quad \dots(2.32)$$

Dividing by g , we have, for unit weight of fluid,

$$\Delta \frac{u^2}{2g\alpha} + \Delta z + \frac{1}{g} \int_1^2 v \cdot dP + \frac{W}{g} + \frac{F}{g} = 0 \quad \dots(2.42)$$

Here each of the terms has the dimension of length and is referred to as a fluid head. Thus, for instance, the term $\frac{u^2}{2g\alpha}$ is known as the velocity head and F/g as the friction head h_f (say).

For an incompressible fluid flowing in a horizontal pipe, of constant cross section, if no work is done on the surroundings, the pressure change due to frictional effects, ΔP_f , is given by

$$\frac{1}{g} v \cdot \Delta P_f + \frac{F}{g} = 0$$

$$\text{i.e.} \quad -\Delta P_f = \frac{F}{v} = h_f \cdot \rho \cdot g \quad \dots(2.43)$$

where ρ is the density of the fluid.

Again, if equation 2.31 is multiplied throughout by the fluid density, it then relates to unit volume of fluid and each of the terms has the dimensions of pressure. The differential form of the equation must be used here because ρ is a function of pressure for a gas.

$$d \left(\rho \frac{u^2}{2\alpha} \right) + \rho \cdot g \cdot dz + dP + \rho \delta W + \rho \delta F = 0 \quad \dots(2.44)$$

CHAPTER 3

Friction in Pipes and Channels

THE first important measurements on the drop in pressure for liquids flowing in pipes under widely differing conditions were made in 1883 by OSBORNE REYNOLDS⁽¹¹⁾. He plotted the loss in head per unit length of pipe (the hydraulic gradient, i) against the velocity, u (see Fig. 3.1). At low velocities the hydraulic gradient was directly proportional to the velocity.

A logarithmic plot of his data gave a curve of the form shown in Fig. 3.2, which is conveniently considered in three sections. At low velocities the data were represented by the straight line PB of slope equal to 1. At high rates the straight line CQ , of slope about 1.8, was obtained. At intermediate velocities, there was some instability in the flow but the line BC

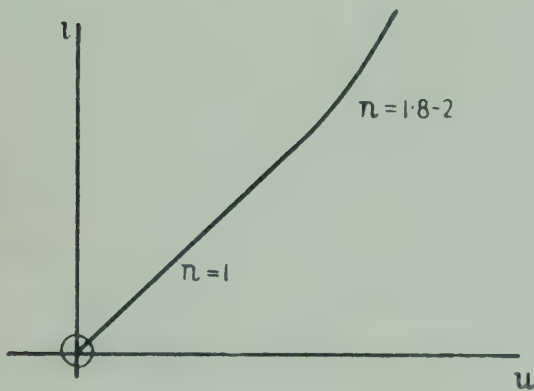


Fig. 3.1. Hydraulic gradient v . velocity

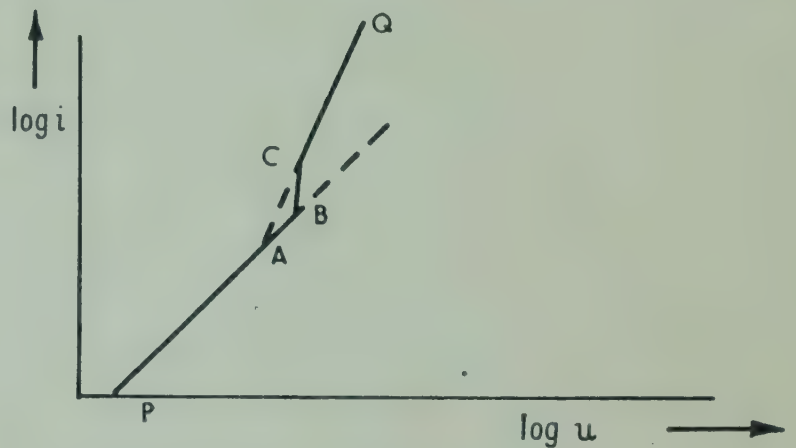


Fig. 3.2. Critical velocities

could be taken to give a fair representation of the data. When produced QC was found to cut PB at A . The velocity corresponding to the point A was referred to as the lower critical velocity and that corresponding to the point B as the upper critical velocity. Comparison of the data from experiments on pipes of various sizes showed that the critical velocities were inversely proportional to the pipe diameter. It was further shown that they were reduced by increasing the temperature of the water. In another series of experiments with a glass pipe REYNOLDS injected a thin filament of coloured water into the main stream at the pipe axis and observed its behaviour. At velocities below the lower critical velocity the injected coloured fluid retained its identity and travelled along the pipe axis. At higher rates of flow the coloured stream was broken up and mixed with the bulk of the fluid; the rate of mixing increased as the velocity was raised.

REYNOLDS found that the dimensionless group, $\frac{ud\rho}{\mu}$ (where d is the diameter of the pipe and ρ and μ are the density and viscosity of the fluid), provided a criterion by which the behaviour of the fluid could be predicted. This group

has since been termed the Reynolds Number (Re). The lower critical velocity occurred at a Reynolds Number of about 2000 and the upper critical velocity corresponded to a value of about 2500. The experiments with the filament of coloured liquid showed that at Reynolds Numbers below 2000 the flow was streamline, i.e. there was no bulk motion of fluid perpendicular to the pipe axis, whereas at higher values eddy currents were responsible for transverse motion and mixing of the fluid streams.

If a fluid flowing under turbulent conditions passes into a pipe where the Reynolds Number is less than 2000, the flow will become streamline at some distance from the entry. On the other hand, if the fluid is initially flowing under streamline conditions ($Re. < 2000$), the diameter of the pipe can be gradually altered so that the Reynolds Number exceeds 2000 and yet streamline flow will persist in the absence of any disturbance. Unstable streamline flow has been obtained in this manner at Reynolds Numbers up to 40,000. Under streamline conditions there is no velocity component at right angles to the direction of flow and the velocity can be gradually increased without eddy formation unless some disturbance arises.

Even when the flow in the main part of the cross section is turbulent, laminar flow persists in the neighbourhood of the pipe walls. The existence of this laminar sub-layer, as it is called, can be demonstrated by introducing a thin filament of coloured fluid into the main stream near the walls. It will be found to move in streamline flow close to the surface of the pipe and not to mix with the bulk of the liquid.

STANTON and PANNELL⁽¹⁴⁾ measured the drop in pressure due to friction for a number of fluids flowing in pipes of various diameters and surface roughnesses. They were able to express their results by plotting the Reynolds Number against the dimensionless group $R/\rho u^2$, where R is the resistance to flow per unit area of pipe surface. For a given surface a single curve was found to express the results for all fluids, pipe diameters, and velocities. As with the results of Reynolds (Fig. 3.2) the curve was in three parts. At low values of Reynolds Number (< 2000), $R/\rho u^2$ was independent of the surface roughness but at high values ($Re. > 2500$), $R/\rho u^2$ varied with the surface roughness. At very high Reynolds Numbers the friction factor became independent of Re . and a function of the surface roughness only. Over the transition region of Re ., from 2000 to 2500, $R/\rho u^2$ increased very rapidly showing the great increase in friction as soon as turbulent motion commenced.

This general relationship is one of the most widely used in all problems associated with fluid motion, heat transfer, and mass transfer. Later work by LANDER and by workers in America, such as MOODY, has extended the earlier work but the fundamental pattern is due to this early work in England.

MOODY⁽²⁵⁾ expressed the group $R/\rho u^2$ (or a quantity proportional to it) as a function of two dimensionless groups, the Reynolds Number and the relative roughness e/d of the surface, where e is a linear quantity representing the roughness of the pipe surface: the effect of surface roughness will be considered

in greater detail in Chapter 9. A single curve of $R/\rho u^2$ versus the Reynolds Number is used to correlate the data for all pipes with the same relative roughness, e/d . Such a curve is reproduced in Fig. 3.3.* Four separate regions can be distinguished.

Region 1 ($Re. < 2000$) corresponds to streamline motion and a single curve represents all the data, irrespective of the roughness of the pipe surface. The equation of the curve is $R/\rho u^2 = 8 Re.^{-1}$.

Region 2 ($2000 < Re. < 3000$) is a transition region between streamline and turbulent flow conditions. Reproducible values of pressure drop cannot be obtained in this region, but the value of $R/\rho u^2$ is considerably higher than that in the streamline region. If an unstable form of streamline flow does persist at a Reynolds Number greater than 2000, the frictional force will correspond to that given by the curve $R/\rho u^2 = 8 Re.^{-1}$, produced.

Region 3 ($Re. > 3000$) corresponds to turbulent motion of the fluid and $R/\rho u^2$ is a function of both $Re.$ and e/d , with rough pipes giving high values of $R/\rho u^2$. For smooth pipes there is a lower limit below which $R/\rho u^2$ does not fall for any particular value of $Re.$ It will be noted that the curves for very rough surfaces never follow the one for a smooth pipe. Moderately rough pipes, however, are found to behave as smooth pipes up to a certain value of $Re.$: the rougher the pipe, the lower is this value of $Re.$

Region 4. Here $R/\rho u^2$ is independent of Reynolds Number and a function of relative roughness only. The lowest Reynolds Numbers at which this occurs depends on the roughness of the surface. In this region the frictional force is proportional to the square of the fluid velocity.

Various expressions have been given⁽²⁵⁾ for the calculation of $R/\rho u^2$, for turbulent flow. Thus, for Reynolds Numbers from 3×10^3 to 10^7 , the data for a smooth pipe may be represented by

$$\left(\frac{R}{\rho u^2}\right)^{-\frac{1}{2}} = 2.5 \ln \left\{ Re. \cdot \left(\frac{R}{\rho u^2}\right)^{\frac{1}{2}} \right\} + 0.3 \quad \dots(3.1)$$

For a rough pipe, where $R/\rho u^2$ is a function of both $Re.$ and e/d ,

$$\left(\frac{R}{\rho u^2}\right)^{-\frac{1}{2}} = -2.5 \ln \left(\frac{e/d}{3.7} + \frac{1}{1.13 Re. \sqrt{\frac{R}{\rho u^2}}} \right) \quad \dots(3.2)$$

For a rough pipe, where $R/\rho u^2$ is a function only of e/d ,

$$\left(\frac{R}{\rho u^2}\right)^{-\frac{1}{2}} = 3.2 - 2.5 \ln \frac{e}{d} \quad \dots(3.3)$$

CALCULATION OF PRESSURE DROP IN A PIPE

For the flow of fluid in a short length dl of pipe of diameter d , the total frictional force at the walls is the product of the shear stress R and the surface area of the pipe.

* See folded chart facing page 334.

Thus, total frictional force

$$= R \cdot \pi \cdot d \cdot dl$$

Suppose that this frictional force results in a change in pressure dP_f , then

$$R \cdot \pi \cdot d \cdot dl = -dP_f \cdot \pi \frac{d^2}{4}$$

$$\text{i.e.} \quad -dP_f = \frac{4R \cdot dl}{d} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{dl}{d} \cdot \rho u^2 \quad \dots (3.4)$$

If the head lost due to friction is dh_f ,

$$dh_f = \frac{-dP_f}{\rho \cdot g} = \frac{4R \cdot dl}{\rho \cdot g \cdot d} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{dl}{d} \cdot \frac{u^2}{g} \quad \dots (3.5)$$

Further, the energy per unit mass not available because of the irreversibility of the process, δF , is given by

$$\delta F = \frac{-dP_f}{\rho} = \frac{4R \cdot dl}{d \cdot \rho} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{dl}{d} \cdot u^2 \quad \dots (3.6)$$

(see equation 2.43).

For an incompressible fluid flowing in a pipe of constant cross-sectional area, u is not a function of pressure or l and the preceding equations can be integrated directly. Thus in a length l of pipe in which the pressure change due to friction is ΔP_f , the head lost due to friction is h_f and the energy lost per unit mass is F , we have

$$-\Delta P_f = \frac{4R \cdot l}{d} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \rho u^2 \quad \dots (3.7)$$

$$h_f = \frac{4R \cdot l}{\rho \cdot g \cdot d} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \frac{u^2}{g} \quad \dots (3.8)$$

and

$$F = \frac{4R \cdot l}{d \cdot \rho} = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot u^2 \quad \dots (3.9)$$

In order to determine the pressure drop due to friction for an incompressible fluid, the relative roughness is calculated from the data in Table 3.1, and the value of Re for the flow is determined. The corresponding value of $R/\rho u^2$ is then read off from the diagram (Fig. 3.3) and the frictional pressure drop or the head lost is then calculated from equation 3.7 or 3.8.

In many reference books and textbooks of fluid mechanics, friction factors other than $R/\rho u^2$ are quoted. Thus FANNING's friction factor and DARCY's friction factor are both equal to $2R/\rho u^2$. Another friction factor which is often used is equal to $8R/\rho u^2$.

A variation of the above problem is the calculation of the rate of flow of an incompressible fluid through a pipe when a given pressure difference exists. The appropriate value of $R/\rho u^2$ cannot be obtained immediately, since it is a function of the Reynolds Number which is itself proportional to the unknown velocity. Two methods are available for solving a problem of this type. The first method

consists in estimating a value of $R/\rho u^2$ and then calculating the velocity of flow and the Reynolds Number on this assumption. A check is then made to find whether this is a sufficiently accurate estimate. If not, a second estimate of the value of $R/\rho u^2$ is made and the procedure is repeated until a sufficiently accurate result is obtained. This trial and error method of solution can be avoided by plotting a series of curves of the dimensionless group $R/\rho u^2 \cdot \left(\frac{ud\rho}{\mu}\right)^2$, which is independent of velocity, to a base of Reynolds Number (Fig. 3.4).*

Now

$$\frac{R}{\rho u^2} \left(\frac{ud\rho}{\mu}\right)^2 = \frac{R \cdot d^2 \cdot \rho}{\mu^2} = \frac{-\Delta P_f d^3 \rho}{4l\mu^2} \dots\dots(3.10)$$

Therefore, its value can be calculated immediately if the pressure drop in the pipe is known and the Reynolds Number can be read off from Fig. 3.4, and the velocity calculated.

Roughness of Pipe Surfaces

The estimation of the roughness of the surface of the pipe normally presents considerable difficulty. The use of an incorrect value, however, will not be serious for turbulent flow at low Reynolds Numbers because the pressure drop is not critically dependent on the roughness in this region. However, at high values of Reynolds Number, the effect of pipe roughness is considerable, as can be seen from the graph of $R/\rho u^2$ against Reynolds Number. The values of the absolute roughness have been measured for a number of materials and the values, in inches, are quoted below. Where the value for the pipe surface in question is not given, it is necessary to take an approximate value, estimated from the known figures. Where pipes have become corroded, the value of the roughness will commonly be increased, up to tenfold.

TABLE 3.1. *Values of absolute roughness e, in feet*

Drawn tubing	0.000005
Commercial steel and wrought iron .	0.00015
Asphalted cast iron	0.0004
Galvanized iron	0.0005
Cast iron	0.00085
Wood stave	0.0006–0.003
Concrete	0.001–0.01
Riveted steel	0.003–0.03

Example. 98% sulphuric acid is pumped at 10,000 lb/hr through a 1-in. diameter pipe, 100 ft long, to a reservoir 40 ft higher than the feed point. Calculate the power required to pump the acid.

Viscosity of acid = 25 centipoises.
Density of acid = 1.84 g./cc.

Solution.

Reynolds Number, $Re. = \frac{ud\rho}{\mu} = \frac{4G}{\pi\mu d}$

$$= \frac{4 \times 10,000 \times 454}{\pi \times 3600 \times 0.25 \times 2.54} \cdot \frac{\text{lb}}{\text{hr}} \cdot \frac{\text{g}}{\text{lb}} \cdot \frac{\text{hr}}{\text{sec}} \cdot \frac{\text{cm sec}}{\text{g}} \cdot \frac{1}{\text{cm}}$$

$$= 2530$$

* See folded chart facing page 360

For a mild steel pipe which has been used for conveying acid, the roughness, e , will be between about 0.00015 and 0.0015 ft,

Relative roughness, $\frac{e}{d} = 0.0018$ to 0.018

From Fig. 3.3, $\frac{R}{\rho u^2} = 0.006$ over this range of $\frac{e}{d}$

$$\begin{aligned} \text{Velocity, } u &= \frac{G}{\rho A} = \frac{10,000 \times 4 \times 144}{3600 \times 1.84 \times 62.5 \times \pi} \frac{\text{lb}}{\text{hr}} \cdot \frac{\text{hr}}{\text{sec}} \cdot \frac{\text{cu ft}}{\text{lb}} \cdot \frac{1}{\text{in.}^2} \cdot \frac{\text{in.}^2}{\text{ft}^2} \\ &= 4.43 \text{ f.p.s.} \end{aligned}$$

This velocity will be dissipated when the liquid enters the reservoir.

The pressure drop can now be calculated from the energy balance equation 2.34a and equation 3.9. For turbulent flow of an incompressible fluid,

$$\Delta \frac{u^2}{2} + g\Delta z + v(P_2 - P_1) + 4 \left(\frac{R}{\rho u^2} \right) \left(\frac{l}{d} \right) u^2 = 0$$

$$\begin{aligned} \therefore P_1 - P_2 &= \rho \left\{ \left[\frac{1}{2} + 4 \left(\frac{R}{\rho u^2} \right) \frac{l}{d} \right] u^2 + g\Delta z \right\} \\ &= 1.84 \times 62.5 \{ (0.5 + 4 \times 0.006 \times 1200) 4.43^2 + 32.2 \times 40 \} \\ &= 214,000 \text{ poundals/sq ft} \end{aligned}$$

$$\begin{aligned} \text{Power required} &= 214,000 \times \frac{10,000}{3600 \times 1.84 \times 62.5} \times \frac{1}{32.2 \times 550} \\ &\quad \text{(pressure diff.)} \quad \text{(vol. rate of flow)} \quad \text{(conversion factor)} \\ &= \underline{\underline{0.3 \text{ H.P.}}} \end{aligned}$$

Example. A cylindrical tank, 16 ft in diameter, discharges through a horizontal mild steel pipe 300 ft long and 9 in. in diameter connected to the base. Find the time taken for the water level in the tank to drop from 9 to 4 ft above the bottom. Take the viscosity of water as 1 centipoise.

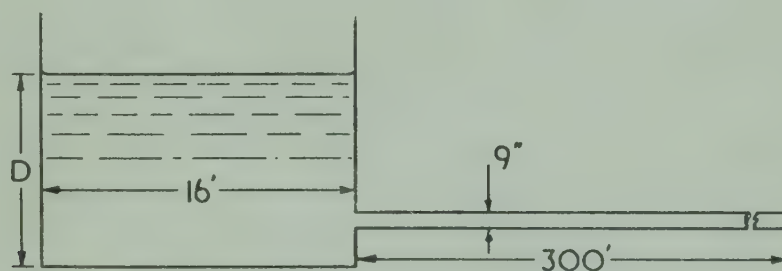


Fig. 3.5. Discharge from tank

Solution. At time t , let the liquid be at a height D in the tank (Fig. 3.5).

Take section 1 as liquid surface in tank and section 2 as pipe outlet. Applying energy balance equation (2.34a) for turbulent flow

$$\Delta \frac{u^2}{2} + g \cdot \Delta z + v(P_2 - P_1) + F = 0$$

Now

$$P_1 = P_2 = \text{atmospheric pressure} \checkmark$$

$$\frac{u_1}{u_2} = \left(\frac{3/4}{16} \right)^2 = 0.0022$$

i.e. u_1 can be neglected.

Since

$$\Delta z = -D,$$

$$\frac{u_2^2}{2} - D \cdot g + 4 \left(\frac{R}{\rho u^2} \right) \frac{l}{d} u_2^2 = 0$$

\therefore

$$\begin{aligned} u_2 &= \sqrt{\frac{D \cdot g}{\frac{1}{2} + 4 \left(\frac{R}{\rho u^2} \right) \frac{l}{d}}} \\ &= \frac{8\sqrt{D}}{\sqrt{1 + 3220 \left(\frac{R}{\rho u^2} \right)}} \text{ f.p.s.} \end{aligned}$$

As the height of liquid in tank changes from D to $D + dD$, the quantity of water discharged

$$= \frac{\pi}{4} \cdot 16^2 (-dD) = -64\pi dD$$

Time taken for level to change by amount dD , is given by,

$$\begin{aligned} dt &= \frac{-64\pi dD}{\frac{\pi}{4} \cdot \left(\frac{3}{4}\right)^2 8 \sqrt{\frac{D}{1 + 3220 \left(\frac{R}{\rho u^2} \right)}}} \\ &= \frac{-56.9 \sqrt{1 + 3220 \left(\frac{R}{\rho u^2} \right)}}{D^{\frac{1}{2}}} dD \text{ sec} \end{aligned}$$

Total time,

$$\begin{aligned} t &= - \int_9^4 56.9 \sqrt{1 + 3220 \left(\frac{R}{\rho u^2} \right)} D^{-\frac{1}{2}} dD \\ &= 114 \sqrt{1 + 3220 \left(\frac{R}{\rho u^2} \right)} \text{ sec} \end{aligned}$$

(if $R/\rho u^2$ can be taken as constant over the range of flow rates considered).

An approximate value of $R/\rho u^2$ will be calculated on the assumption that the kinetic energy of the liquid is small compared with the frictional losses.

$$\text{Pressure drop along pipe} = D\rho g = \frac{4Rl}{d}$$

$$\begin{aligned} \left(\frac{R}{\rho u^2} \right) Re.^2 &= \frac{Rd^2\rho}{\mu^2} = \frac{Dg\rho^2 d^3}{4l\mu^2} \\ &= \frac{D \times 32.2 \times 62.4^2 \times \left(\frac{3}{4}\right)^3}{4 \times 300 \times \left(\frac{0.01}{454} \times 30.48\right)^2} \\ &= 0.98 \times 10^8 D \end{aligned}$$

As D varies from 9 ft to 4 ft, $(R/\rho u^2) \cdot Re.^2$ varies from 8.8×10^8 to 3.9×10^8 .

It is interesting to consider whether the roughness of a new or an old pipe makes a significant difference.

For a mild steel pipe,

new, $e = 0.00015$ ft, $e/d = 0.0002$, $Re. = 7.0$ to 4.4×10^5 (from Fig. 3.4)

old, $e = 0.0015$ ft, $e/d = 0.002$, $Re. = 6.0$ to 3.9×10^5 (from Fig. 3.4)

For a new pipe, $R/\rho u^2$ therefore varies from 0.0019 to 0.0020 and for an old pipe $\frac{R}{\rho u^2} = 0.0029$ from (Fig. 3.3).

Taking a constant value of 0.002 for a new pipe,

$$t = 114\sqrt{1 + 3200 \times 0.002} = 114\sqrt{7.4} = 310 \text{ sec}$$

Pressure drop due to friction is therefore $\frac{6.4}{7.4}$, approximately, of the total pressure drop, due to friction plus kinetic energy change.

Thus $(R/\rho u^2) \cdot Re.^2$ varies from about 7.6×10^8 to 3.4×10^8

$Re.$ varies from about 6.4×10^5 to 4.2×10^5

and $(R/\rho u^2)$ varies from about 0.0020 to 0.0019

Thus the original value of $R/\rho u^2$ is sufficiently close.

The time taken for the level to fall is therefore about 310 sec.

Example. Two storage tanks, A and B, containing a petroleum product, discharge through pipes, each 1 ft in diameter and 1 mile long to a junction at D. From D the liquid is carried by

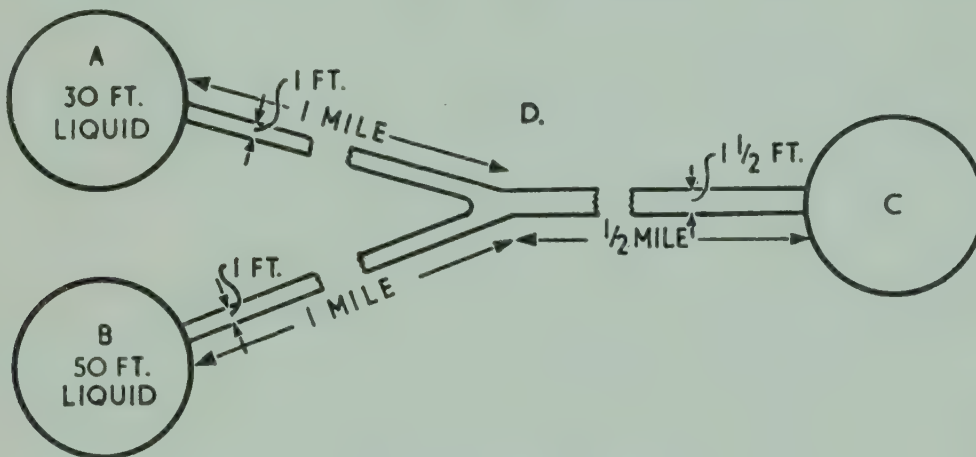


Fig. 3.6. Tank layout

an 18 in. pipe to a third storage tank, C, half a mile away, as shown in Fig. 3.6. The surface of the liquid in A is initially 30 ft above that in C and the liquid level in B is 20 ft higher than that in A. Calculate the initial rate of discharge of liquid into tank C if the pipes are of mild steel. Take the density of the liquid as 0.87 g/cc and the viscosity as 0.7 centipoises.

Solution. Because the pipes are long, we may neglect the kinetic energy of the fluid and minor losses at entry to the pipe.

Let u_1, u_2, u_3 be the velocities in pipes AD, BD, DC: assume, as a first approximation, that the value of $R/\rho u^2$ can be taken as the same in each pipe.

Let the pressure at D be P_D above atmospheric.

Take the liquid level in C as the datum for the calculation of potential energy and let D be a height z_d above this.

Applying the energy balance equation 2.34a between D and the liquid level in each of the tanks,

$$\text{A-D} \quad (z_d - 30) \cdot g + v \cdot P_D + 4 \left(\frac{R}{\rho u^2} \right) \cdot 5280 u_1^2 = 0 \quad (1)$$

$$\text{B-D} \quad (z_d - 50) \cdot g + v \cdot P_D + 4 \left(\frac{R}{\rho u^2} \right) \cdot 5280 u_2^2 = 0 \quad (2)$$

$$\text{D-C} \quad -z_d \cdot g + v \cdot (-P_D) + 4 \left(\frac{R}{\rho u^2} \right) \cdot 1760 u_3^2 = 0 \quad (3)$$

From (1) and (2),
$$20g + \left(\frac{R}{\rho u^2}\right) 21120(u_1^2 - u_2^2) = 0 \quad (4)$$

From (2) and (3),
$$-50g + \left(\frac{R}{\rho u^2}\right) 21120(u_2^2 + \frac{1}{3}u_3^2) = 0 \quad (5)$$

Take the roughness e for the mild steel pipe as 0.00015 ft,

e/d therefore varies from 0.00010 to 0.00015

As a first approximation, take Reynolds Number Re as 2×10^5 in each pipe. Then from Fig. 3.3, $R/\rho u^2 = 0.0021$.

Thus, substituting this value in equations (4) and (5).

$$640 + 44.3(u_1^2 - u_2^2) = 0 \quad (6)$$

and

$$-1600 + 44.3(u_2^2 + \frac{1}{3}u_3^2) = 0 \quad (7)$$

Now the flow rate in DC is equal to the sum of the flow rates in AD and BD.

Thus
$$u_1 + u_2 = \frac{9}{4} u_3 \quad (8)$$

From equation (6),
$$u_1^2 = u_2^2 - 14.5 \quad (9)$$

From equations (7), (8), (9),

$$-1600 + 44.3\{u_2^2 + \frac{1}{3} \cdot (\frac{4}{9})^2(u_2^2 + u_2^2 - 14.5 + 2u_2\sqrt{u_2^2 - 14.5})\} = 0$$

$$\therefore -1600 + 44.3u_2^2 + 2.9u_2^2 + 2.9u_2^2 - 42 + 5.83u_2\sqrt{u_2^2 - 14.5} = 0$$

$$\therefore 5.83u_2\sqrt{u_2^2 - 14.5} = 1642 - 50.13u_2^2$$

$$\therefore u_2\sqrt{u_2^2 - 14.5} = 281.5 - 8.59u_2^2$$

squaring,
$$u_2^4 - 14.5u_2^2 = 79200 - 4837u_2^2 + 73.84u_2^4$$

$$\therefore 72.8u_2^4 - 4823u_2^2 + 79200 = 0$$

$$\therefore u_2^4 - 66.2u_2^2 + 1087 = 0$$

$$\therefore u_2^2 = 33.10 \pm \sqrt{1096 - 1087}$$

$$= 35.94 \text{ or } 30.26$$

$$\therefore u_2 = 5.99 \text{ or } 5.50 \text{ f.p.s.}$$

Substitution in equation (9) gives

$$u_1^2 = 21.49 \text{ or } 15.81$$

$$\therefore u_1 = 4.63 \text{ or } 3.98 \text{ f.p.s.}$$

The corresponding values of u_3 are obtained by substitution in equation (8),

$$\begin{aligned} u_3 &= \frac{4}{9} (4.63 + 5.99) \text{ or } \frac{4}{9} (5.50 + 3.98) \\ &= 4.72 \text{ or } 4.22 \text{ f.p.s.} \end{aligned}$$

If these values of u_1 , u_2 , and u_3 are substituted back into equation (7) it is found that the lower set of velocities satisfies the equation but the higher ones do not; they were introduced as false roots during the squaring.

Thus

$$u_1 = 3.98 \text{ f.p.s.}$$

$$u_2 = 5.50 \text{ f.p.s.}$$

$$u_3 = 4.22 \text{ f.p.s.}$$

It is now necessary to check whether the estimate of 2×10^5 for the value of Reynolds Number was sufficiently accurate.

For pipe AD,
$$Re. = \frac{3.98 \times 30.48 \times 30.48 \times 0.87}{0.007} = 4.6 \times 10^5$$

For pipe BD, $Re. = \frac{5.50 \times 30.48 \times 30.48 \times 0.87}{0.007} = 6.3 \times 10^5$

For pipe DC, $Re. = \frac{4.22 \times 30.48 \times 45.72 \times 0.87}{0.007} = 7.3 \times 10^5$

For this range of Reynolds Numbers, $R/\rho u^2$ varies from 0.0019 to 0.0017. Take a mean value of 0.0018 and substitute again in equations (4) and (5).

$$640 + 38.0(u_1^2 - u_2^2) = 0 \quad (10)$$

and $-1600 + 38.0(u_2^2 + \frac{1}{3}u_3^2) = 0 \quad (11)$

From equation (10), $u_1^2 = u_2^2 - 16.9 \quad (12)$

From equations (8), (11), (12),

$$-1600 + 38.0\{u_2^2 + \frac{1}{3}(\frac{4}{9})^2(u_2^2 + u_2^2 - 16.9 + 2u_2\sqrt{u_2^2 - 16.9})\} = 0$$

$$\therefore -1600 + 38.0u_2^2 + 2.50u_2^2 + 2.50u_2^2 - 42 + 5.0\sqrt{u_2^2 - 16.9} = 0$$

$$\therefore 5.0u_2\sqrt{u_2^2 - 16.9} = 1642 - 43u_2^2$$

$$\therefore u_2\sqrt{u_2^2 - 16.9} = 328 - 8.60u_2^2$$

$$\therefore u_2^4 - 16.9u_2^2 = 107,500 - 5630u_2^2 + 73.96u_2^4$$

$$\therefore 72.96u_2^4 - 5613u_2^2 + 107,500 = 0$$

$$\therefore u_2^4 - 77.3u_2^2 + 1472 = 0$$

$$\therefore u_2^2 = 38.7 \pm \sqrt{1494 - 1472}$$

$$= 43.4 \text{ or } 34.0$$

$$\therefore u_2 = 6.6 \text{ or } 5.8 \text{ f.p.s.}$$

Again, the lower velocity is taken.

Substitution in equation (12) gives,

$$u_1^2 = 17.1$$

$$\therefore u_1 = 4.15 \text{ f.p.s.}$$

Substitution in equation (8) gives

$$u_3 = \frac{4}{9}(4.14 + 5.8)$$

$$= 4.3 \text{ f.p.s.}$$

The corresponding values of the Reynolds Numbers for the flows in pipes AD, BD, DC are 4.75×10^5 , 6.7×10^5 , and 7.5×10^5 , respectively.

$R/\rho u^2$ varies from 0.0017 to 0.0019 over this range of Reynolds numbers. This final solution is, therefore, acceptable.

The velocities in the three pipes are, therefore,

$$\text{AD } 4.2 \text{ f.p.s.}$$

$$\text{BD } 5.8 \text{ f.p.s.}$$

$$\text{DC } 4.3 \text{ f.p.s.}$$

The volumetric rate of flow is therefore $\frac{\pi}{4}(1.5)^2 \cdot 4.3$

$$= \underline{\underline{7.6 \text{ cu. ft/sec}}}$$

TYPES OF FLOW

In addition to streamline and turbulent flow already mentioned a third type, known as molecular flow, occurs in gases at very low pressures where the mean

free path of the molecules is comparable with the dimensions of the duct; this type will not be discussed further.

In steady streamline flow the direction and velocity of flow at any point remain constant and show no variation with time. There is no bulk motion of fluid at right angles to the main direction of flow. Under these conditions, the shear stress R_y at a point where the velocity gradient at right angles to the direction of flow is $\frac{\partial u_y}{\partial y}$, is given by the relation

$$R_y = -\mu \frac{\partial u_y}{\partial y} = -\frac{\mu}{\rho} \frac{\partial(\rho u_y)}{\partial y} \quad \dots(3.11)$$

where μ is the viscosity and $\frac{\mu}{\rho}$ is the kinematic viscosity of the fluid.

When the fluid is in turbulent motion, small random fluctuations in the velocity and direction of flow at a point will occur though the mean velocity and direction will remain constant. A certain bulk transfer of fluid takes place at right angles to the main direction of flow because of the presence of circulating or eddy currents in the fluid. Momentum transfer within the fluid therefore occurs at a very much greater rate and the consequent shear stresses are greater and velocity gradients are smaller, except close to boundaries. We can therefore write

$$R_y = -\left(\frac{\mu}{\rho} + E\right) \cdot \frac{\partial(\rho u_y)}{\partial y} \quad \dots(3.12)$$

where E is known as the eddy kinematic viscosity of the fluid.

The value of E will depend on the degree of turbulence in the fluid and will vary from point to point: it is therefore not a physical property of the fluid. It will generally be small near a boundary surface and increase with increase in the distance from the surface. In highly turbulent flow, E will be large compared with $\frac{\mu}{\rho}$, which can then be neglected.

Significance of the Reynolds Number

The Reynolds Number $\frac{ud\rho}{\mu}$ has been used as the basis for the calculation of the pressure gradient in a pipe through which a fluid is flowing, and it has been shown that it can be used as a criterion by which to determine whether the stable form of flow in a pipe in given conditions is streamline or turbulent. It will be shown below that the Reynolds Number is proportional to the ratio of the inertia or non-viscous forces (i.e. the forces attributable to the momentum of the fluid) to the viscous forces. When the Reynolds Numbers for two geometrically similar systems are the same, dynamic similarity exists and the flow pattern is the same.

The Reynolds group will often be used where we are concerned with a moving fluid. Thus the drag produced as a fluid flows past a particle is related to the

Reynolds Number in which the diameter of the particle is used in place of the diameter of the pipe. Under these conditions the transition from streamline to turbulent flow occurs at a very much lower value. Again for the flow of fluid through a bed composed of granular particles, a mean dimension of the particles is used and the velocity is usually calculated by dividing the flow rate by the total area of the bed. In this case there is no sharp transition from streamline to turbulent flow because the sizes of the individual flow passages vary.

If the surface over which the fluid is flowing contains a series of relatively large projections, turbulence may arise at a very low Reynolds Number. Under these conditions, the frictional force will be increased but so will the coefficients for heat transfer and mass transfer and therefore turbulence is often purposely induced by this method.

Consider an element of fluid of dimensions dx by dy by dz moving with a velocity u_x in the X -direction (Fig. 3.7). In all cases where there is a solid boundary within a finite distance of the element there will be relative motion in the fluid because of the existence, either of a stable velocity distribution, or of eddy currents. Thus the velocity of the element in the X -direction will be changing as it moves from one point to another.

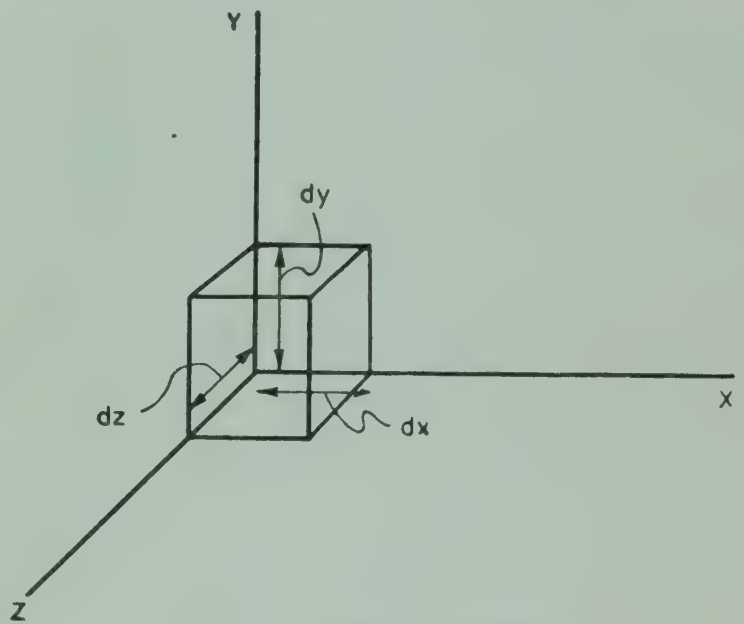


Fig. 3.7. Element of fluid

The net force acting on the element is equal to its rate of change of momentum. Thus the net force in the X -direction

$$\begin{aligned} &= \rho dx \cdot dy \cdot dz \cdot \frac{\partial u_x}{\partial t} \\ &= \rho dx \cdot dy \cdot dz \cdot u_x \cdot \frac{\partial u_x}{\partial x} \end{aligned}$$

ρ is the density of the fluid.

The shear stresses acting on the two faces of the element which lie in X - Z planes are R_y (say) and $R_y + \frac{\partial R_y}{\partial y} dy$.

Thus the net shear force acting on these faces

$$\begin{aligned} &= - \frac{\partial R_y}{\partial y} dy \cdot dx \cdot dz \\ &= \mu \cdot \frac{\partial^2 u_x}{\partial y^2} dx \cdot dy \cdot dz \quad (\text{from 3.11}) \end{aligned}$$

Similarly the net shear force acting on the two faces in the X - Y planes

$$= \mu \frac{\partial^2 u_x}{\partial z^2} dx \cdot dy \cdot dz$$

These two forces are referred to as the viscous forces in the fluid. The resultant force will produce a pressure gradient in the X -direction.

The force acting on the element because of the pressure gradient

$$= \left(-\frac{\partial P}{\partial x} dx \right) \cdot dy \cdot dz$$

Then equating the net force acting in the X -direction to the rate of change of momentum in that direction,

$$\rho u_x \cdot \frac{\partial u_x}{\partial x} dx \cdot dy \cdot dz = \mu \frac{\partial^2 u_x}{\partial y^2} dx \cdot dy \cdot dz + \mu \frac{\partial^2 u_x}{\partial z^2} dx \cdot dy \cdot dz + \left(-\frac{\partial P}{\partial x} \right) dx \cdot dy \cdot dz$$

$$\text{i.e.} \quad \rho u_x \frac{\partial u_x}{\partial x} = \mu \frac{\partial^2 u_x}{\partial y^2} + \mu \frac{\partial^2 u_x}{\partial z^2} - \frac{\partial P}{\partial x}$$

It is now convenient to work in terms of dimensionless derivatives of the length, pressure, and velocity. These are obtained by dividing the actual length, pressure, and velocity by a reference length, pressure, and velocity respectively.

Let the reference length $= L$, some characteristic dimension of the system, the reference velocity $= u_0$, the velocity at some particular point, and the reference pressure $= \rho u_0^2$, which is proportional to the kinetic energy per unit volume of fluid at that point.

Then the dimensionless velocity, $u' = u/u_0$ the dimensionless lengths x' , y' , and z' , $= x/L$, y/L , and z/L respectively, and the dimensionless pressure $P' = P/\rho u_0^2$. The equation can therefore be written,

$$\rho \cdot \frac{u_0^2}{L} \cdot u' \cdot \frac{\partial u'}{\partial x'} = \mu \frac{u_0}{L^2} \left(\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2} \right) - \frac{1}{L} \cdot \rho u_0^2 \frac{\partial P'}{\partial x'}$$

$$\text{i.e.} \quad \underbrace{\frac{\partial P'}{\partial x'} + u' \frac{\partial u'}{\partial x'}}_{\text{(inertia forces)}} = \underbrace{\frac{\mu}{u_0 L \rho} \left(\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2} \right)}_{\text{(viscous forces)}}$$

$\frac{u_0 L \rho}{\mu}$ is a form of Reynolds Number, the dimensionless group whose value determines the nature of the flow of a fluid. It is shown above to be proportional to the ratio of the non-viscous or inertia forces to the viscous forces acting on the element of fluid. At low Reynolds Numbers, the inertia forces are small compared with the viscous forces and the flow is streamline. At high Reynolds Numbers the inertia forces predominate and turbulent flow is obtained. At very high Reynolds Numbers, the viscous forces can be neglected and the fluid behaves as an ideal fluid of zero viscosity; this condition is important in aerodynamics, where the flow of fluid remote from solid boundaries is considered.

Streamline Motion

The pressure drop due to friction and the velocity distribution resulting from the shear stresses within a fluid in streamline flow will be considered for three cases: (1) the flow through a pipe of circular cross section, (2) the flow between two parallel plates, and (3) the flow through an annulus. The velocity at any distance from the boundary surfaces will be calculated and the mean velocity of the fluid will be related to the pressure gradient in the system. For flow through a circular pipe, the kinetic energy of the fluid will be calculated in terms of the mean velocity of flow.

Pipe of Circular Cross Section. For a fluid of viscosity μ and density ρ , flowing under steady conditions in a pipe of diameter d , let the change in pressure due to friction over a small length dl of pipe be dP_f (Fig. 3.8).

Consider the motion of a cylinder of fluid of radius s , at the outer edge of which the velocity of the fluid is u_x . The force resulting from the shear stress at the boundary of this cylinder of fluid will cause the change in pressure dP_f over the small length of pipe dl . Since the viscosity μ of the fluid is the shear stress arising from a unit velocity gradient $\frac{du_x}{ds}$, the shearing force on the boundary of the cylinder of fluid

$$\mu \cdot 2\pi \cdot s \, dl \cdot \frac{du_x}{ds} = dP_f \cdot \pi \cdot s^2$$

The velocity at any distance s from the axis of the pipe can now be found by integrating this expression.

$$\text{Thus } u_x = \frac{dP_f}{2\mu dl} \cdot \frac{s^2}{2} + \text{constant}$$

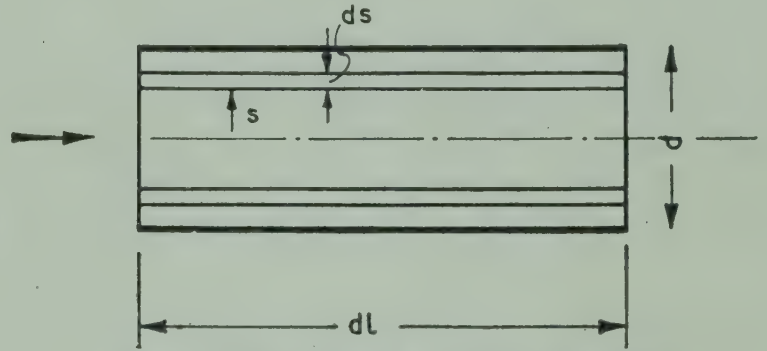


Fig. 3.8. Streamline flow through pipe

At the walls of the pipe (i.e. where $s = d/2$) the velocity u_x must be zero; otherwise there would be an infinite velocity gradient and consequently an infinite shear stress at the walls of the pipe. Substituting the value $u_x = 0$, when $s = d/2$

$$\text{constant} = -\frac{dP_f}{2\mu dl} \cdot \frac{d^2}{8}$$

and therefore

$$u_x = -\frac{dP_f}{4\mu dl} \left(\frac{1}{4}d^2 - s^2 \right) \dots (3.13)$$

Thus the velocity over the cross section varies in a parabolic manner with the distance from the axis of the pipe. The velocity of flow is seen to be a maximum when $s = 0$, i.e. at the pipe axis.

$$\text{Maximum value of velocity } u_s = \frac{-dP_f d^2}{16\mu dl} \dots (3.14)$$

Volumetric Rate of Flow and Average Velocity

If the velocity is taken as constant over an annulus of radii s and $s + ds$, the volumetric rate of flow through the annulus dQ is given by

$$\begin{aligned} dQ &= 2\pi s \cdot ds \cdot u_x \\ &= \frac{-dP_f}{4\mu dl} \left(\frac{d^2}{4} - s^2 \right) 2\pi s \cdot ds \quad (\text{from 3.13}) \\ &= \frac{-dP_f \cdot \pi}{2\mu dl} \left(\frac{d^2 s}{4} - s^3 \right) ds \end{aligned}$$

Then the total flow over the whole cross section Q is given by

$$\begin{aligned} Q &= -\frac{\pi d P_f}{2\mu dl} d^4 \left(\frac{1}{32} - \frac{1}{64} \right) \\ &= -\frac{\pi d P_f d^4}{128\mu dl} \end{aligned} \quad \dots(3.15)$$

This relation was derived experimentally by HAGEN⁽⁹⁾ in 1839 and independently by POISEUILLE⁽¹⁰⁾ in 1840.

Thus the average velocity,

$$\begin{aligned} u &= \frac{Q}{\frac{\pi}{4} d^2} \\ &= \frac{-dP_f d^2}{32\mu dl} \\ &= \frac{1}{2} u_s \end{aligned} \quad \dots(3.16)$$

The velocity at the pipe axis is thus seen to be twice the mean velocity. The pressure drop $-dP_f$ in the pipe can be written

$$-dP_f = \frac{32\mu dl \cdot u}{d^2} \quad \dots(3.17)$$

Kinetic Energy of Fluid

The kinetic energy of the fluid flowing per unit time in the annulus between s and $s + ds$

$$\begin{aligned} &= \frac{1}{2} u_x^2 \rho dQ \\ &= \frac{1}{2} \rho u_x^3 \cdot 2\pi s \cdot ds \\ &= \pi \rho s \cdot ds \left(\frac{-dP_f}{4\mu dl} \right)^3 \left(\frac{1}{4} d^2 - s^2 \right)^3 \quad (\text{from 3.13}) \end{aligned}$$

Thus the total kinetic energy per unit time of the fluid in the pipe

$$\begin{aligned} &= \left(\frac{-dP_f}{4\mu dl} \right)^3 \pi \rho \int_0^{\frac{d}{2}} \left(\frac{1}{4} d^2 - s^2 \right)^3 \cdot -\frac{1}{2} d \left(\frac{1}{4} d^2 - s^2 \right) \\ &= \frac{\pi d^8 \rho}{2048} \left(\frac{-dP_f}{4\mu dl} \right)^3 \end{aligned}$$

The kinetic energy per unit mass will then

$$\begin{aligned} &= \frac{\frac{\pi d^8 \rho}{2048} \left(\frac{-dP_f}{4\mu dl} \right)^3}{\frac{\pi d P_f d^4}{128\mu dl} \rho} \quad (\text{from 3.15}) \\ &= \frac{d^4}{64} \left(\frac{-dP_f}{4\mu dl} \right)^2 \\ &= u^2 \quad (\text{from 3.16}) \end{aligned}$$

In equation 2.31 the kinetic energy per unit mass was expressed as $\frac{u^2}{2\alpha}$ and thus $\alpha = \frac{1}{2}$ for the streamline flow of a fluid in a round pipe.

$$\frac{32\mu dl \cdot u}{d^2} = -dP_f = 4 \left(\frac{R}{\rho u^2} \right) \frac{dl}{d} \cdot \rho u^2 \text{ (from 3.4 and 3.17)}$$

so that
$$\frac{R}{\rho u^2} = \frac{8\mu}{u d \rho} = 8 Re^{-1} \quad \dots (3.18)$$

as already indicated on page 30.

Flow between Two Parallel Plates. Consider the flow of fluid between two plates of unit width, a distance f apart (Fig. 3.9). The force on the fluid contained within a distance s on either side of the centre plane is the shearing force on its boundary planes; this produces the pressure difference dP_f , so that over the distance dl ,

$$dP_f \cdot 2s = \mu \cdot \frac{du_x}{ds} \cdot 2dl$$

where u_x is the velocity at a distance s from the centre plane.

$$\therefore du_x = \frac{dP_f}{\mu \cdot dl} \cdot s \cdot ds$$

and
$$u_x = \frac{dP_f}{\mu \cdot dl} \cdot \frac{s^2}{2} + \text{constant}$$

When $s = f/2$, $u_x = 0$,

$$\therefore \text{constant} = -f^2/8 \cdot \frac{dP_f}{\mu \cdot dl}$$

i.e.
$$u_x = \frac{-dP_f}{2\mu dl} \left(\frac{1}{4}f^2 - s^2 \right) \quad \dots (3.19)$$

The total rate of flow of fluid between the plates is obtained by calculating the flow through two laminae of thickness ds and situated at a distance s from the centre plane and then integrating. Flow through laminae dQ

$$= \frac{-dP_f}{2\mu dl} \left(\frac{1}{4}f^2 - s^2 \right) \cdot 2 \cdot ds$$

Total rate of flow,
$$Q = \frac{-dP_f}{\mu dl} \cdot (f^3/8 - f^3/24)$$

$$= \frac{-dP_f \cdot f^3}{12\mu dl} \quad \dots (3.20)$$

The average velocity of the fluid,

$$u = \frac{Q}{f \times 1}$$

$$= \frac{-dP_f \cdot f^2}{12\mu dl} \quad \dots (3.21)$$

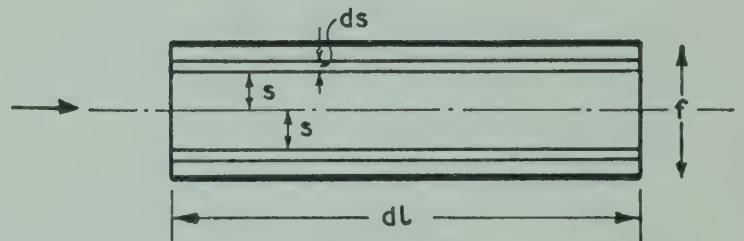


Fig. 3.9. Streamline flow between parallel plates

The maximum velocity occurs at the centre plane and its value is obtained by putting $s = 0$ in equation (3.19).

$$\text{Maximum velocity} = \frac{-dP_f \cdot f^2}{8\mu dl} = 1.5u \quad \dots (3.22)$$

It has been assumed that the width of the plates is large compared with the distance between them so that the flow can be considered as unidirectional.

Flow through an Annulus

The velocity distribution and the mean velocity of a fluid flowing through an annulus of outer radius r and inner radius r_i is more complex (Fig. 3.10). If the pressure changes by an amount dP_f as a result of friction in a length dl of annulus, we can equate the resulting force to the shearing force acting on the fluid. Consider the flow of the fluid situated at a distance not greater than s from the centre line of the pipes. The shear force acting on this fluid consists of two parts; one is the drag on its outer surface; this

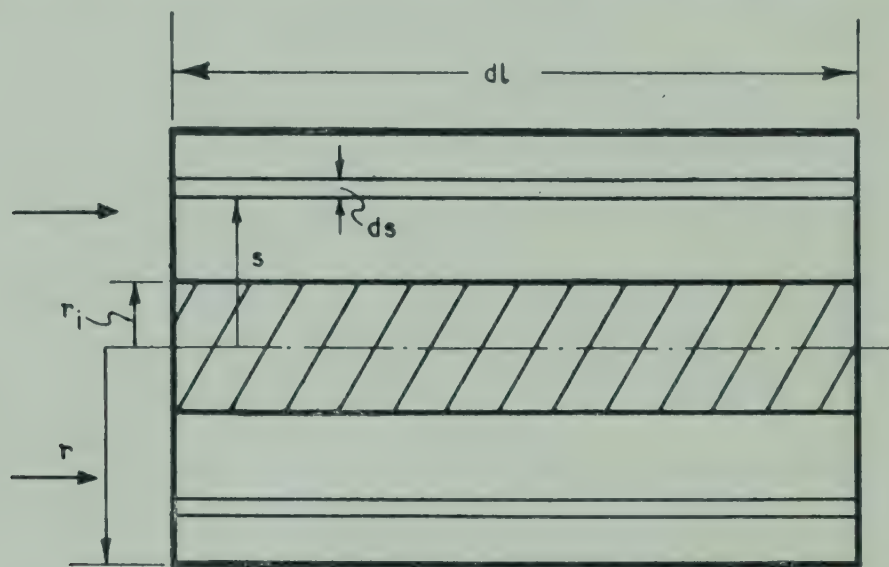


Fig. 3.10. Streamline flow through annulus

can be expressed in terms of the viscosity of the fluid and the velocity gradient at that radius: the other is the drag occurring at the inner boundary of the annulus; this cannot be estimated at present and will be denoted by the symbol λ .

$$\text{Then} \quad dP_f \cdot \pi(s^2 - r_i^2) = \mu 2\pi s dl \frac{du_x}{ds} + \lambda$$

where u_x is the velocity of the fluid at radius, s .

$$\therefore \quad du_x = \frac{dP_f(s^2 - r_i^2)}{2\mu s dl} ds - \lambda \cdot \frac{ds}{2\pi\mu s dl}$$

$$\text{Integrating,} \quad u_x = \frac{dP_f}{2\mu dl} \left(\frac{1}{2}s^2 - r_i^2 \ln s \right) - \frac{\lambda}{2\pi\mu dl} \ln s + u_c \quad \dots (3.23)$$

where u_c is an integration constant with dimensions of velocity.

Substituting the boundary conditions $s = r_i$, $u_x = 0$, and $s = r$, $u_x = 0$ in equation 3.23 and solving for λ and u_c , we have

$$\lambda = \pi dP_f \left\{ \frac{r^2 - r_i^2}{2 \ln \frac{r}{r_i}} - r_i^2 \right\}$$

and

$$u_c = \frac{-dP_f}{2\mu dl} \left(\frac{1}{2}r^2 - \frac{r^2 - r_i^2}{2 \ln \frac{r}{r_i}} \ln r \right)$$

Now substituting these values of λ and u_c in equation (3.23), and simplifying,

$$u_x = \frac{-dP_f}{4\mu dl} \left(r^2 - s^2 + \frac{r^2 - r_i^2}{\ln \frac{r}{r_i}} \ln \frac{s}{r} \right) \dots (3.24)$$

The rate of flow of fluid through a small annulus of inner radius, s , and outer radius, $s + ds$, is given by,

$$\begin{aligned} dQ &= 2\pi s ds u_x \\ &= -\frac{\pi dP_f}{2\mu dl} \left(r^2 s - s^3 + \frac{r^2 - r_i^2}{\ln \frac{r}{r_i}} s \cdot \ln \frac{s}{r} \right) ds \end{aligned}$$

Integrating between the limits $s = r_i$ and $s = r$.

$$Q = \frac{-\pi dP_f}{8\mu dl} \left\{ r^2 + r_i^2 - \frac{r^2 - r_i^2}{\ln \frac{r}{r_i}} \right\} (r^2 - r_i^2) \dots (3.25)$$

The average velocity,

$$\begin{aligned} u &= \frac{Q}{\pi(r^2 - r_i^2)} \\ &= \frac{-dP_f}{8\mu dl} \left\{ r^2 + r_i^2 - \frac{r^2 - r_i^2}{\ln \frac{r}{r_i}} \right\} \dots (3.26) \end{aligned}$$

Turbulent Flow

No exact mathematical analysis of the conditions within a turbulent fluid has yet been developed, though a number of semi-theoretical expressions for the shear stress at the walls of a pipe of circular cross section have been suggested. The simplest of these is the BLASIUS equation⁽¹²⁾ for the shear stress in a hydrodynamically smooth pipe.

$$R = 0.0396 \rho u^2 \cdot Re^{-0.25} \dots (3.27)$$

$$\text{or} \quad \frac{R}{\rho u^2} = 0.0396 \cdot Re^{-0.25} \dots (3.27a)$$

This expression is applicable for turbulent flow at Reynolds Numbers up to about 10^5 .

The shear stresses within the fluid are responsible for the frictional force at the walls and the velocity distribution over the cross section. A given assumption for the shear stress at the walls therefore implies some particular velocity distribution. It will be shown in Chapter 9 that the velocity at any point in the cross section will be proportional to the one-seventh power of the distance from the walls if the shear stress is given by the Blasius equation. This may be expressed as follows,

$$\frac{u_x}{u_s} = \left(\frac{y}{r} \right)^{1/7} \dots (3.28)$$

where u_x is the velocity at a distance y from the walls,

u_s is the velocity at the axis of the pipe, and

r is the radius of the pipe.

This equation is sometimes referred to as the Prandtl one-seventh power law.

Mean Velocity

In a thin annulus of inner radius s and outer radius $s + ds$, the velocity u_x may be taken as constant (Fig. 3.11).

$$\begin{aligned} \therefore dQ &= 2\pi s ds u_x \\ &= -2\pi(r - y)dy u_x \quad (\text{since } s + y = r) \quad \dots(3.29) \\ &= -2\pi(r - y)dy u_s \left(\frac{y}{r}\right)^{1/7} \end{aligned}$$

$$\begin{aligned} \therefore Q &= \int_{y=r}^{y=0} -2\pi r^2 \left(1 - \frac{y}{r}\right) \left(\frac{y}{r}\right)^{1/7} d\left(\frac{y}{r}\right) \cdot u_s \\ &= 2\pi r^2 \left(\frac{7}{8} - \frac{7}{15}\right) u_s \\ &= \frac{49}{60} \pi r^2 u_s \end{aligned}$$

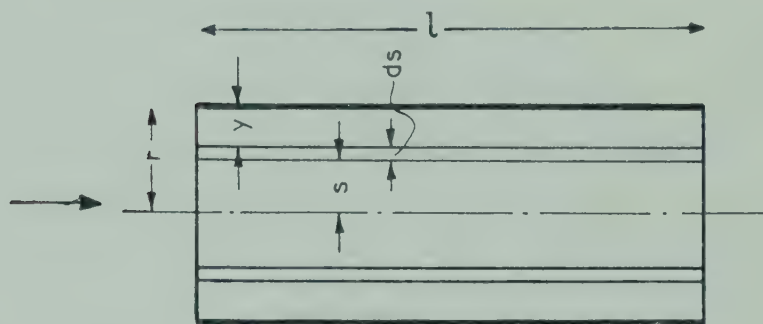


Fig. 3.11. Turbulent flow through pipe

The mean velocity of flow, $u = \frac{Q}{\pi r^2} = 0.82u_s \quad \dots(3.30)$

This relation holds provided that the one-seventh power law can be assumed to apply over the whole of the cross section of the pipe. This is strictly so only at high Reynolds Numbers when the thickness of the laminar sub-layer is small.

Kinetic Energy

Since $dQ = -2\pi(r - y)dy \cdot u_x \quad \dots(3.29)$

the kinetic energy per unit time of the fluid passing through the annulus

$$\begin{aligned} &= -2\pi(r - y)\rho dy \cdot u_x \cdot \frac{1}{2}u_x^2 \\ &= -\pi u_s^3 \rho r^2 \left(1 - \frac{y}{r}\right) \cdot \left(\frac{y}{r}\right)^{3/7} d\left(\frac{y}{r}\right) \end{aligned}$$

Total kinetic energy per unit time,

$$\begin{aligned} &= \pi u_s^3 \rho r^2 \left(\frac{7}{10} - \frac{7}{17}\right) \\ &= \frac{49}{170} \pi r^2 \rho \cdot u_s^3 \end{aligned}$$

Mean kinetic energy per unit mass of fluid

$$\begin{aligned}
 & \frac{49}{170} \pi r^2 \rho u_s^3 \\
 &= \frac{\pi r^2 \rho \cdot \frac{49}{60} u_s}{\frac{49}{60} u_s} \\
 &= \frac{6}{17} u_s^2 \\
 &= \frac{6}{17} \times \left(\frac{60}{49}\right)^2 u^2 \\
 &= 0.53u^2 \approx \frac{1}{2}u^2 \quad \dots(3.31) \\
 &= \frac{u^2}{2\alpha} \text{ (by definition of } \alpha \text{)}
 \end{aligned}$$

Thus for turbulent flow at high Reynolds Numbers, where the thickness of the laminar sub-layer can be neglected, $\alpha \approx 1$.

When the thickness of the laminar sub-layer cannot be neglected, α will be slightly less than 1.

Flow in Non-circular Ducts. For turbulent flow in a duct of non-circular cross section, the hydraulic mean diameter may be used in place of the pipe diameter and the formulae for circular pipes can then be applied without introducing a large error. This method of approach is entirely empirical.

The hydraulic mean diameter d_m is defined as four times the cross-sectional area divided by the wetted perimeter: some examples are given. For a circular pipe,

$$d_m = \frac{4 \cdot \frac{1}{4}\pi d^2}{\pi d} = d \quad \dots(3.32a)$$

For an annulus of outer radius r and inner radius r_i ,

$$d_m = \frac{4 \cdot \pi(r^2 - r_i^2)}{2\pi(r + r_i)} = 2(r - r_i) \quad \dots(3.32b)$$

For a duct of rectangular cross section d_a by d_b ,

$$\begin{aligned}
 d_m &= \frac{4d_a d_b}{2(d_a + d_b)} \\
 &= \frac{2d_a d_b}{d_a + d_b} \quad \dots(3.32c)
 \end{aligned}$$

For streamline flow this method is not applicable and exact expressions relating the pressure drop to the velocity can be obtained for ducts of certain shapes only. Two examples of this have already been given.

Flow Through Curved Pipes

If the pipe is not straight, the velocity distribution over the section is altered and the direction of flow of the fluid is continuously changing. The frictional losses are therefore somewhat greater than for a straight pipe of the same length.

If the radius of the pipe divided by the radius of the bend is less than about 1/500, however, the effects of the curvature are negligible.

It has been found⁽¹⁶⁾ that stable streamline flow persists at higher values of the Reynolds Number in coiled pipes. Thus, for instance, when the ratio of the diameter of the pipe to the diameter of the coil is 1 to 15, the transition occurs at a Reynolds Number of about 8000.

Miscellaneous Friction Losses for Incompressible Fluids

The friction losses occurring as a result of a sudden enlargement or contraction in the cross section of the pipe, and the resistance of various standard pipe fittings will now be considered.

Sudden Enlargement. If the diameter of the pipe suddenly increases, as shown in Fig. 3.12, the effective area available for flow will gradually increase

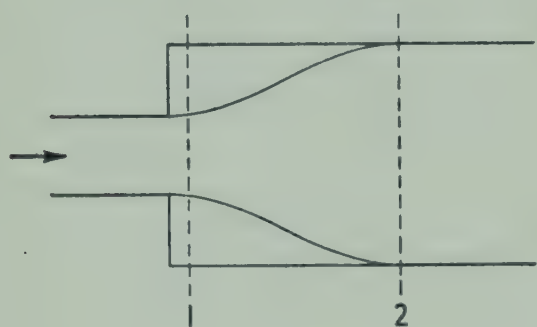


Fig. 3.12. Sudden enlargement

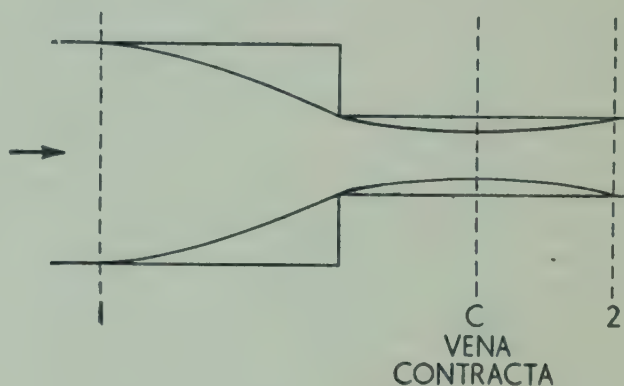


Fig. 3.13. Sudden contraction

from that of the smaller pipe to that of the larger one and the velocity of flow will progressively decrease. Thus fluid with a relatively high velocity will be injected into relatively slow moving fluid; turbulence will be set up and most of the excess kinetic energy will be converted into heat and therefore wasted. If the change of cross section is gradual, the kinetic energy can be recovered as pressure energy.

For the fluid flowing as shown in Fig. 3.12, from section 1 (the pressure just inside the enlargement is found to be equal to that at the end of the smaller pipe) to section 2, the net force = the rate of change of momentum, i.e.

$$P_1 A_2 - P_2 A_2 = \rho_2 A_2 u_2 (u_2 - u_1)$$

$$\begin{aligned} \text{i.e.} \quad (P_1 - P_2) v_2 &= u_2^2 - u_1 u_2 \\ &= - \int_1^2 v \, dP \text{ for an incompressible fluid} \end{aligned}$$

Applying equation 2.32 between the two sections,

$$\frac{u_1^2}{2\alpha_1} = \frac{u_2^2}{2\alpha_2} + \int_1^2 v \, dP + F$$

$$\text{i.e.} \quad F = \frac{u_1^2}{2\alpha_1} - \frac{u_2^2}{2\alpha_2} + u_2^2 - u_1 u_2 \quad \dots (3.33)$$

For fully turbulent flow $\alpha_1 = \alpha_2 = 1$ and

$$F = \frac{(u_1 - u_2)^2}{2} \quad \dots(3.33a)$$

The change in pressure ΔP_f due to friction is therefore $-\frac{\rho \cdot (u_1 - u_2)^2}{2}$

and the loss in head h_f is $\frac{(u_1 - u_2)^2}{2g}$

Sudden Contraction. The effective area for flow gradually decreases as the sudden contraction is approached and then continues to decrease, for a short distance, to what is known as the “vena contracta.” After the vena contracta the flow area gradually approaches that of the smaller pipe. As the fluid moves towards the vena contracta, it is accelerated and pressure energy is converted into kinetic energy; this process does not give rise to eddy formation and losses are very small. However, beyond the vena contracta the velocity falls as the flow area increases and conditions are equivalent to those for a sudden enlargement. The expression for the loss at a sudden enlargement can, therefore, be applied for the fluid flowing from the vena contracta to some section a small distance away, where the whole of the cross section of the pipe is available for flow.

Applying equation (3.33a) between sections c and 2, as shown in Fig. 3.13, the frictional loss per unit mass of fluid is given by,

$$\begin{aligned} F &= \frac{(u_c - u_2)^2}{2} \\ &= \frac{u_2^2}{2} \left(\frac{u_c}{u_2} - 1 \right)^2 \end{aligned}$$

Denoting the ratio of the area at section c to that at section 2 by a coefficient of contraction C_c ,

$$F = \frac{u_2^2}{2} \left(\frac{1}{C_c} - 1 \right)^2 \quad \dots(3.34)$$

Thus the change in pressure ΔP_f is $-\frac{\rho u_2^2}{2} \left(\frac{1}{C_c} - 1 \right)^2$ and the head lost is $\frac{u_2^2}{2g} \left(\frac{1}{C_c} - 1 \right)^2$.

C_c varies from about 0.6 to 1.0 as the ratio of the pipe diameters varies from 0 to 1. For the usual value of C_c of $\frac{2}{3}$,

$$F = \frac{u_2^2}{8} \quad \dots(3.34a)$$

It should be noted that the maximum possible frictional loss which can occur at a change in cross section is the whole of the kinetic energy of the fluid.

Pipe Fittings. Some average figures are given below (Table 3.2) for the friction losses in various pipe fittings for the turbulent flow of fluid. They are

expressed in terms of the equivalent length of straight pipe with the same resistance. Considerable variation will occur according to the exact construction of the fittings.

TABLE 3.2

45° elbows (a)	15 pipe diameters
90° elbows (standard radius) (b)	30-40 " "
90° square elbows (c)	60 " "
Entry from leg of T-piece (d)	60 " "
Entry into leg of T-piece (d)	90 " "
Unions and couplings (e)	Generally very small
Globe valves fully open	60-300 pipe diameters
Gate valves: fully open	7 " "
$\frac{3}{4}$ open	40 " "
$\frac{1}{2}$ open	200 " "
$\frac{1}{4}$ open	800 " "

Typical fittings are shown in Fig. 3.14 and 3.15.

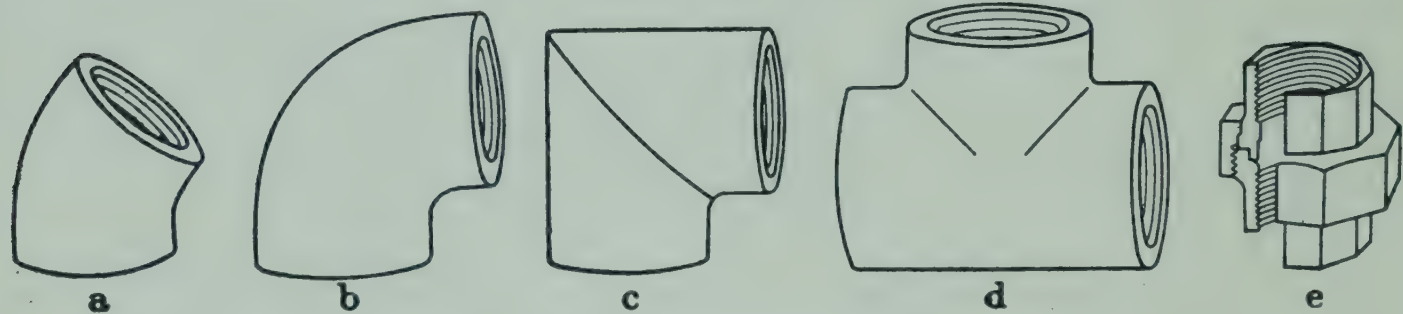


Fig. 3.14. Standard pipe fittings

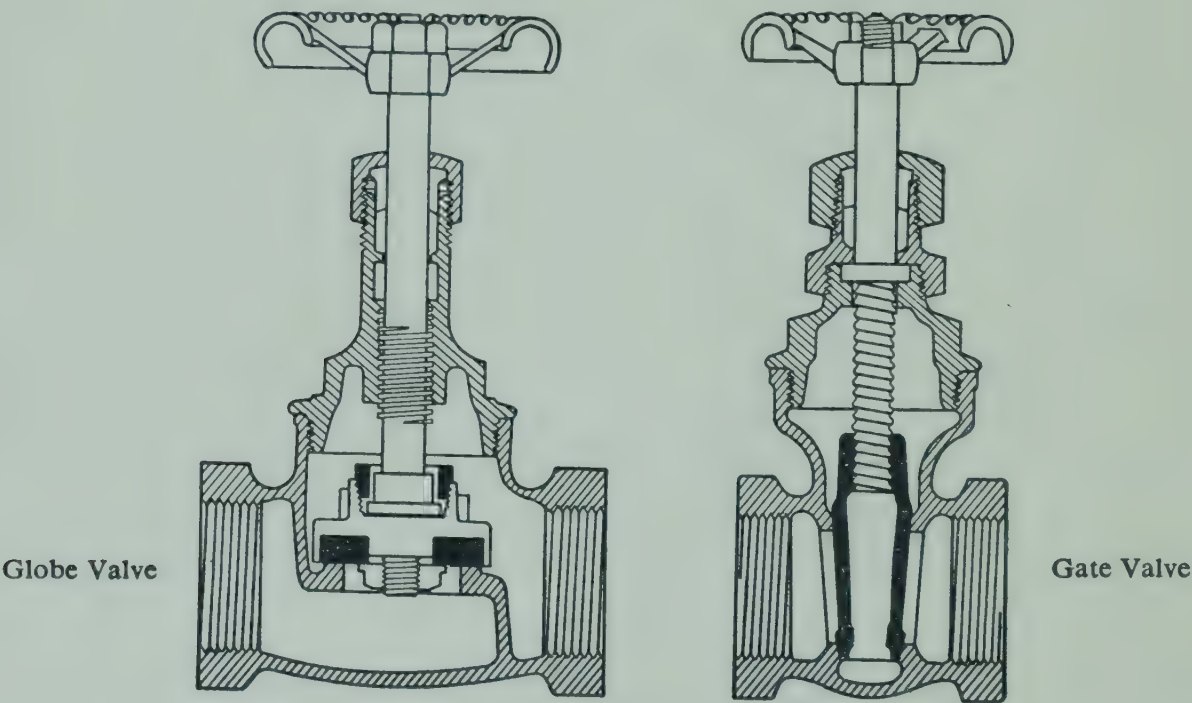


Fig. 3.15. Standard valves

Flow over Banks of Tubes

The frictional loss for a fluid flowing parallel to the axes of the tubes can be calculated in the normal manner by considering the hydraulic mean diameter of the system. This is applicable only to turbulent flow.

For flow at right angles to the axes of the tubes, the cross sectional area is continually changing and the problem can be treated as one involving a series of sudden enlargements and sudden contractions. Thus the friction loss would be

expected to be directly proportional to the number of banks of pipes j in the direction of flow and to the kinetic energy of the fluid. Thus the pressure drop $-\Delta P_f$ can be written

$$-\Delta P_f = \frac{1}{8} C_f \cdot j \cdot \rho \cdot u_t^2 \quad \dots (3.35)$$

where C_f is a coefficient dependent on the arrangement of the tubes and the Reynolds Number. Values of C_f are given in Chapter 6.

u_t is the velocity of flow at the narrowest cross section.

SEPARATION

It should be noted that the energy and mass balance equations assume that the fluid is continuous. This is so, in the case of a liquid, provided that the pressure does not fall to such a low value that boiling, or the evolution of dissolved gases, takes place. For water at normal temperatures the pressure should not be allowed to fall below the equivalent of a head of 4 ft of liquid. With gases, there is no lower limit to the pressures at which the fluid remains continuous but the various equations which are derived need modification if the pressures are so low that the linear dimensions of the channels become comparable with the mean free path of the molecules, i.e. when the so-called molecular flow sets in.

FLOW OF COMPRESSIBLE FLUIDS IN PIPES

The energy balance equation for a small change in a flowing fluid is

$$d \left(\frac{u^2}{2\alpha} \right) + g \cdot dz + v \cdot dP + \delta W + \delta F = 0 \quad \dots (2.31)$$

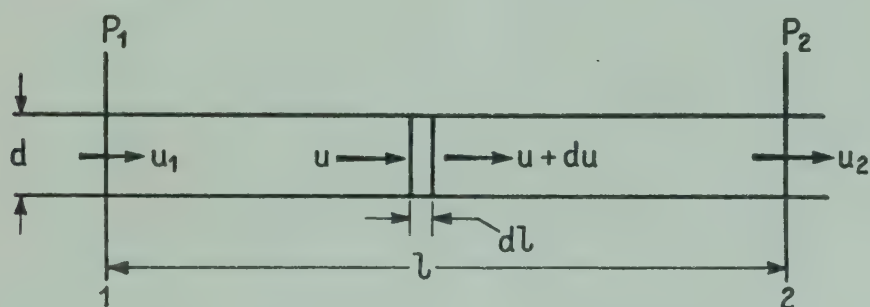


Fig. 3.16. Flow of compressible fluid

For a fluid flowing through a length dl of pipe of constant cross sectional area A ,

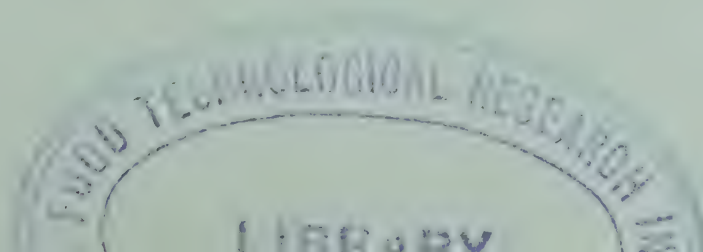
$$d \left(\frac{u^2}{2\alpha} \right) + g \cdot dz + v \cdot dP + 4 \left(\frac{R}{\rho u^2} \right) \frac{dl}{d} \cdot u^2 = 0 \quad \dots (3.36)$$

This equation cannot be integrated directly because the velocity u increases as the pressure falls and is, therefore, a function of l (Fig. 3.16). It is, therefore, convenient to use the mass flow G which remains constant throughout the length of pipe.

Since

$$G = \frac{u \cdot A}{v} \quad \dots (2.29b)$$

$$u = \frac{G \cdot v}{A} = \frac{4 \cdot Gv}{\pi \cdot d^2} \quad \dots (3.37)$$



The friction factor $R/\rho u^2$ is a function of the Reynolds Number and of the relative roughness e/d which will be taken as constant throughout the pipe.

Thus
$$Re. = \frac{ud\rho}{\mu} = \frac{4G}{\pi d\mu} \text{ (from 3.37)}$$

and will be affected only by changes in the viscosity of the fluid. Unless the temperature variation is large μ can be taken as constant and hence $Re.$ will be constant.

The Reynolds Number for the flow of gases is usually high, because the viscosities are low, and therefore small changes in the value of $Re.$ will result in negligible changes in $R/\rho u^2$. By substitution from equation 2.29b in equation (3.36)

$$\left(\frac{G}{A}\right)^2 v \cdot dv + g \cdot dz + v \cdot dP + 4 \left(\frac{R}{\rho u^2}\right) \frac{dl}{d} \left(\frac{G}{A}\right)^2 v^2 = 0 \quad \dots(3.38)$$

where α has been taken as unity for turbulent flow.

For a horizontal pipe $dz = 0$ and

$$\left(\frac{G}{A}\right)^2 v \cdot dv + v \cdot dP + 4 \left(\frac{R}{\rho u^2}\right) \frac{dl}{d} \left(\frac{G}{A}\right)^2 v^2 = 0 \quad \dots(3.39)$$

Dividing through by v^2 and integrating over a length, l , of pipe

$$\left(\frac{G}{A}\right)^2 \ln \frac{v_2}{v_1} + \int_1^2 \frac{dP}{v} + 4 \left(\frac{R}{\rho u^2}\right) \cdot \frac{l}{d} \cdot \left(\frac{G}{A}\right)^2 = 0 \quad \dots(3.39a)$$

The relation between v and P must be known before $\int_1^2 \frac{dP}{v}$ can be evaluated. Several different conditions will be considered.

Isothermal Flow of an Ideal Gas in a Horizontal Pipe

$$\int_1^2 \frac{dP}{v} = \frac{1}{P_1 v_1} \int_1^2 P \cdot dP = \frac{P_2^2 - P_1^2}{2P_1 v_1} \quad \dots(3.40)$$

and therefore, substituting in 3.39,

$$\left(\frac{G}{A}\right)^2 \ln \frac{P_1}{P_2} + \frac{P_2^2 - P_1^2}{2P_1 v_1} + 4 \left(\frac{R}{\rho u^2}\right) \cdot \frac{l}{d} \cdot \left(\frac{G}{A}\right)^2 = 0 \quad \dots(3.41)$$

Since v_m , the specific volume at the mean pressure in the pipe is given by,

$$\frac{P_1 + P_2}{2} \cdot v_m = P_1 v_1$$

$$\left(\frac{G}{A}\right)^2 \ln \frac{P_1}{P_2} + (P_2 - P_1) \cdot \frac{1}{v_m} + 4 \left(\frac{R}{\rho u^2}\right) \cdot \frac{l}{d} \cdot \left(\frac{G}{A}\right)^2 = 0 \quad \dots(3.42)$$

If the pressure drop in the pipe is a small proportion of the total pressure at

the upstream end, the first term in the above expression, representing the increase in the kinetic energy of the fluid, will be small. In this case

$$\frac{P_2 - P_1}{v_m} + 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \left(\frac{G}{A} \right)^2 = 0$$

$$\begin{aligned} \therefore P_1 - P_2 &= 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \frac{u_m^2}{v_m^2} \cdot v_m \\ &= 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \rho_m \cdot u_m^2 \end{aligned} \quad \dots (3.43)$$

where u_m is the velocity and ρ_m is the density of the fluid at the pressure $\frac{P_2 + P_1}{2}$

It will be seen that this expression is of the same form as that obtained for the friction loss for an incompressible fluid. It is applicable only when the pressure drop is small. For large pressure drops equation 3.41 or 3.42 must be used.

For a constant upstream pressure P_1 , the rate of flow G changes as the downstream pressure is varied. From equation (3.41), when $P_1 = P_2$, $G = 0$; also when $P_2 = 0$, $G = 0$. At some intermediate value of P_2 , the flow must therefore be a maximum.

Multiplying equation 3.41 by $\left(\frac{A}{G} \right)^2$,

$$-\ln \frac{P_2}{P_1} + \left(\frac{A}{G} \right)^2 \cdot \frac{(P_2^2 - P_1^2)}{2P_1 v_1} + 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} = 0 \quad \dots (3.44)$$

Differentiating with respect to P_2 , for a constant value of P_1 ,

$$-\frac{P_1}{P_2} \cdot \frac{1}{P_1} + \left(\frac{A}{G} \right)^2 \cdot \frac{2P_2}{2P_1 v_1} + \frac{A^2}{2P_1 v_1} (P_2^2 - P_1^2) \cdot \frac{-2}{G^3} \cdot \frac{\partial G}{\partial P_2} = 0$$

The rate of flow is a maximum when $\frac{\partial G}{\partial P_2} = 0$. Denoting conditions at the downstream end of the pipe by suffix w , when the flow is a maximum, we have by differentiation

$$\frac{1}{P_w} = \left(\frac{A}{G} \right)^2 \cdot \frac{P_w}{P_1 v_1} \quad \dots (3.45)$$

i.e.
$$\left(\frac{G}{A} \right)^2 = \frac{P_w}{v_w}$$

i.e.
$$u_w = \sqrt{P_w v_w} \quad \dots (3.45a)$$

It will be shown later that u_w would be the velocity of transmission of a pressure wave in the fluid at the pressure P_w if heat could be transferred sufficiently rapidly to maintain isothermal conditions. If the pressure at the downstream end of the pipe were P_w , the fluid there would then be moving with the velocity of a pressure wave, and therefore a wave could not be transmitted through the fluid in the opposite direction because its velocity relative to the

pipe would be zero. If, at the downstream end, the pipe were connected to a reservoir in which the pressure was reduced below P_w , the flow conditions within the pipe would be unaffected and the pressure at the exit of the pipe would remain at the value P_w (Fig. 3.17). The drop in pressure from P_w to P_2 would then take place by virtue of lateral expansion of the gas beyond the end of the pipe. If the pressure P_2 in the reservoir at the downstream end were gradually reduced from P_1 , the rate of flow would increase until the pressure reached P_w ; it would then remain constant at this maximum value as the pressure was further reduced.

From equation (3.45) the maximum rate of flow through the pipe is given

by substitution in equation 3.44, $G_w = AP_w \sqrt{\frac{1}{P_1 v_1}}$ (3.45b)

$$\ln \frac{P_1}{P_w} + \frac{v_w}{P_w} \cdot \frac{1}{P_w v_w} \cdot \frac{P_w^2 - P_1^2}{2} + 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} = 0$$

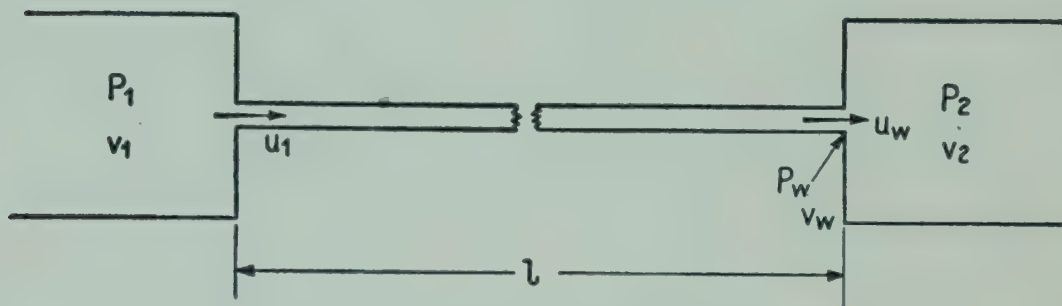


Fig. 3.17. Maximum flow conditions

Hence P_w is given by

$$\ln \left(\frac{P_1}{P_w} \right)^2 + 1 - \left(\frac{P_1}{P_w} \right)^2 + 8 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} = 0 \quad \dots(3.46)$$

Heat Flow required to maintain Isothermal Conditions. As the pressure in the pipe falls, the kinetic energy of the fluid increases at the expense of the internal energy and the temperature tends to fall. The maintenance of isothermal conditions therefore depends on the transfer of an adequate amount of heat from the surroundings. For a small change in the system, the energy balance is given by equation 2.26 as,

$$\delta q - \delta W = dH + g \cdot dz + u du \quad \dots(2.26)$$

For a horizontal pipe, $dz = 0$ and for isothermal expansion of an ideal gas $dH = 0$. Thus if the system does no work on the surroundings,

$$\delta q = u \cdot du \quad \dots(3.47)$$

and the required transfer of heat (in mechanical energy units) per unit mass is $\Delta \frac{1}{2} u^2$. Thus the amount of heat required is equivalent to the increase in the kinetic energy of the fluid. If the mass rate of flow is G , the total heat to be transferred per unit time is $G \cdot \Delta \frac{1}{2} u^2$. In cases where the change in the kinetic energy is small, the required flow of heat is correspondingly small, and conditions are almost adiabatic.

Non-isothermal Flow of an Ideal Gas in a Horizontal Pipe

In general, where an ideal gas expands or is compressed, the relation between the pressure P and the specific volume v can be represented approximately by the expression,

$$P \cdot v^k = \text{a constant} = P_1 \cdot v_1^k$$

where k will depend on the heat transfer to the surroundings.

Thus

$$\int_1^2 \frac{dP}{v} = \frac{k}{k+1} \frac{P_1}{v_1} \cdot \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} - 1 \right\}$$

Inserting this value in equation 3.39a,

$$\left(\frac{G}{A} \right)^2 \frac{1}{k} \ln \frac{P_1}{P_2} + \frac{k}{k+1} \frac{P_1}{v_1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} - 1 \right\} + 4 \left(\frac{R}{\rho u^2} \right) \frac{l}{d} \cdot \left(\frac{G}{A} \right)^2 = 0 \quad \dots (3.48)$$

For a given upstream pressure, P_1 , the maximum flow rate occurs when $u_2 = \sqrt{kP_2v_2}$, the velocity of transmission of a pressure wave under these conditions. Flow under adiabatic conditions is considered in detail below but an approximate result is obtained by putting k equal to γ in equation 3.48.

Adiabatic Flow of an Ideal Gas in a Horizontal Pipe^(24, 26)

The conditions existing during the adiabatic flow in a pipe can be calculated using the approximate expression $Pv^k = \text{a constant}$ to give the relation between the pressure and the specific volume of the fluid. In general, however, the value of the index k may not be known for an irreversible adiabatic process. An alternative approach to the problem is therefore desirable.

For a fluid flowing under turbulent conditions in a pipe, $\delta W = 0$ and

$$\delta q = dH + g dz + u du \quad (\text{from 2.26})$$

In an adiabatic process, $\delta q = 0$ and the equation can then be written as follows, for the flow in a pipe of constant cross-sectional area, A ,

$$\left(\frac{G}{A} \right)^2 v dv + g \cdot dz + dH = 0 \quad (\text{from equation 3.37}) \quad \dots (3.49)$$

Now $dH = dU + d(Pv)$

$= C_v dT + d(Pv)$, for an ideal gas (from equation 2.21)

Further $C_p dT = C_v dT + d(Pv)$, for an ideal gas (from equation 2.22)

$$\therefore dT = \frac{d(Pv)}{C_p - C_v}$$

so that

$$\begin{aligned} dH &= d(Pv) \left\{ \frac{C_v}{C_p - C_v} + 1 \right\} \\ &= \frac{\gamma}{\gamma - 1} d(Pv) \end{aligned}$$

Substituting this value of dH in equation 3.49,

$$\left(\frac{G}{A} \right)^2 v \cdot dv + g \cdot dz + \frac{\gamma}{\gamma - 1} d(Pv) = 0 \quad \dots (3.50)$$

Integrating, we have a relation between P and v for adiabatic flow in a pipe,

$$\frac{1}{2} \left(\frac{G}{A} \right)^2 \cdot v^2 + gz + \frac{\gamma}{\gamma - 1} Pv = \frac{1}{2} \left(\frac{G}{A} \right)^2 v_1^2 + gz_1 + \frac{\gamma}{\gamma - 1} P_1 v_1 = \text{constant}, K \text{ (say)} \quad \dots(3.51)$$

From equations 3.50 and 3.51, for a horizontal pipe,

$$\frac{dP}{v} = - \frac{\gamma - 1}{\gamma} \left(\frac{G}{A} \right)^2 \frac{dv}{v} + \frac{\gamma - 1}{\gamma} \left\{ \frac{1}{2} \left(\frac{G}{A} \right)^2 \cdot \frac{1}{v} - \frac{K}{v^3} \right\} dv$$

On integration, we have

$$\int_1^2 \frac{dP}{v} = \frac{\gamma - 1}{2\gamma} \left(\frac{G}{A} \right)^2 \left\{ \ln \frac{v_1}{v_2} + \frac{v_1^2}{2v_2^2} - \frac{1}{2} \right\} + \frac{P_1 v_1}{2v_2^2} - \frac{P_1}{2v_1} \quad \dots(3.52)$$

Substituting from equation 3.52 into equation 3.39a, and simplifying,

$$8 \left(\frac{R}{\rho u^2} \right) \frac{l}{d} = \left\{ \frac{\gamma - 1}{2\gamma} + \frac{P_1}{v_1} \left(\frac{A}{G} \right)^2 \right\} \left\{ 1 - \left(\frac{v_1}{v_2} \right)^2 \right\} + \frac{\gamma + 1}{\gamma} \ln \frac{v_2}{v_1} \quad \dots(3.53)$$

This expression enables v_2 , the specific volume at the downstream end of the pipe, to be calculated for the fluid flowing at a mass rate G , from an upstream pressure P_1 . Alternatively the mass rate of flow G can be calculated in terms of the specific volume of the fluid at the two pressures, P_1 and P_2 .

The pressure P_2 at the downstream end of the pipe is obtained by substituting the value of v_2 in equation 3.51, giving

$$P_2 = \frac{\gamma - 1}{2\gamma} \left(\frac{G}{A} \right)^2 \cdot \frac{1}{v_2} (v_1^2 - v_2^2) + P_1 \frac{v_1}{v_2} \quad \dots(3.54)$$

It will be noted that equation 3.53 is different in form from equation 3.48, which was derived on the assumption that Pv^k was a constant. This shows that k cannot in fact be taken as constant over the whole length of pipe and that equation 3.48 gives only an approximate result.

From equation 3.53, it is seen that the flow is zero when v_2 is equal to v_1 ($P_2 = P_1$) or to infinity ($P_2 = 0$). The flow is therefore a maximum at some intermediate value of v_2 ; this occurs when $\frac{\partial G}{\partial v_2} = 0$. Differentiating equation 3.53 with respect to v_2 and putting $\frac{\partial G}{\partial v_2} = 0$, and simplifying,

$$\left(\frac{G}{A} \right)^2 = \gamma \frac{P_w}{v_w} \quad \dots(3.55)$$

where suffix w denotes maximum flow conditions

Hence the maximum velocity of the fluid at the downstream end of the pipe u_w is equal to $\sqrt{\gamma P_w v_w}$, the velocity of transmission of a pressure wave under isentropic conditions.

This is shown on page 58 to be the sonic velocity, and thus supersonic velocities cannot be obtained in a pipe of constant cross-section. The ratio of the actual velocity to the sonic velocity is known as the Mach Number.

If this maximum value of G/A is inserted in equations 3.53 and 3.51, two simultaneous equations are obtained; solution of these equations gives the value of the downstream pressure P_w at which the maximum flow occurs. The equations are,

$$8 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \gamma = \left\{ 1 - \left(\frac{v_1}{v_w} \right)^2 \right\} \left\{ \frac{P_1}{P_w} \frac{v_w}{v_1} + \frac{\gamma - 1}{2} \right\} - (\gamma + 1) \ln \frac{v_w}{v_1} \quad \dots(3.56a)$$

and

$$\frac{P_w}{P_1} \left\{ (\gamma + 1) - (\gamma - 1) \left(\frac{v_1}{v_w} \right)^2 \right\} = \frac{2v_1}{v_w} \quad \dots(3.56b)$$

If the downstream end of the pipe is connected to a reservoir at a pressure P_2 , the flow rate will remain at its maximum value if P_2 is reduced below the critical value P_w given by equations 3.56a and 3.56b.

The rate of flow of gas under adiabatic conditions is never more than 20% greater than that obtained for the same pressure difference with isothermal conditions. For pipes of length at least 1000 diameters, the difference does not exceed about 5%. In practice the rate of flow may be limited, not by the conditions in the pipe itself but by the development of sonic velocity at some valve or other constriction in the pipe. Care should, therefore, be taken in the selection of fittings for pipes conveying gases at high velocities.

LAPPLE⁽²⁴⁾ has given a graphical method for the calculation of the pressure drop for a gas flowing in a pipe. He plots $\frac{P_2}{P_1}$ against the ratio of the actual flow rate to that which would be obtained under conditions of maximum discharge for isothermal flow of gas through a nozzle of the same diameter as the pipe. Separate curves are given for a number of values of $\left(\frac{R}{\rho u^2}\right) \cdot \frac{l}{d}$. For high values of this group, e.g. for long pipes, conditions of maximum flow are obtained only at very low values of the pressure ratio, $\frac{P_2}{P_1}$.

Flow of Non-Ideal Gases

Methods have been given for the calculation of the pressure drop for the flow of an incompressible fluid, and for a compressible fluid which behaves as an ideal gas. If the fluid is compressible and deviations from the ideal gas law are appreciable, one of the approximate equations of state, such as Van der Waals' equation, can be used in place of the law $PV = nRT$ to give the relation between temperature, pressure, and volume. Alternatively, if the enthalpy of the gas is known over a range of temperature and pressure, the energy balance equation 2.32a, which involves a term representing the change in the enthalpy, can be employed.

$$\Delta \frac{u^2}{2\alpha} + g \cdot \Delta z + \Delta H = q - W \quad \dots(2.32a)$$

This method of approach is useful in considering the flow of steam at high pressures.

VELOCITY OF PROPAGATION OF A PRESSURE WAVE

When the pressure at some point in a fluid is changed the new condition takes a finite time to be transmitted to some other point in the fluid because the state of each intervening element of fluid has to be changed. The velocity of propagation is a function of the bulk modulus of elasticity ε , where ε is defined by the relation

$$\varepsilon = \frac{\text{increase of stress within the fluid}}{\text{resulting volumetric strain}} = \frac{dP}{\frac{dv}{v}} = -v \frac{dP}{dv} \quad \dots(3.57)$$

Suppose a pressure wave to be transmitted at a velocity u_w over a distance dl in a fluid of cross-sectional area A , from section A to section B (Fig. 3.18).

Now imagine the pressure wave to be brought to rest by causing the fluid to flow at a velocity u_w in the opposite direction. Let the pressure and specific volume at B be P and v , and at A ($P + dP$) and $(v + dv)$, respectively. As a result of the change in pressure, the velocity of the fluid changes from u_w at B to $u_w + du_w$ at A .

The mass rate of flow of fluid,

$$G = \frac{u_w \cdot A}{v} = \frac{(u_w + du_w) \cdot A}{v + dv} \quad (\text{from equation 2.29b})$$

The net force acting on the fluid between sections A and B is equal to the rate of change of momentum of the fluid, i.e.

$$P \cdot A - (P + dP) \cdot A = G \cdot du_w$$

$$\text{i.e.} \quad -A \cdot dP = G \cdot \frac{G}{A} \cdot dv$$

$$\begin{aligned} \therefore \quad -\frac{dP}{dv} &= \frac{G^2}{A^2} \\ &= \frac{\epsilon}{v} \quad (\text{from equation 3.57}) \end{aligned}$$

$$\therefore \quad u_w = \sqrt{\epsilon v} \quad \dots (3.58)$$

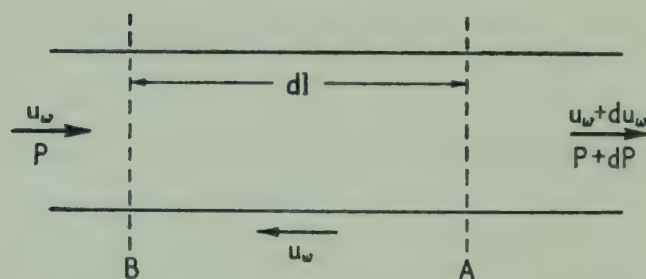


Fig. 3.18. Propagation of pressure wave

For an ideal gas, ϵ can be calculated from the equation of state. Under isothermal conditions,

$$Pv = \text{constant}$$

$$\therefore \quad -\frac{dP}{dv} = \frac{P}{v}$$

$$\therefore \quad \epsilon = P \quad \dots (3.59)$$

and

$$u_w = \sqrt{Pv} \quad \dots (3.60)$$

Under isentropic conditions,

$$Pv^\gamma = \text{constant}$$

$$\therefore \quad -\frac{dP}{dv} = \frac{\gamma P}{v}$$

$$\therefore \quad \epsilon = \gamma P \quad \dots (3.61)$$

and

$$u_w = \sqrt{\gamma Pv} \quad \dots (3.62)$$

This value of u_w is found to correspond closely to the velocity of sound in the fluid. That is, for normal conditions of transmission of a pressure wave, the process is almost isentropic. When the relation between pressure and volume is $Pv^k = \text{constant}$,

$$u_w = \sqrt{kPv} \quad \dots (3.62a)$$

It will be noted that these values of u_w correspond to the velocity of the fluid at the downstream end of a pipe under conditions of maximum flow.

It will now be shown from purely thermodynamic considerations that for adiabatic conditions supersonic flow cannot be obtained in a pipe of constant cross-sectional area, because the fluid is in a condition of maximum entropy when flowing at the sonic velocity. The condition of the gas at any point in the pipe where the pressure is P is given by the equations

$$P \cdot v = \frac{1}{M} RT \quad \dots(2.2a)$$

and
$$\frac{\gamma}{\gamma - 1} P \cdot v + \frac{1}{2} \left(\frac{G}{A} \right)^2 \cdot v^2 = K \quad \dots(3.51a)$$

It will be noted that, if the changes in the kinetic energy of the fluid are small, the process is almost isothermal.

Eliminating v , we have an expression for T ,

$$\frac{\gamma}{\gamma - 1} \cdot \frac{RT}{M} + \frac{1}{2} \left(\frac{G}{A} \right)^2 \frac{R^2 T^2}{P^2 M^2} = K \quad \dots(3.63)$$

The corresponding value of the entropy is obtained as follows.

$$dH = TdS + vdP = C_p dT, \text{ for an ideal gas (from 2.37 and 2.22)}$$

$$\therefore dS = C_p \frac{dT}{T} - \frac{R}{MP} \cdot dP$$

$$\therefore S = C_p \ln \frac{T}{T_0} - \frac{R}{M} \ln \frac{P}{P_0} \text{ (if } C_p \text{ is constant)} \quad \dots(3.64)$$

where T_0, P_0 represents the condition of the gas at which the entropy is arbitrarily taken as zero.

The temperature of the gas can then be plotted to a base of entropy as the pressure falls in the pipe. We then obtain a "Fanno Line"⁽⁶⁾ (Fig. 3.19). For small changes in pressure and entropy, the kinetic energy of the gas increases only very slowly and therefore the temperature remains almost constant. As the pressure is further reduced, the kinetic energy changes become important and the rate of fall of temperature increases and eventually $\frac{dT}{dS}$ becomes infinite. Any further reduction of the

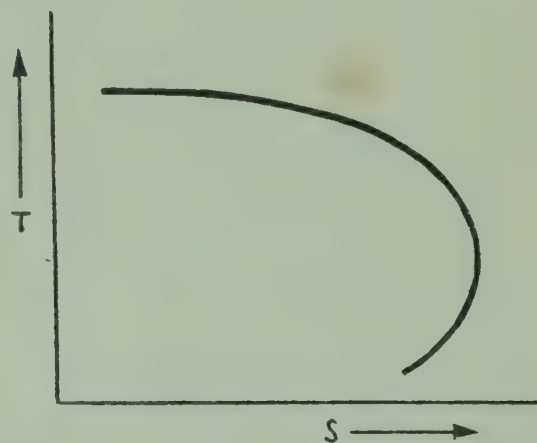


Fig. 3.19. Fanno Line

pressure would cause a decrease in the entropy of the fluid and is, therefore, impossible.

The condition of maximum entropy occurs when $\frac{dS}{dT} = 0$.

$$\frac{dS}{dT} = \frac{C_p}{T} - \frac{R}{MP} \frac{dP}{dT} \text{ (from equation 3.64)}$$

The entropy is, therefore, a maximum when

$$\frac{dP}{dT} = \frac{MPC_p}{RT} \quad \dots(3.65)$$

Now, for an ideal gas,

$$C_p - C_v = \frac{dH}{dT} - \frac{dU}{dT} = \frac{d(Pv)}{dT} = \frac{R}{M} \text{ (from equations 2.22, 2.21, and 2.40b)}$$

Substituting in equation 3.65,

$$\frac{dP}{dT} = \frac{PC_p}{(C_p - C_v)T} = \frac{P}{T} \cdot \frac{\gamma}{\gamma - 1} \quad \dots(3.66)$$

The general value of $\frac{dP}{dT}$ can be obtained by differentiating equation 3.63 with respect to T .

$$\text{Then} \quad \frac{R}{M} + \frac{\gamma - 1}{2\gamma} \left(\frac{G}{A}\right)^2 \frac{R^2}{M^2} \left\{ \frac{P^2 \cdot 2T - T^2 \cdot 2P \cdot \frac{dP}{dT}}{P^4} \right\} = 0$$

$$\therefore \quad 1 + \frac{\gamma - 1}{\gamma} \left(\frac{G}{A}\right)^2 \frac{R}{M} \left(\frac{T}{P^2} - \frac{T^2}{P^3} \frac{dP}{dT} \right) = 0$$

$$\therefore \quad \frac{dP}{dT} = \frac{P}{T} + \frac{\gamma}{\gamma - 1} \left(\frac{A}{G}\right)^2 \frac{M}{R} \cdot \frac{P^3}{T^2} \quad \dots(3.67)$$

The maximum value of the entropy occurs when the values of $\frac{dP}{dT}$ given by equations 3.66 and 3.67 are the same,

$$\text{i.e. when} \quad \frac{\gamma}{\gamma - 1} \frac{P}{T} = \frac{P}{T} + \frac{\gamma}{\gamma - 1} \left(\frac{A}{G}\right)^2 \cdot \frac{M}{R} \frac{P^3}{T^2}$$

$$\begin{aligned} \text{i.e. when} \quad \left(\frac{G}{A}\right)^2 &= \frac{\gamma}{\gamma - 1} \cdot \frac{M}{R} \cdot \frac{P^2}{T} (\gamma - 1) \\ &= \gamma \frac{P^2}{T} \cdot \frac{T}{Pv} = \gamma \frac{P}{v} \end{aligned}$$

$$\text{i.e. when} \quad u = \sqrt{\gamma Pv} = u_w \quad (\text{from equation 3.62})$$

which has been shown to be the velocity of propagation of a pressure wave.

Thus the gas can expand until its velocity is equal to the velocity of propagation of a pressure wave.

POWER REQUIREMENTS FOR PUMPING

A fluid will flow of its own accord so long as the energy per unit mass of fluid decreases in the direction of flow. It can be made to flow in the opposite direction only by the action of some external agent, such as a pump.

The energy balance equation is

$$\Delta \frac{u^2}{2\alpha} + g\Delta z + \int_1^2 v \cdot dP + W + F = 0 \quad \dots(2.32)$$

The work done on unit mass of fluid is $-W$ and the total rate at which energy must be transferred to the fluid is $-G \cdot W$, when the mass rate of flow is G .

$$-G \cdot W = G \left\{ \Delta \frac{u^2}{2\alpha} + g \cdot \Delta z + \int_1^2 v \cdot dP + F \right\} \quad \dots(3.68)$$

Liquids

F can be calculated directly for the flow of liquid through a uniform pipe. If a liquid is pumped through a height Δz from one open tank to another and

none of the kinetic energy is recoverable as pressure energy, the fluid pressure is the same at both ends of the system and $\int_1^2 v \cdot dP$ is zero. The power requirement, is, therefore,

$$G \left\{ \frac{u^2}{2\alpha} + g \cdot \Delta z + F \right\} \quad \dots (3.69)$$

Gases

If a gas is pumped under turbulent flow conditions from a reservoir at a pressure P_1 to a second reservoir at a higher pressure P_2 through a uniform pipe of cross-sectional area A by means of a pump situated at the upstream end, the power required is

$$G \left\{ \frac{1}{2} u_2^2 + g \Delta z + \int_1^2 v \cdot dP + F \right\} \quad \dots (3.68a)$$

In order to make the gas flow, the pump must raise the pressure at the upstream end of the pipe to a pressure P_3 , which is greater than P_2 ; P_3 can be calculated from equation 2.32 (putting $W = 0$), since the pressure P_2 at the downstream end and the mass rate of flow G are known. The friction loss in the pipe F is then evaluated by substituting this value of P_3 back into equation 2.32.

$$\text{i.e.} \quad \Delta \frac{1}{2} u^2 + g \Delta z + \int_3^2 v \cdot dP + F = 0$$

$$\therefore F = - \left\{ \left(\frac{G}{A} \right)^2 \cdot \frac{1}{2} (v_2^2 - v_3^2) + g \Delta z + \int_3^2 v \cdot dP \right\} \quad \dots (3.70)$$

Substituting this value of F in equation 3.68a, the power required

$$\begin{aligned} &= G \left\{ \frac{1}{2} \left(\frac{G}{A} \right)^2 v_2^2 + g \Delta z + \int_1^2 v \cdot dP - \left(\frac{G}{A} \right)^2 \frac{1}{2} (v_2^2 - v_3^2) - g \Delta z - \int_3^2 v \cdot dP \right\} \\ &= G \left\{ \left(\frac{G}{A} \right)^2 \cdot \frac{v_3^2}{2} + \int_1^3 v \cdot dP \right\} \quad \dots (3.71) \end{aligned}$$

Strictly, conditions in the compressor determine the pressure-volume relation in this integral.

For isothermal conditions, power required

$$= G \left\{ \left(\frac{G}{A} \right)^2 \cdot \frac{v_3^2}{2} + P_1 v_1 \ln \frac{P_3}{P_1} \right\} \quad (\text{from equation 2.36}) \quad \dots (3.71a)$$

If a fluid is to be pumped between two points, the diameter of the pipeline should be chosen so that the overall cost of operation is a minimum. The smaller the diameter, the lower is the initial cost of the line but the greater is the cost of pumping; an economic balance must, therefore, be achieved.

The initial cost of a pipeline and the depreciation and maintenance costs will be approximately proportional to the diameter raised to a power of between 1.0 and 1.5. The power for pumping and incompressible fluid at a given rate G is made up of two parts.

(1) That necessitated by the difference in static pressure and vertical height at the two ends of the system. This is independent of the diameter of the pipe.

(2) That attributable to the kinetic energy of the fluid and the work done against friction. If the kinetic energy is small, this is equal to

$$G \times 4 \left(\frac{R}{\rho u^2} \right) \frac{l}{d} u^2$$

which is proportional to $d^{-4.5}$ to -5 for turbulent flow, since $u \propto d^{-2}$ and $\frac{R}{\rho u^2} \propto u^{-0.25}$ to 0 , according to the roughness of the pipe.

The power requirement can, therefore, be calculated as a function of d and the cost obtained. The total cost per annum is then plotted against the diameter of pipe and the optimum conditions are given by the minimum on the curve.

Example. Hydrogen is pumped from a reservoir at 20 atmospheres pressure absolute through a clean horizontal mild steel pipe 2 in. in diameter and 1600 ft long. The downstream pressure is also 20 atmospheres and the pressure of the gas is raised to 25 atmospheres by a pump at the upstream end of the pipe. The conditions of flow are isothermal and the temperature of the gas is 20°C. What is the flow rate and what is the effective rate of working of the pump?

Viscosity of hydrogen = 0.009 centipoises at 20°C.

Solution. Viscosity of hydrogen

$$\begin{aligned} &= 9 \times 10^{-5} \text{ g/cm-sec} \\ &= 9 \times 10^{-5} \times \frac{30.48}{454} = 6.05 \times 10^{-6} \text{ lb/ft-sec} \end{aligned}$$

Density of hydrogen at mean pressure, 22.5 atmospheres,

$$= \frac{2 \times 22.5}{359} \times \frac{273}{293} = 0.117 \text{ lb/cu ft}$$

We shall first obtain an approximate value of G by neglecting the kinetic energy of the fluid. Take P_1 and P_2 as the pressures at the upstream and downstream ends of the pipe, i.e. 25 and 20 atmospheres. Then, from equation 3.43,

$$P_1 - P_2 = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{l}{d} \cdot \rho_m \cdot u_m^2$$

$$\text{i.e.} \quad 5 \times 14.7 \times 144 \times 32.2 = 4 \left(\frac{R}{\rho u^2} \right) \cdot \frac{1600}{1/6} \cdot 0.117 \cdot u_m^2 \text{ pounds/sq. ft}$$

$$\therefore \left(\frac{R}{\rho u^2} \right) \cdot u_m^2 = 75.8$$

$$\begin{aligned} \therefore \left(\frac{R}{\rho u^2} \right) \cdot Re^2 &= 75.8 \times \frac{(1/6)^2 \cdot (0.117)^2}{6.05^2 \times 10^{-12}} \\ &= 7.88 \times 10^8 \end{aligned}$$

Take the roughness e of the pipe surface as 0.00015 ft.

Then $e/d = 0.0009$ and $Re. = 5.7 \times 10^5$ from Fig. 3.4,

$$\therefore \frac{4G}{\pi \mu d} = 5.7 \times 10^5$$

$$\therefore G = 5.7 \times 10^5 \times \frac{\pi}{4} \times \frac{1}{6} \times 6.05 \times 10^{-6} \\ = 0.45 \text{ lb/sec}$$

From Fig. 3.3, $R/\rho u^2 = 0.0024$.

Taking the kinetic energy of the fluid into account, we must apply equation 3.42.

$$\left(\frac{G}{A}\right)^2 \ln \frac{P_1}{P_2} + (P_2 - P_1) \cdot \rho_m + 4 \left(\frac{R}{\rho u^2}\right) \cdot \frac{l}{d} \cdot \left(\frac{G}{A}\right)^2 = 0$$

Using the value of $R/\rho u^2$ obtained by neglecting the kinetic energy,

$$\left(\frac{G}{A}\right)^2 \ln \frac{25}{20} - 5 \times 14.7 \times 144 \times 32.2 \times 0.117 + 4 \times 0.0024 \times \frac{1600}{1/6} \left(\frac{G}{A}\right)^2 = 0$$

$$\therefore 0.223 \left(\frac{G}{A}\right)^2 - 39700 + 92.0 \left(\frac{G}{A}\right)^2 = 0$$

$$\therefore \frac{G}{A} = 20.8$$

$$\therefore G = 20.8 \times \frac{\pi}{4} \times \left(\frac{1}{6}\right)^2 = 0.45 \text{ lb/sec}$$

Thus, as is commonly the case when the pressure drop is a fairly small proportion of the total pressure, the change in kinetic energy is negligible compared with the frictional losses. This would no longer have been true had the pressure drop been much greater.

The power required to pump the gas through the pipe can be calculated from equation 3.71a.

$$\begin{aligned} \text{Power} &= G \left\{ \left(\frac{G}{A}\right)^2 \frac{v_1^2}{2} + P_1 v_1 \ln \frac{P_1}{P_2} \right\} \\ &= 0.45 \left\{ 25.4^2 \cdot \frac{1}{2} \cdot \frac{1}{0.117^2} \frac{22.5^2}{25^2} + \frac{1}{0.117} \times 22.5 \times 14.7 \times 144 \times 32.2 \times 0.223 \right\} \\ &= 0.45 \{ 19,200 + 2,920,000 \} \\ &= 1,323,000 \text{ ft-poundals/sec} \\ &= \frac{1,323,000}{32.2 \times 550} = \underline{\underline{75 \text{ H.P.}}} \end{aligned}$$

The actual power supplied to the pump can then be calculated if the isothermal efficiency of the pump is known.

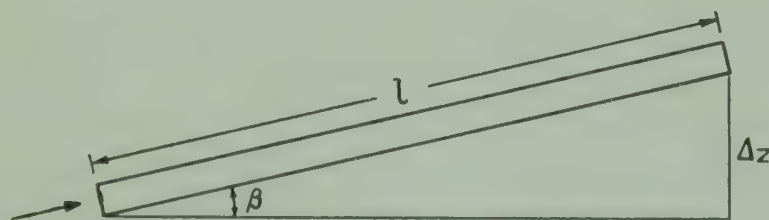


Fig. 3.20. Flow through inclined pipe

Exercise. An ideal gas flows from a pressure P_1 to a pressure P_2 through a pipe of length l , inclined upwards at an angle β to the horizontal (Fig. 3.20). Show that the mass rate of flow G is given by

$$l = \left\{ \frac{d}{8 \left(\frac{R}{\rho u^2}\right)} + \frac{P_1 v_1}{2g \sin \beta} \right\} \ln \frac{4 \left(\frac{R}{\rho u^2}\right) \left(\frac{G}{A}\right)^2 v_1^2 + dg \sin \beta}{4 \left(\frac{R}{\rho u^2}\right) \left(\frac{G}{A}\right)^2 v_2^2 + dg \sin \beta} + \frac{P_1 v_1}{g \sin \beta} \ln \frac{P_1}{P_2} \dots (3.72)$$

CONVERGING-DIVERGING NOZZLES FOR GASES

Converging-diverging nozzles, sometimes known as Laval nozzles, are used for the expansion of gases where the pressure drop is large. If the nozzle is carefully designed so that the contours closely follow the lines of flow, the resulting expansion of the gas is almost reversible. Because the rate of flow is large for a high pressure differential, there will be little time for heat transfer to take place between the gas and the surroundings and the expansion can therefore be considered as isentropic. The specific volume v_2 at a downstream pressure P_2 , is given by

$$v_2 = v_1 \left(\frac{P_1}{P_2} \right)^{1/\gamma} = v_1 \left(\frac{P_2}{P_1} \right)^{-1/\gamma} \quad \dots(3.73)$$

If gas flows under turbulent conditions from a reservoir at a pressure P_1 , through a horizontal nozzle, the velocity of flow u_2 , at the pressure P_2 is given by

$$\frac{u_2^2}{2} + \int_1^2 v \cdot dP = 0 \quad (\text{from equation 2.32})$$

$$\text{Thus} \quad u_2^2 = \frac{2\gamma}{\gamma - 1} P_1 v_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} \quad \dots(3.74)$$

$$\text{Since,} \quad A_2 = G \frac{v_2}{u_2} \quad (\text{from equation 2.29b}) \quad \dots(3.75)$$

the required cross-sectional area for flow can be found.

As the pressure is reduced from P_1 the velocity increases rapidly but the specific volume increases at a very much slower rate; the required area therefore decreases. When the pressure ratio is high, however, the velocity changes much less rapidly than the specific volume and the area must therefore increase again. Thus the area of the nozzle will pass through a minimum if the pressure ratio is high. A_2^2 can be expressed in terms of P_2 from equations 3.73, 3.74, and 3.75.

$$\begin{aligned} \text{Thus} \quad A_2^2 &= G^2 \cdot \frac{v_1^2 \left(\frac{P_2}{P_1} \right)^{-2/\gamma}}{\frac{2\gamma}{\gamma - 1} P_1 v_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} \right\}} \\ &= \frac{G^2 \cdot v_1(\gamma - 1)}{2P_1 \cdot \gamma} \cdot \frac{w^{-2/\gamma}}{1 - w^{(\gamma-1)/\gamma}} \quad \dots(3.76) \end{aligned}$$

For a given rate of flow, G , A_2 decreases from infinity at pressure, P_1 , where the gas is at rest, to a minimum, which occurs when

$$\frac{dA_2^2}{dw} = 0$$

$$\text{i.e. when} \quad (1 - w^{(\gamma-1)/\gamma}) \cdot \frac{-2}{\gamma} \cdot w^{-1-2/\gamma} - w^{-2/\gamma} \cdot \frac{1 - \gamma}{\gamma} w^{-1/\gamma} = 0$$

$$\text{i.e. when} \quad w = \left\{ \frac{2}{\gamma + 1} \right\}^{\frac{\gamma}{\gamma-1}} \quad \dots(3.77)$$

It will be shown on page 86 that $\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} = w_c$, the critical pressure ratio.

The velocity at the throat is therefore the velocity of propagation of a pressure wave and the flow in the converging section of the nozzle is independent of the downstream

conditions. If the pressure at the downstream end of the nozzle is greater than the throat pressure, the kinetic energy of the gas will be decreased and an isentropic compression will take place; the flow rate of the gas will be subsonic. On the other hand, if the pressure at the exit of the nozzle is less than the throat pressure, a further expansion will take place and the velocity will be supersonic. Thus for a given area beyond the throat of the nozzle, the pressure may have one of two different values for optimum conditions. For the upper of these two pressures the flow is subsonic and is similar to that obtained in a venturi meter (see page 90); for the lower the flow is supersonic.

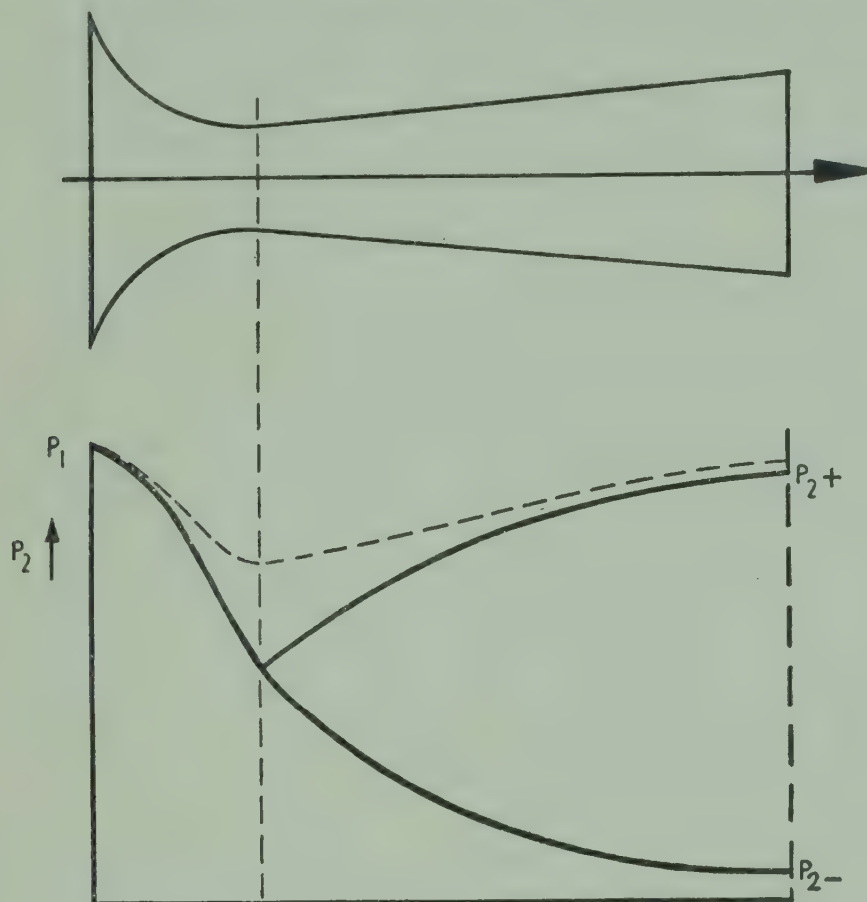


Fig. 3.21. Converging-diverging nozzle

As an example, when γ is 1.4 we have

$$v_2 = v_1 \cdot w^{-0.71} \quad \dots (3.78)$$

and

$$u_2^2 = 7P_1v_1(1 - w^{0.29}) \quad \dots (3.79)$$

Thus

$$A_2^2 = \frac{v_1 \cdot w^{-1.42}}{7P_1(1 - w^{0.29})} G^2 \quad \dots (3.80)$$

In Fig. 3.22, values of v_2 , u_2 , and A_2 are plotted as abscissae against P_2 as ordinate. It is seen that the specific volume at first increases very much more slowly than the velocity. The area, A_2 , consequently decreases to a minimum and then increases again; this minimum occurs at a value of P_2 corresponding to the critical pressure ratio. On the same curve is plotted the velocity of propagation of a pressure wave at the pressure, P_2 ($\sqrt{\gamma P_2 v_2}$). It is seen that the velocity of the fluid is subsonic for pressures above the critical and that the velocity is supersonic for lower pressures.

If the area at any point is reduced below the minimum value of A_2 the rate of flow will fall off below the desired value G . It has been shown that a given nozzle is correctly designed for two different exit pressures. We will now investigate the flow conditions when the outlet pressure is altered from one of these two values.

First, if the outlet pressure is less than the lower design pressure, the fluid expands down to the lower design pressure in the diverging portion of the nozzle. A further expansion then occurs beyond the end of the nozzle to the pressure in the downstream reservoir. Secondly, if the outlet pressure is above the upper design pressure, sonic

conditions are not reached at the throat and the rate of flow is below the desired value G . Thirdly, the outlet pressure may be intermediate between the two design pressures; in that case the outlet area is too large and, in the diverging section, the fluid tends to separate from the walls and thereby artificially to reduce the cross section for flow.

A converging-diverging nozzle is sketched in Fig. 3.21 and, for a given upstream pressure P_1 , the ideal pressure distribution along the axis of the nozzle is plotted: the required values of the pressure P_2 are read off from the curve of area versus pressure in Fig. 3.22. Provided that the exit pressure is less than the upper design pressure of the nozzle, the required rate of flow G is obtained. The velocity at the throat is equal to the sonic velocity and the area of the throat corresponds to the minimum value for A_2 on Fig. 3.22. The fluid expands in the converging section

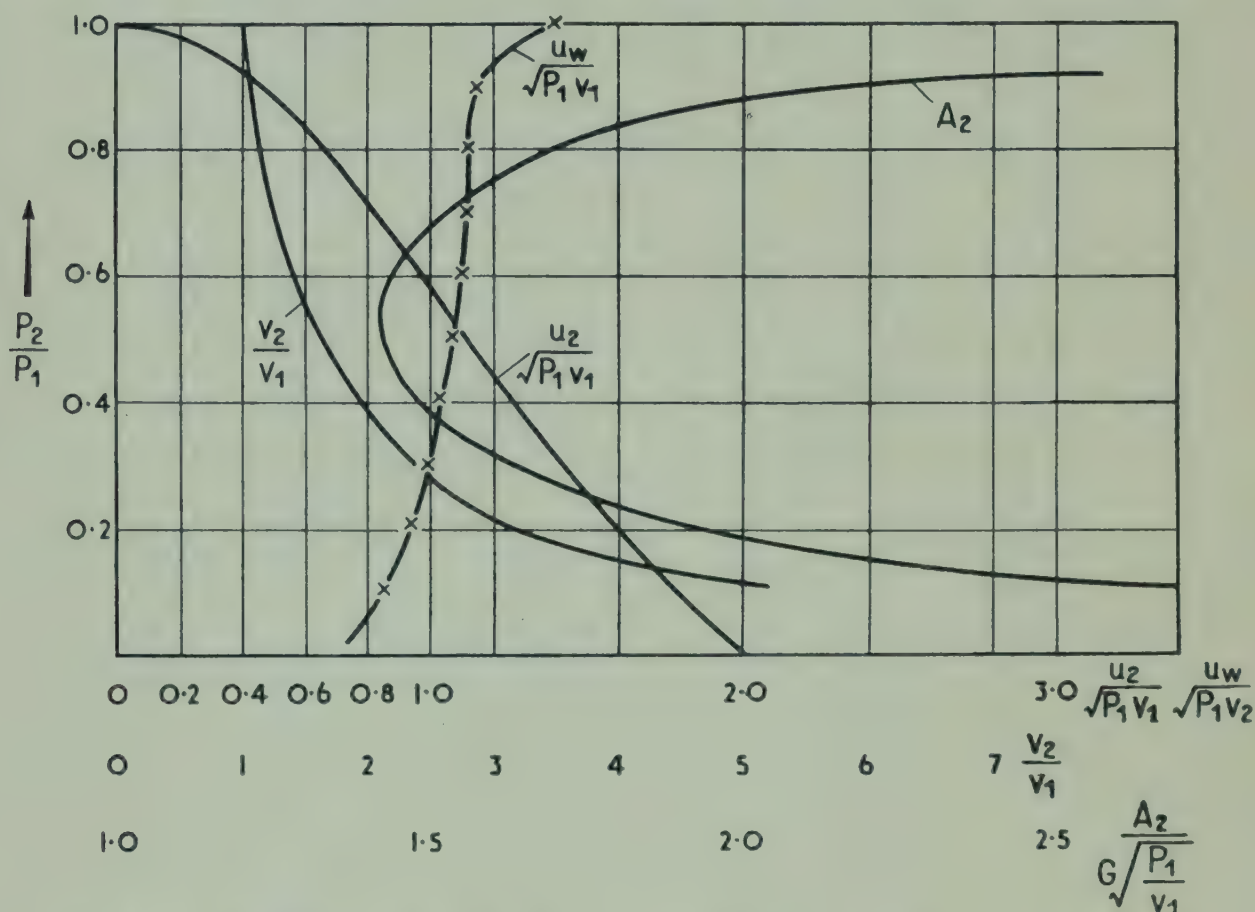


Fig. 3.22. Specific volume, velocity, and nozzle area as a function of pressure

and flows at a subsonic velocity; the flow conditions are not affected by the pressure distribution in the diverging portion because sonic velocity exists at the throat. If the nozzle is operating under design conditions, the pressure distribution in the diverging cone may follow either the upper curve A or the lower curve B , according to the particular value of the downstream pressure (P_2^+ or P_2^-). For a downstream pressure greater than P_2^+ , a typical pressure distribution is given by the dotted curve; it will be noted that the pressure at the throat does not correspond to the critical pressure ratio. When the downstream pressure lies between the two design pressures, the flow tends to separate from the walls in the diverging section and energy losses result as the nozzle becomes inefficient.

FLOW IN OPEN CHANNELS

The chief characteristic of flow in an open channel is that the pressure at the surface is everywhere equal to atmospheric pressure and the channel may be either partly or completely full. The flow of liquid may be either streamline or turbulent, but streamline flow occurs, in practice, only when the liquid is present as a thin film, and will be considered in Chapter 6: we shall therefore confine

our attention to turbulent flow. The transition from streamline to turbulent flow occurs over the range of Reynolds Numbers, $\frac{ud_m\rho}{\mu}$, of 4000 to 11,000, where d_m is the hydraulic mean diameter (see page 47).

Three different types of turbulent flow can be obtained in open channels. They are tranquil flow, rapid flow, and critical flow. In tranquil flow, the velocity is less than that at which some disturbance, such as a surge wave, will be transmitted. Therefore the flow is influenced by conditions at both the upstream and the downstream end of the channel. In rapid flow, the velocity of the fluid is greater than the velocity of a surge wave and therefore the conditions at the downstream end do not influence the flow. Critical flow occurs when the velocity is exactly equal to the velocity of a surge wave.

Uniform Flow

For a channel of constant cross section and slope, the flow is said to be uniform when the depth of the liquid D is constant throughout the length of the channel. For these conditions, as indicated in Fig. 3.23, for a length l of channel the accelerating force acting on liquid

$$= l \cdot A \cdot \rho \cdot g \cdot \sin \theta$$

The force resisting the motion

$$= R_m \cdot p \cdot l$$

where R_m is the mean value of the shear stress at the surface of the channel,

p is the wetted perimeter, and

A is the cross-sectional area of the flowing liquid.

For uniform motion,

$$l \cdot A \cdot \rho \cdot g \cdot \sin \theta = R_m \cdot p \cdot l$$

$$\begin{aligned} \therefore R_m &= \frac{A}{p} \cdot \rho \cdot g \cdot \sin \theta \\ &= \frac{d_m}{4} \rho \cdot g \cdot \sin \theta \end{aligned} \quad \dots(3.81)$$

Dividing both sides of the equation by ρu^2 , where u is the mean velocity in the channel,

$$\frac{R_m}{\rho u^2} = \frac{d_m}{4} \frac{g \cdot \sin \theta}{u^2} \quad \dots(3.82)$$

$$\text{i.e.} \quad u^2 = \frac{d_m}{4 \left(\frac{R_m}{\rho u^2} \right)} g \cdot \sin \theta \quad \dots(3.83)$$

For turbulent flow, $\frac{R_m}{\rho u^2}$ is almost independent of velocity but is a function of

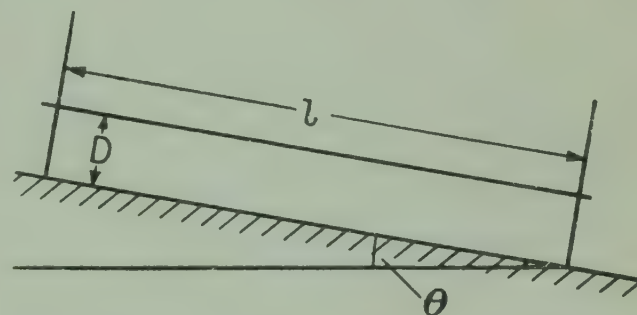


Fig. 3.23. Uniform flow in open channel

the surface roughness of the channel. Thus the resistance force is proportional to the square of the velocity. $\frac{R_m}{\rho u^2}$ is found experimentally to be proportional to the one-third power of the relative roughness of the channel surface and can conveniently be written,

$$\frac{R_m}{\rho u^2} = \frac{1}{16} \left(\frac{e}{d_m} \right)^{1/3} \dots\dots(3.84)$$

for the values of the roughness e given in Table 3.3.

$$u^2 = 4d_m \left(\frac{d_m}{e} \right)^{1/3} \cdot g \cdot \sin \theta \dots\dots(3.85)$$

$$= 4d_m^{4/3} e^{-1/3} \cdot g \cdot \sin \theta \dots\dots(3.85a)$$

The volumetric rate of flow, Q , is then given by

$$\begin{aligned} Q &= u \cdot A \\ &= 2Ad_m^{2/3} e^{-1/6} \sqrt{g \sin \theta} \dots\dots(3.86) \end{aligned}$$

The loss of energy due to friction for unit mass of fluid flowing isothermally through a length l of channel is equal to its loss of potential energy because the other forms of energy remain unchanged,

i.e.
$$F = g \times l \sin \theta = 4 \left(\frac{R_m}{\rho u^2} \right) \left(\frac{l}{d_m} \right) \cdot u^2 \dots\dots(3.87)$$

An empirical equation for the calculation of the velocity of flow in an open channel is the Chezy equation, which can be expressed as follows,

$$u = C \sqrt{d_m} \cdot \sin \theta \dots\dots(3.88)$$

where the value of the coefficient C is a function of the units of the other quantities in the equation. This expression takes no account of the effect of surface roughness on the velocity of flow.

TABLE 3.3. Values of the roughness e , for use in equation 3.84

	feet	inches
Planned wood or finished concrete .	0.00015	0.0020
Unplanned wood	0.00024	0.0028
Unfinished concrete	0.00037	0.0044
Cast iron	0.00056	0.0067
Brick	0.00082	0.0098
Riveted steel	0.0017	0.019
Corrugated metal	0.0055	0.066
Rubble	0.012	0.14

The velocity of the liquid varies over the cross section; it is usually a maximum at a depth of between $0.05D$ and $0.25D$ below the surface, at the centre line of the channel. The velocity distribution can be measured by means of a pitot tube; this is described in Chapter 4.

The shape and the proportions of the channel can be chosen for any given flow rate so that the perimeter, and hence the cost of the channel, is a minimum. We have

$$Q = 2Ad_m^{2/3}e^{-1/6}\sqrt{g \sin \theta} \quad \dots(3.86)$$

Assuming that slope and the roughness of the channel are fixed we have, for a given flow rate Q ,

$$A \cdot d_m^{2/3} = \text{constant}$$

$$\therefore A^{5/3} \cdot p^{-2/3} = \text{constant}$$

The perimeter is therefore a minimum when the cross section for flow is a minimum. For a rectangular channel, of depth D and width B ,

$$A = D \cdot B = D(p - 2D)$$

$$\therefore D^{5/3}(p - 2D)^{5/3}p^{-2/3} = \text{constant}$$

$$\therefore D \cdot p^{3/5} - 2D^2 \cdot p^{-2/5} = \text{constant}$$

p is a minimum when $\frac{dp}{dD} = 0$. Differ-

entiating with respect to D and putting

$\frac{dp}{dD}$ equal to 0,

$$p^{3/5} \cdot 1 - p^{-2/5} \cdot 4D = 0$$

$$\therefore 4D = p = 2D + B$$

$$\text{i.e.} \quad B = 2D$$

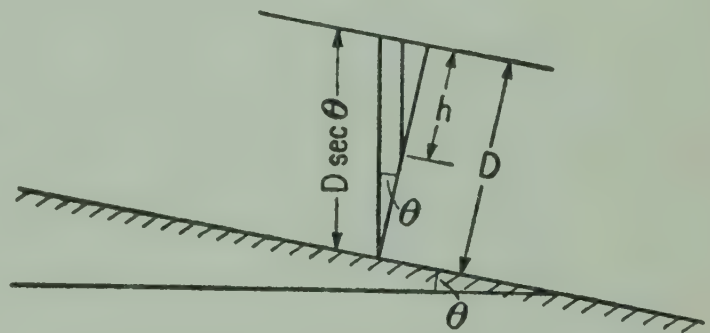


Fig. 3.24. Non-uniform flow in open channel

Thus the most economical rectangular section is such that the width is equal to twice the depth of liquid flowing. The most economical proportions for other shapes can be determined in a similar manner.

Non-uniform Flow

The flow is said to be non-uniform when the velocity of flow or the depth of liquid in the channel varies along its length. Consider a liquid flowing in a channel inclined at an angle θ to the horizontal (Fig. 3.24). The depth of liquid D measured at right angles to the bottom of the channel is changing along its length and consequently the area for flow and the velocity are both varying. Consider the energy and mass balance equations for the fluid. The various energies possessed by unit mass of fluid at a depth h below the liquid surface (measured at right angles to the bottom of the channel) are as follows.

Internal energy U

Pressure energy $(P_a + h \sec \theta \rho \cdot g)v = P_a v + h \cdot g \sec \theta$

where P_a is atmospheric pressure.

Potential energy $z \cdot g + (D - h) \cdot \sec \theta \cdot g$

where z is the height of the bottom of the channel above the datum level at which the potential energy is reckoned as zero.

Kinetic energy $\frac{1}{2}u^2$

The total energy per unit mass is therefore

$$U + P_a v + z \cdot g + D \cdot g \sec \theta + \frac{1}{2}u^2 \quad \dots(3.89)$$

It will be seen that at any cross section the total energy is independent of the depth h , below the liquid surface. As the depth is increased, the pressure energy increases at the same rate as the potential energy decreases. If the fluid flows through a length dl of channel, the net change in energy per unit mass is given by

$$\delta q - \delta W = dU + g \cdot dz + g \cdot \sec \theta \cdot dD + u \cdot du \quad \dots(3.90)$$

$$\text{For an irreversible process, } dU = T \cdot dS - P \cdot dv \quad \dots(2.8)$$

$$= \delta q + \delta F \quad (\text{from 2.6})$$

if the fluid is incompressible, since $dv = 0$.

If no work is done on the surroundings, $\delta W = 0$ and

$$g \cdot dz + g \cdot \sec \theta \cdot dD + u \cdot du + \delta F = 0 \quad \dots(3.91)$$

Substituting for δF from equation 3.87, and writing $u = \frac{Q}{A}$,

$$-g \cdot \sin \theta \cdot dl + g \cdot \sec \theta \cdot dD + \frac{Q}{A} \cdot d\left(\frac{Q}{A}\right) + 4 \left(\frac{R_m}{\rho u^2}\right) \cdot \frac{dl}{d_m} \left(\frac{Q}{A}\right)^2 = 0 \quad \dots(3.92)$$

For a rectangular channel of small slope ($\sec \theta \rightarrow 1$).

$$\therefore \quad dl = \frac{1 - \frac{Q^2}{A^2 D g}}{\sin \theta - 4 \left(\frac{R_m}{\rho u^2}\right) \frac{1}{g d_m} \left(\frac{Q}{A}\right)^2} dD \quad \dots(3.93)$$

This expression can be integrated to give D as a function of l , since both d_m and A are functions of D . It will be found that D will become almost constant after a certain point and the flow is then approximately uniform.

Specific Energy of Liquid

From equation 3.89 the total energy per unit mass of fluid flowing in a rectangular duct inclined at an angle θ to the horizontal is equal to

$$U + P_a \cdot v + z \cdot g + D \cdot g \sec \theta + \frac{1}{2}u^2 \quad \dots(3.89)$$

For a fluid at a constant temperature, the first three terms in this expression are independent of the flow conditions within the channel. On the other hand, the last two terms are functions of the velocity and the depth of liquid. The specific energy of the fluid is defined by the relation,

$$J = D \cdot g \cdot \sec \theta + \frac{1}{2}u^2 \quad \dots(3.94)$$

For a horizontal channel, rectangular in section,

$$J = D \cdot g + \frac{1}{2}u^2 = D \cdot g + \frac{1}{2} \frac{Q^2}{B^2 D^2}$$

The specific energy will vary with the velocity of the liquid and will be a minimum for some critical value of D ; for a given rate of flow Q the minimum will occur when $\frac{\partial J}{\partial D} = 0$,

i.e. when
$$g + \frac{-2Q^2}{2B^2D^3} = 0$$

i.e. when
$$\frac{u^2}{D} = g$$

i.e. when
$$u = \sqrt{g \cdot D} \quad \dots (3.95)$$

This value of u is known as the critical velocity.

The corresponding values of D and E are given by,

$$D = \sqrt[3]{\frac{Q^2}{B^2g}} \quad \dots (3.96)$$

the “critical depth”,

and
$$J = D \cdot g + \frac{1}{2}D \cdot g = \frac{3}{2}D \cdot g \quad \dots (3.97)$$

It can similarly be shown that, at the critical conditions, the flow rate is a maximum for a given value of the specific energy J . At the critical velocity,

$\frac{u}{\sqrt{gD}}$ is equal to unity. This dimensionless group is known as the Froude

Number. For velocities greater than the critical velocity it is greater than unity, and vice versa. We shall now show that the velocity with which a small disturbance is transmitted through a liquid in an open channel is equal to the critical velocity. Thus the Froude Number is the criterion by which the type of flow, tranquil or rapid, is determined. Tranquil flow occurs when the Froude Number is less than unity and rapid flow when it is greater than unity.

Velocity of Transmission of a Wave

Consider a liquid which is flowing with a velocity u in a rectangular channel of width B . Initially, the depth of liquid is D_1 but, as a result of a change in conditions at the downstream end of the channel, the level there is suddenly increased to some value D_2 . A wave therefore tends to move upstream against the motion of the oncoming fluid. Consider two sections, 1 and 2, one on each side of the wave at any instant (Fig. 3.25).

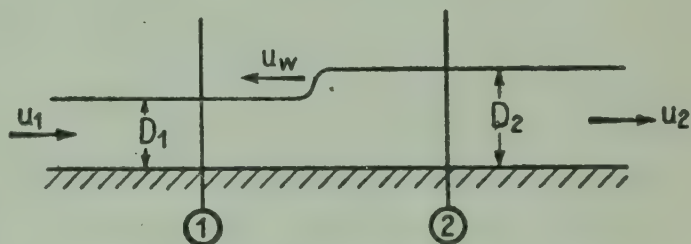


Fig. 3.25. Transmission of a wave

The rate of accumulation of fluid between the two sections is

$$u_1 D_1 B - u_2 D_2 B$$

This accumulation of fluid results from the propagation of the wave and is therefore equal to $u_w \cdot B(D_2 - D_1)$.

Thus
$$u_1 D_1 - u_2 D_2 = u_w(D_2 - D_1)$$

$$\therefore u_2 = \frac{1}{D_2} \{u_w(D_1 - D_2) + u_1 D_1\} \quad \dots (3.98)$$

The velocity of the fluid is changed from u_1 to u_2 by the passage of the wave. The rate of travel of the wave relative to the upstream liquid is $(u_1 + u_w)$ and therefore the mass of fluid whose velocity is changed in unit time

$$= (u_1 + u_w) \cdot B \cdot D_1 \cdot \rho \quad \dots (3.99)$$

The force acting on the fluid at any section where the liquid depth is D is equal to

$$\int_0^D (h \cdot \rho \cdot g) \cdot B \cdot dh = \frac{1}{2} B \cdot \rho \cdot g \cdot D^2 \quad \dots (3.100)$$

where h is any depth below the liquid surface.

The net force acting on the fluid between sections 1 and 2 is

$$\frac{1}{2} B \cdot \rho \cdot g (D_1^2 - D_2^2) \text{ in the direction of flow}$$

Then, neglecting the frictional drag of the walls of the channel between sections 1 and 2, the net force can be equated to the rate of increase of momentum, i.e.

$$\frac{1}{2} B \cdot \rho \cdot g (D_1^2 - D_2^2) = (u_1 + u_w) \cdot B \cdot D_1 \cdot \rho (u_2 - u_1)$$

$$\therefore (u_1 + u_w) \cdot D_1 \left\{ \frac{1}{D_2} [u_w (D_1 - D_2) + u_1 D_1] - u_1 \right\} = \frac{1}{2} g (D_1^2 - D_2^2)$$

$$\begin{aligned} \therefore (u_1 + u_w)^2 &= \frac{D_2}{D_1} (D_1 + D_2) \cdot \frac{g}{2} \\ &= \frac{g D_2}{2} \left(1 + \frac{D_2}{D_1} \right) \quad \dots (3.101) \end{aligned}$$

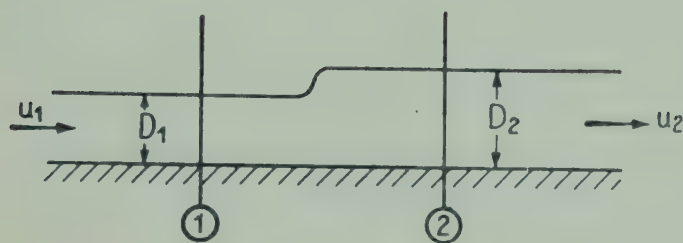


Fig. 3.26. Hydraulic jump

$u_1 + u_w$ is the velocity of the wave relative to the oncoming fluid. For a very small wave, $D_1 \rightarrow D_2$ and

$$u_1 + u_w = \sqrt{g \cdot D_2} \quad \dots (3.102)$$

It is thus seen that the velocity of an elementary wave is equal to the critical velocity, at which the specific energy of the fluid is a minimum for a given flow rate. The criterion for critical conditions is therefore

that the Froude Number, $\frac{u}{\sqrt{gD}}$, is equal to unity.

Hydraulic Jump

If a liquid enters a channel under a gate, it will flow at a high velocity through and just beyond the gate and the depth will be correspondingly low. This is an unstable condition, and at some point the depth of the liquid may suddenly increase and the velocity fall. This change is known as the hydraulic jump, and is accompanied by a reduction of the specific energy of the liquid as the flow changes from rapid to tranquil; the excess energy is dissipated as a result of turbulence.

Consider a liquid flowing in a rectangular channel in which a hydraulic jump occurs between sections 1 and 2 (Fig. 3.26). Then the conditions after the jump can be determined by equating the net force acting on the liquid between the sections to the rate of change of momentum; frictional forces at the walls of the channel are neglected.

Net force acting on the fluid

$$= \frac{1}{2} B \cdot \rho \cdot g (D_1^2 - D_2^2)$$

Rate of change of momentum of fluid

$$= u_1 \cdot B \cdot D_1 \cdot \rho (u_2 - u_1)$$

$$\therefore \frac{1}{2} g (D_1^2 - D_2^2) = u_1 \cdot D_1 (u_2 - u_1)$$

The volumetric rate of flow of the fluid is the same at sections 1 and 2.

$$\therefore Q = u_1 \cdot B \cdot D_1 = u_2 \cdot B \cdot D_2$$

$$\therefore \frac{1}{2}g(D_1^2 - D_2^2) = u_1^2 \cdot D_1 \left(\frac{D_1}{D_2} - 1 \right)$$

$$\text{If } D_1 \neq D_2, \quad \frac{1}{2}g(D_1 + D_2) = u_1^2 \cdot \frac{D_1}{D_2}$$

$$\therefore D_2^2 + D_1 D_2 - \frac{2u_1^2 D_1}{g} = 0 \quad \dots(3.103)$$

$$\therefore D_2 = \frac{1}{2} \left\{ -D_1 \pm \sqrt{D_1^2 + \frac{8u_1^2 D_1}{g}} \right\} \quad \dots(3.104)$$

This expression gives D_2 as a function of the conditions at the upstream side of the hydraulic jump. The corresponding velocity u_2 is obtained by substituting in the equation,

$$u_1 D_1 = u_2 D_2$$

Corresponding values of D_1 and D_2 are referred to as conjugate depths.

The minimum depth at which a hydraulic jump can occur is found by putting $D_1 = D_2 = D$ in equation (3.103),

$$\text{i.e.} \quad 2D^2 = \frac{2u_1^2 D}{g}$$

$$\therefore D = \frac{u_1^2}{g} \quad \dots(3.105)$$

This value of D corresponds to the critical depth for flow in a channel.

Thus a hydraulic jump can occur provided that the depth of the liquid is less than the critical depth. After the jump it will be greater than the critical depth, the flow having changed from rapid to tranquil.

The energy loss associated with the hydraulic jump will now be calculated. For a small change in the flow of a fluid in an open channel,

$$u \cdot du + g \cdot dz + g \cdot \sec \theta \cdot dD + \delta F = 0 \quad \dots(3.91)$$

For a horizontal channel, therefore,

$$F = g(D_1 - D_2) + \frac{1}{2}(u_1^2 - u_2^2) \quad \dots(3.106)$$

$$\text{From equation 3.103,} \quad u_1^2 = \frac{1}{2}g \frac{D_2}{D_1} (D_1 + D_2)$$

$$\text{Similarly,} \quad u_2^2 = \frac{1}{2}g \frac{D_1}{D_2} (D_1 + D_2)$$

$$\begin{aligned} \therefore F &= g(D_1 - D_2) + \frac{1}{4}g(D_1 + D_2) \left(\frac{D_2}{D_1} - \frac{D_1}{D_2} \right) \\ &= \frac{1}{4}g(D_1 - D_2) \left\{ 4 + (D_1 + D_2) \cdot \frac{-(D_1 + D_2)}{D_1 \cdot D_2} \right\} \\ &= \frac{(D_2 - D_1)^3 \cdot g}{4D_1 D_2} \quad \dots(3.107) \end{aligned}$$

CHAPTER 4

Flow Measurement

FLUID PRESSURE

IN a stationary fluid, the pressure is exerted equally in all directions and is referred to as the static pressure; in a moving fluid, the static pressure is exerted on any plane parallel to the direction of motion. The pressure exerted on a plane at right angles to the direction of flow is greater than the static pressure because the surface has, in addition, to exert sufficient force to bring the fluid to rest. This additional pressure is proportional to the kinetic energy of the fluid; it cannot be measured independently of the static pressure.

Static Pressure

The energy balance equation (2.31) can be applied between any two sections in a continuous fluid. If the fluid is not moving the kinetic energy and the frictional loss are both zero, and therefore

$$v \cdot dP + g \cdot dz = 0$$

For an incompressible fluid,

$$\begin{aligned} v(P_2 - P_1) + g(z_2 - z_1) &= 0 \\ \text{i.e.} \quad P_2 - P_1 &= -(z_2 - z_1) \cdot \rho \cdot g \end{aligned} \quad \dots(4.1)$$

Thus the pressure difference can be expressed in terms of the height of a vertical column of fluid.

If the fluid is compressible and behaves as an ideal gas, we have for isothermal conditions,

$$\begin{aligned} P_1 v_1 \ln \frac{P_2}{P_1} + g(z_2 - z_1) &= 0 \\ \therefore \quad \frac{P_2}{P_1} &= e^{-\frac{gM}{RT}(z_2 - z_1)} \end{aligned} \quad \dots(4.2)$$

This expression enables the pressure distribution within an ideal gas to be calculated for isothermal conditions.

When the static pressure in a moving fluid is to be determined, the measuring surface must be parallel to the direction of flow so that no kinetic energy is converted into pressure energy at the surface. If the fluid is flowing in a circular pipe, the measuring surface must, therefore, be perpendicular to the radial direction, at any point. The pressure connection, which is known as a “piezo-meter tube,” should terminate flush with the wall of the pipe so that the flow is not disturbed: the pressure is then measured near the walls where the velocity is a minimum and the reading would be subject only to a small error if the surface

were not quite parallel to the direction of flow. A “piezometer tube” of narrow diameter is used for accurate measurements.

The static pressure should always be measured at a distance of not less than fifty diameters from bends or other obstructions, so that the flow lines are almost parallel to the walls of the tube. If there are likely to be large cross currents or eddies a piezometer ring should be used. This consists of four pressure tapings equally spaced at 90° intervals round the circumference of the tube; they are joined by a circular tube which is connected to the pressure measuring device. By this means false readings due to irregular flow are avoided. If the pressure on one side of the tube is relatively high, the pressure on the opposite side is generally correspondingly low; with the piezometer ring a

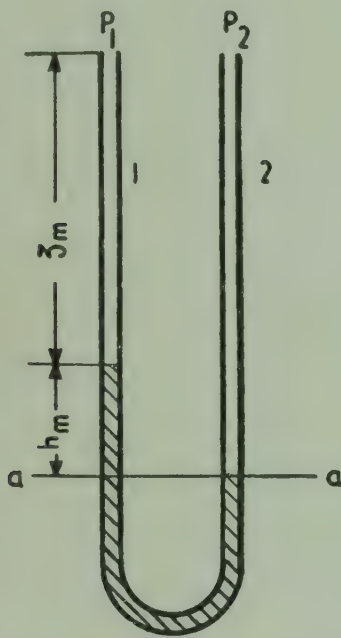


Fig. 4.1. Simple manometer

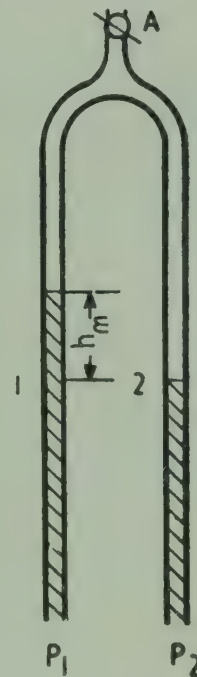


Fig. 4.2. Inverted manometer

mean value is obtained. The cross section of the piezometer tubes and ring should be small to prevent any appreciable circulation of the fluid.

Pressure Measuring Devices

(a) *The simple manometer* is shown in Fig. 4.1. It consists of a transparent U-tube containing the fluid A whose pressure is to be measured and an immiscible fluid B of higher density. The limbs are connected to the two points between which the pressure difference $P_2 - P_1$ is required; the connecting leads should be completely full of fluid A . If P_2 is greater than P_1 , the interface between the two liquids in limb 2 will be depressed a distance h_m (say) below that in limb 1. The pressure at the level $a - a$ must be the same in each of the limbs and, therefore,

$$P_2 + z_m \rho g = P_1 + (z_m - h_m) \rho g + h_m \rho_m g$$

$$\therefore \Delta P = P_2 - P_1 = h_m (\rho_m - \rho) \cdot g \quad \dots (4.3)$$

If fluid A is a gas ρ will normally be small compared with ρ_m so that we can write

$$\Delta P = h_m \rho_m g \quad \dots (4.3a)$$

(b) *The inverted manometer* (Fig. 4.2) is used for measuring pressure differences

in liquids. The space above the liquid in the manometer is filled with air which can be admitted or expelled through the tap *A* in order to adjust the level of the liquid in the manometer.

(c) *The inclined manometer* (Fig. 4.3). For the measurement of small pressure differences the limbs of the manometer can be inclined at an angle ϕ to the vertical. The distance l_m between the interfaces in the two limbs is then measured in a direction parallel to the limbs and the difference in vertical height is then given by

$$h_m = l_m \cos \phi$$

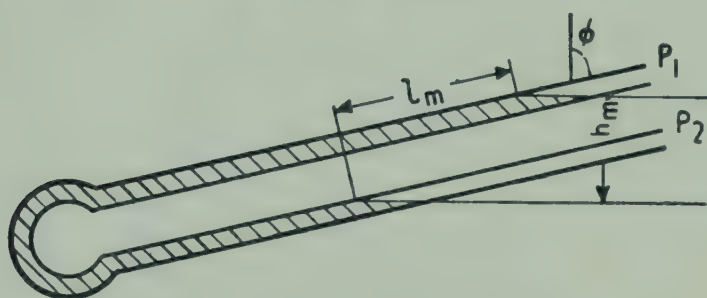


Fig. 4.3. Inclined manometer

(d) *The two liquid manometer*. Small differences in pressure in gases are often measured with a manometer of the form shown in Fig. 4.4. The reservoir at the top of each limb is of a sufficiently large cross section for the liquid level to remain approximately the same on each side of the manometer. The difference in pressure is then given by

$$\Delta P = P_2 - P_1 = h_m(\rho_{m1} - \rho_{m2})g \quad \dots(4.3b)$$

where ρ_{m1} and ρ_{m2} are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the

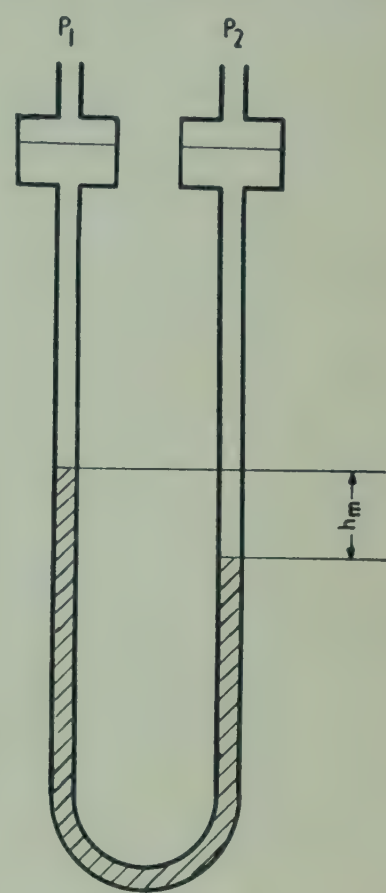


Fig. 4.4. Two liquid manometer

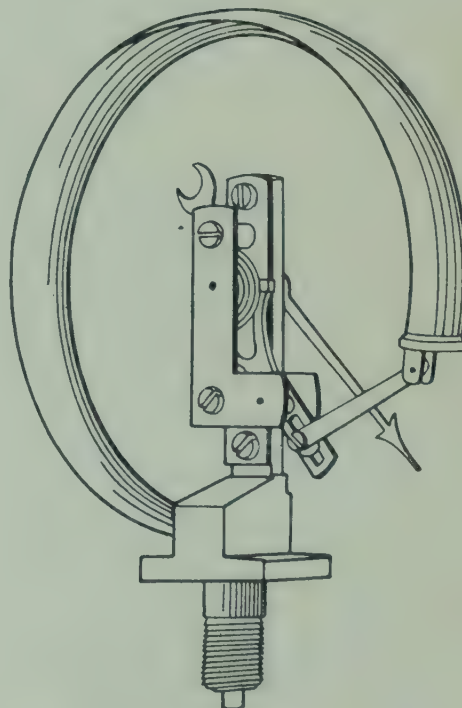
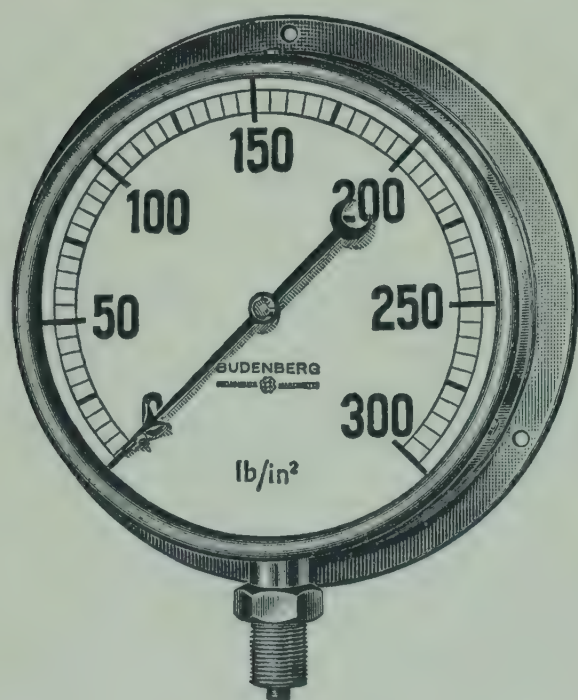


Fig. 4.5. Bourdon gauge
(Courtesy of Budenberg Gauge Co., Manchester)

same. To obtain accurate readings, it is necessary to choose liquids which give sharp interfaces: paraffin oil and industrial alcohol are commonly used.

According to OWER⁽⁸⁾, benzyl alcohol (specific gravity 1.048) and calcium chloride solutions give the most satisfactory results. The difference in density can be varied by altering the concentration of the calcium chloride solution.

(e) *The Bourdon gauge* (Fig. 4.5). The pressure to be measured is applied to a curved tube, oval in cross section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle.

Impact Pressure

The pressure exerted on a plane at right angles to the direction of flow of the fluid consists of two components—

- (a) static pressure, and
- (b) the additional pressure required to bring the fluid to rest at the point.

Consider a fluid flowing between two sections, 1 and 2 (Fig. 4.6), which are sufficiently close for friction losses to be negligible between the two sections; they are a sufficient distance apart, however, for the presence of a small surface at right angles to the direction of flow at section 2 to have negligible effect on the pressure at section 1. These conditions are normally met if the distance between the sections is one pipe diameter.

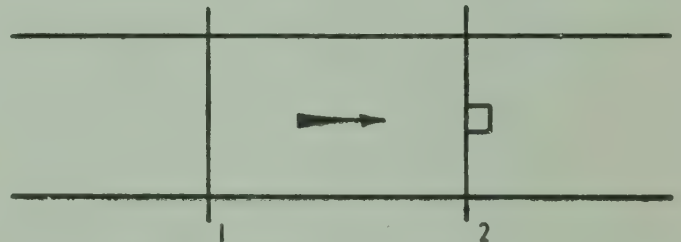


Fig. 4.6. Impact pressure

Now consider a small filament of liquid which is brought to rest at section 2.

Applying the energy balance equation (2.28) between the two sections,

$$\frac{u_1^2}{2} = \frac{u_2^2}{2} + \int_1^2 v \cdot dP$$

If the fluid is incompressible or if the change in the density of the fluid between the sections is negligible, we have (since $u_2 = 0$)

$$\frac{u_1^2}{2} = v(P_2 - P_1) = h_i g$$

$$\text{i.e.} \quad u_1 = \sqrt{2v(P_2 - P_1)} = \sqrt{2gh_i} \quad \dots (4.4)$$

where h_i is the difference between the impact pressure head at section 2 and the static pressure head at section 1.

Little error is introduced if this expression is applied to the flow of a compressible fluid, provided that the velocity is not greater than about 200 ft/sec. When the velocity is high, the equation of state must be used to give the relation between the pressure and the volume of the gas. For non-isothermal flow, we have $P \cdot v^k = \text{a constant}$,

$$\text{and} \quad \int_1^2 v \cdot dP = \frac{k}{k-1} \cdot P_1 v_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \quad \dots (2.41)$$

so that

$$\frac{u_1^2}{2} = \frac{k}{k-1} \cdot P_1 v_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \quad \dots (4.5)$$

$$\therefore P_2 = P_1 \cdot \left\{ 1 + \frac{u_1^2}{2} \cdot \frac{k-1}{k} \cdot \frac{1}{P_1 v_1} \right\}^{\frac{k}{k-1}} \quad \dots (4.6)$$

For isothermal flow,

$$\frac{u_1^2}{2} = P_1 v_1 \ln \frac{P_2}{P_1} \quad (\text{from 2.36})$$

$$\therefore P_2 = P_1 \cdot e^{u_1^2/2P_1v_1} = P_1 \cdot e^{u_1^2 M/2RT} \quad \dots (4.7)$$

Equations 4.6 and 4.7 can be used for the calculation of the fluid velocity and the impact pressure in terms of the static pressure a short distance upstream. The two sections are chosen so that they are sufficiently close together for frictional losses to be negligible. Thus P_1 will be approximately equal to the static pressure at both sections and the equations give the relation between the static and impact pressures, and the velocity, at any point in the fluid.

The temperature of a flowing fluid is normally measured by placing a thermometer in the stream. For a gas flowing at high velocity, the “flowing temperature” is measured by a thermometer moving at the same velocity as the fluid. If the thermometer is at rest it will record a different temperature because the fluid is brought to rest and its pressure is increased from the static pressure to the impact pressure: a corresponding change in temperature takes place and the new value is called the stagnation temperature.

For an ideal gas (where $P \cdot v^k = \text{constant}$),

$$\frac{u_1^2}{2} = \frac{k}{k-1} \cdot P_1 v_1 \cdot \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \quad \dots (4.5)$$

$$= \frac{k}{k-1} \cdot P_1 v_1 \left(\frac{T_2}{T_1} - 1 \right)$$

$$\therefore T_2 = \left\{ 1 + \frac{k-1}{k P_1 v_1} \frac{u_1^2}{2} \right\} T_1 \quad \dots (4.8)$$

This equation gives the stagnation temperature T_2 in terms of the flowing temperature T_1 . If the process takes place under adiabatic conditions, the temperature to which the fluid is raised is known as the total temperature. It is the value of T_2 in equation 4.8 when $k = \gamma$. Thus the total temperature is given by

$$T_2 = \left\{ 1 + \frac{\gamma-1}{\gamma P_1 v_1} \frac{u_1^2}{2} \right\} T_1 \quad \dots (4.9)$$

MEASUREMENT OF FLUID FLOW

The most important class of flowmeter is that in which the fluid is either accelerated or retarded at the measuring section and the change in the kinetic energy is measured by the pressure difference produced. This class includes—

The pitot tube, in which a small element of fluid is brought to rest at an

orifice situated at right angles to the direction of flow. The flow rate is obtained by measuring the difference between the impact and the static pressure.

The orifice meter, in which the fluid is accelerated at a sudden constriction in the flow line. This is a cheap reliable instrument but the overall pressure drop is high because the excess kinetic energy of the fluid is wasted.

The nozzle, in which the fluid is gradually accelerated up to the throat of the instrument. The pressure energy is not recovered and therefore the characteristics are similar to those of the orifice meter.

The venturi meter, in which the fluid is gradually accelerated and then gradually retarded again. A high proportion of the excess kinetic energy is recovered but the instrument is expensive.

The notch or weir, in which the energy of the fluid due to its hydrostatic pressure is converted into kinetic energy.

Various other types of meter such as the anemometer, the hot wire anemometer, and quantity meters will be mentioned.

The Pitot Tube

The pitot tube is used to measure the difference between the impact and static pressures in a fluid. It normally consists of two concentric tubes arranged parallel to the direction of flow; the impact pressure is measured on the open end of the inner tube. The end of the outer concentric tube is sealed and a series of orifices on the curved surface give an accurate indication of the static pressure. The position of these orifices must

be carefully chosen because there are two disturbances which may cause an incorrect reading of the static pressure. They are due to

- (1) the head of the instrument, and
- (2) the portion of the stem which is at right angles to the direction of flow of the fluid.

These two disturbances cause errors in opposite directions, as shown in Fig. 4.7, and the static pressure should therefore be measured at the point where the effects are equal and opposite

If the head and stem are situated at a distance of fourteen diameters from each other (as on the N.P.L. standard instrument), the two disturbances are equal and opposite at a section six diameters from the head and eight from the stem. This is therefore the position at which the static pressure orifices should be located. If the distance between the head and the stem is too great the

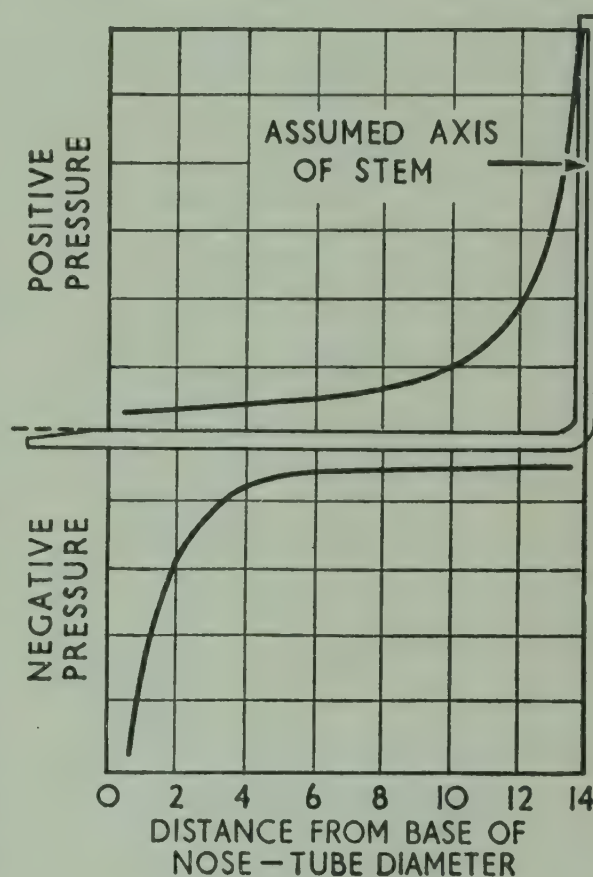


Fig. 4.7. Errors in static pressure due to pitot tube

instrument will be unwieldy; if it is too short the magnitude of each of the disturbances will be relatively great and a small error in the location of the static pressure orifices will appreciably affect the reading.

The two standard N.P.L. instruments are shown; the one with the rounded nose is preferred, since it is less subject to damage (Fig. 4.8 (a) and (b)).

For Reynolds Numbers, of from 500 to 300,000, based on the external diameter of the pitot tube, an error of not more than 1% is obtained with this instrument. A Reynolds Number of 500 with the standard $\frac{5}{16}$ in. pitot tube corresponds to a water velocity of 0.23 ft/sec or an air velocity of 3 ft/sec. Sinusoidal fluctuations in the flow rate up to 20% also do not affect the accuracy by more than 1%. Calibration of the instrument is not necessary.

A very small pressure difference is obtained for low rates of flow of gases and the lower limit of velocity that can be measured is usually set by the minimum

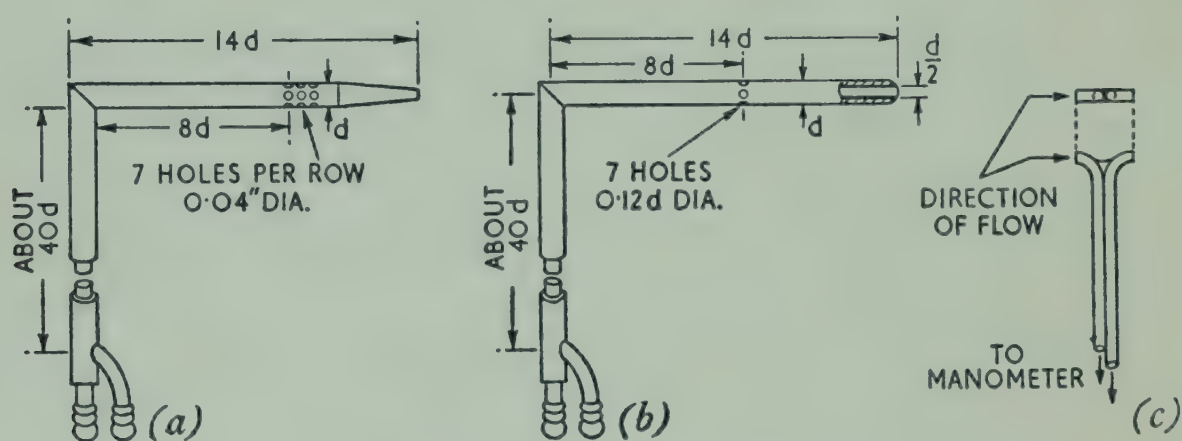


Fig. 4.8 (a), (b), and (c). Pitot tubes

difference in pressure that can be measured. This limitation is serious and various methods have been adopted for increasing the reading of the instrument but they involve the necessity for calibration. One such modified instrument (Fig. 4.8 (c)) incorporates two impact orifices pointing in opposite directions, so that the difference in head which is measured is about twice the velocity head. However, the pressure on the orifice facing downstream does not accurately represent the difference between the static pressure and the pressure due to the velocity head, because the flow is appreciably disturbed.

Correct alignment of the instrument with respect to the direction of flow is important; this is attained when the differential reading is a minimum.

For the flow not to be appreciably disturbed, the diameter of the instrument must not exceed about one-fiftieth of the diameter of the pipe; the standard instrument (diameter $\frac{5}{16}$ in.) should therefore not be used in pipes of less than 16 in. diameter. An accurate measurement of the impact pressure can be obtained using a tube of very small diameter with its open end at right angles to the direction of flow; hypodermic tubing is convenient for this purpose. The static pressure is measured using a single piezometer tube or a piezometer ring, upstream at a distance equal approximately to the diameter of the pipe: measurement should be made at least fifty diameters from any bend or obstruction.

The pitot tube measures the velocity of only a very fine filament of fluid and hence it can be used for exploring the velocity distribution across the pipe section. If, however, it is desired to measure the total flow of fluid through the pipe, the velocity must be measured at various distances from the walls and the results integrated. The total rate of flow can be calculated from a single reading only if the velocity distribution across the section is already known.

The Orifice Meter, Nozzle, and Venturi Meter

In each of these three measuring devices the fluid is accelerated by causing it to flow through a constriction; the kinetic energy is thereby increased and the corresponding change in the pressure energy is measured. The flow rate is obtained in terms of the pressure difference, by the application of the material balance and energy balance equations. The pressure of the fluid is measured a short distance upstream from the constriction where the flow is undisturbed (section 1) and at the position where the area of flow is a minimum (section 2). Application of the energy and material balance equations gives

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} + g(z_2 - z_1) + \int_1^2 v \cdot dP + W + F = 0 \quad \dots(2.32)$$

and
$$G = \frac{u_1 A_1}{v_1} = \frac{u_2 A_2}{v_2} \quad \dots(2.29b)$$

If the frictional losses are neglected, and the fluid does no work on the surroundings,

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} = g(z_1 - z_2) - \int_1^2 v \cdot dP \quad \dots(4.10)$$

Inserting the value of u_1 in terms of u_2 from equation 2.29b in equation 4.10 gives the value of u_2 . G is then calculated from equation 2.29b.

Incompressible Fluid

$$\int_1^2 v \cdot dP = v(P_2 - P_1) \quad \dots(2.33)$$

and
$$u_1 = u_2 \cdot \frac{A_2}{A_1} \text{ (from 2.29b)}$$

Substituting these values in equation 4.10,

$$\begin{aligned} \frac{u_2^2}{2\alpha_2} \left\{ 1 - \frac{\alpha_2}{\alpha_1} \frac{A_2^2}{A_1^2} \right\} &= g(z_1 - z_2) + v(P_1 - P_2) \\ \therefore G &= \frac{u_2 A_2}{v} \\ &= \frac{A_2}{v} \sqrt{\frac{2\alpha_2 \{ z_1 - z_2 \} g + v(P_1 - P_2)}{1 - \frac{\alpha_2}{\alpha_1} \left(\frac{A_2}{A_1} \right)^2}} \quad \dots(4.11) \end{aligned}$$

Ideal Gas—Isothermal Flow

$$\int_1^2 v \cdot dP = P_1 v_1 \ln \frac{P_2}{P_1} \quad \dots(2.36)$$

and
$$u_1 = u_2 \frac{A_2 v_1}{A_1 v_2} \text{ (from 2.29b)}$$

Substituting in equation 4.10,

$$\begin{aligned} \frac{u_2^2}{2\alpha_2} \left\{ 1 - \frac{\alpha_2}{\alpha_1} \left(\frac{v_1 A_2}{v_2 A_1} \right)^2 \right\} &= g(z_1 - z_2) + P_1 v_1 \ln \frac{P_1}{P_2} \\ \therefore G &= \frac{u_2 A_2}{v_2} \\ &= \frac{A_2}{v_2} \sqrt{\frac{2\alpha_2 \left\{ g(z_1 - z_2) + P_1 v_1 \ln \frac{P_1}{P_2} \right\}}{1 - \frac{\alpha_2}{\alpha_1} \left(\frac{v_1 A_2}{v_2 A_1} \right)^2}} \quad \dots(4.12) \end{aligned}$$

Ideal Gas—Non-isothermal Flow

If the pressure and the specific volume of the gas are connected by the relation, $P \cdot v^k = \text{constant}$,

$$\int_1^2 v \cdot dP = P_1 v_1 \frac{k}{k-1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \quad \dots(2.41)$$

Substituting in equation 4.10,

$$\begin{aligned} \frac{u_2^2}{2\alpha_2} \left\{ 1 - \frac{\alpha_2}{\alpha_1} \left(\frac{v_1 A_2}{v_2 A_1} \right)^2 \right\} &= g(z_1 - z_2) - P_1 v_1 \frac{k}{k-1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} - 1 \right\} \\ \therefore G &= \frac{u_2 A_2}{v_2} \\ &= \frac{A_2}{v_2} \sqrt{\frac{2\alpha_2 \left\{ g(z_1 - z_2) + P_1 v_1 \frac{k}{k-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right] \right\}}{1 - \left(\frac{v_1 A_2}{v_2 A_1} \right)^2 \frac{\alpha_2}{\alpha_1}}} \quad \dots(4.13) \end{aligned}$$

Non-ideal Gas

We have $q = \frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} + g(z_2 - z_1) + H_2 - H_1 \text{ (from 2.32a)}$

$$\therefore \frac{u_2^2}{2\alpha_2} \left\{ 1 - \frac{\alpha_2}{\alpha_1} \left(\frac{v_1 A_2}{v_2 A_1} \right)^2 \right\} = g(z_1 - z_2) + H_1 - H_2 + q \quad \dots(4.14)$$

The rate of flow is therefore obtained in terms of the change in enthalpy and the heat flow into the system. Equation 4.14 is also applicable to an ideal gas.

The Orifice Meter. The most important factors influencing the reading of an orifice meter are the size of the orifice and the diameter of the pipe in which it is situated. A number of other factors do, however, affect the reading to some extent. Thus, the exact position and the method of fixing the pressure tapping is of importance because the area of flow, and therefore the velocity of the fluid, gradually changes in the region of the orifice. The upstream tapping should be situated at about one pipe diameter and the downstream tapping at about 0.5 diameter from the orifice. The former will then correspond with the pipe running full and the latter with the vena contracta, the section where the area of flow is a minimum. The flow rate for a given pressure differential will be rather less than that given by equations 4.11, 4.12, 4.13 because of friction losses. These will be

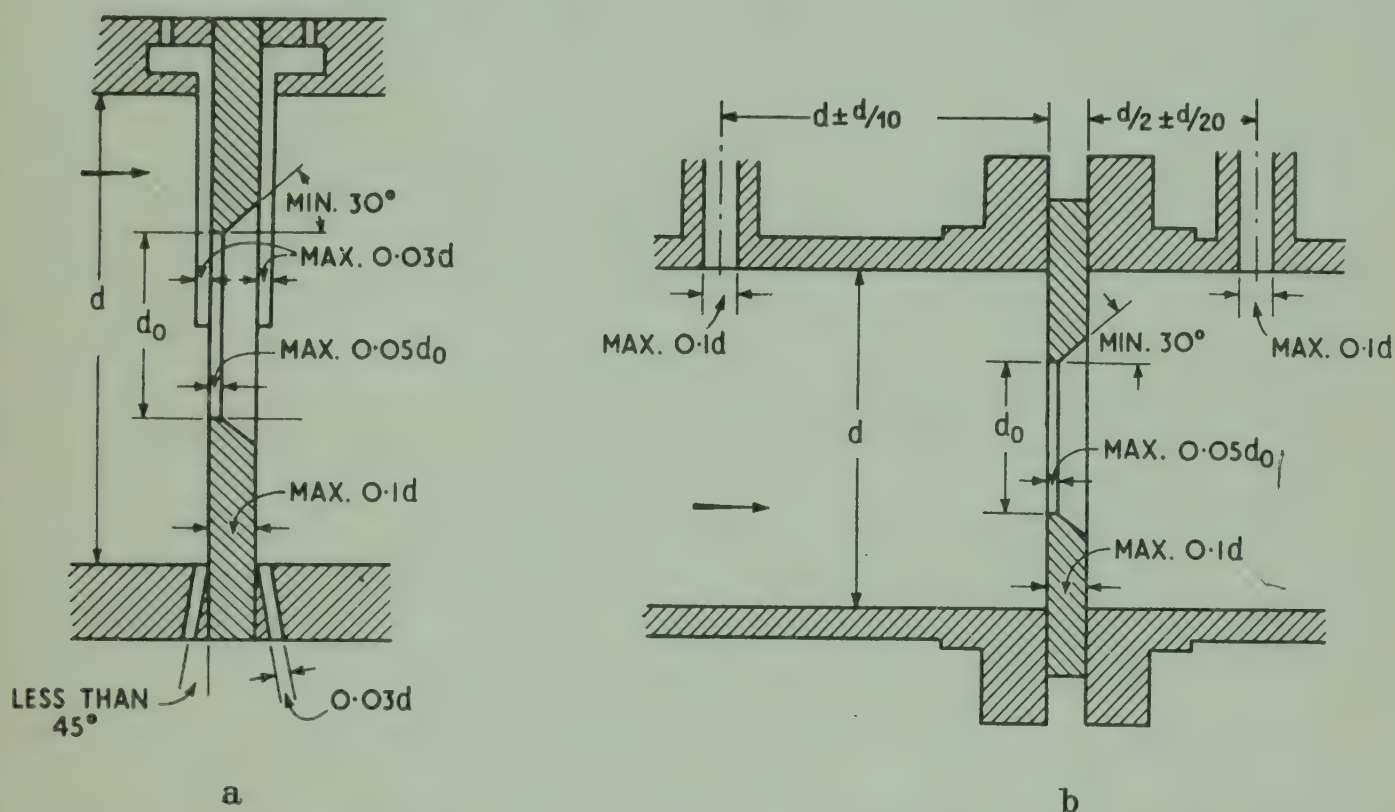


Fig. 4.9. (a) Orifice plate with corner tapplings. Upper half shows construction with piezometer ring. Lower half shows construction with tapplings into pipe flange. (b) Orifice plate with d and $d/2$ tapplings. Nipples must finish flush with wall of pipe without burrs

allowed for in a coefficient of discharge C_D , which will depend on the Reynolds Number, the roughness of the pipe walls, the exact shape of the orifice and the thickness of the orifice plate. The reading will also be influenced if the orifice is situated near a bend or obstruction in the pipe; the meter should be located not less than fifty pipe diameters from any such obstruction. The British Standard 1042⁽³²⁾ gives details of the construction of the standard orifice (Fig. 4.9). Any meter which is not made in accordance with this, or some other, standard specification must be calibrated against a standard instrument.

A simple instrument can be made by inserting a drilled orifice plate between two pipe flanges and arranging suitable pressure connections. The orifice plate should be carefully drilled so that the edges are sharp: a stainless steel is commonly used because it is resistant to corrosion and to the action of abrasive materials. The size of the orifice should be chosen to give a convenient pressure drop. As the flow rate is approximately proportional to the square root of the

pressure drop, accurate readings are obtained, but it is difficult to cover a wide range of flows with any given size of orifice. The effective range can be increased, however, by changing the orifice plate. Unlike the pitot tube, the orifice meter immediately gives the flow rate from a single reading.

The most serious disadvantage of the meter is that most of the pressure drop is not recoverable (i.e. it is inefficient). The velocity of the fluid is increased at the throat without much loss of energy. The fluid is subsequently retarded as it mixes with the relatively slowly moving fluid in the section of pipe ahead of the orifice. A high degree of turbulence is set up and most of the excess kinetic energy is dissipated as heat. Usually only about 5 or 10% of the excess kinetic energy can be recovered as pressure energy. The pressure drop over the orifice meter is therefore high and this may preclude it from being used in a particular instance.

The area of flow decreases from A_1 at section 1 to A_0 at the orifice and then to A_2 at the vena contracta (Fig. 4.10). The area at the vena contracta can be

conveniently related to the area of the orifice by the coefficient of contraction C_c , defined by the relation,

$$C_c = \frac{A_2}{A_0}$$

Inserting the value $A_2 = C_c A_0$ in equation 4.11, we have for an incompressible fluid,

$$G = \frac{C_c A_0}{v} \sqrt{\frac{2\alpha_2 \{(z_1 - z_2)g + v(P_1 - P_2)\}}{1 - \frac{\alpha_2}{\alpha_1} \left(\frac{C_c A_0}{A_1}\right)^2}} \quad \dots (4.15)$$

Using a coefficient of discharge C_D to take account of frictional losses in the orifice meter and of the values of C_c and α_1 and α_2 ,

$$G = \frac{C_D A_0}{v} \sqrt{\frac{2\{(z_1 - z_2)g + v(P_1 - P_2)\}}{1 - \left(\frac{A_0}{A_1}\right)^2}} \quad \dots (4.16)$$

It is seen that C_D is not a simple function and cannot be calculated. Values are plotted as a function of $Re.$ and $\frac{A_0}{A_1}$ in Fig. 4.11.

For a horizontal meter in which the orifice area is small compared with the area of the pipe,

$$z_1 = z_2 \text{ and } \left\{1 - \left(\frac{A_0}{A_1}\right)^2\right\}^{\frac{1}{2}} \rightarrow 1$$

$$\therefore G = \frac{C_D A_0}{v} \sqrt{2v(P_1 - P_2)} \quad \dots (4.16a)$$

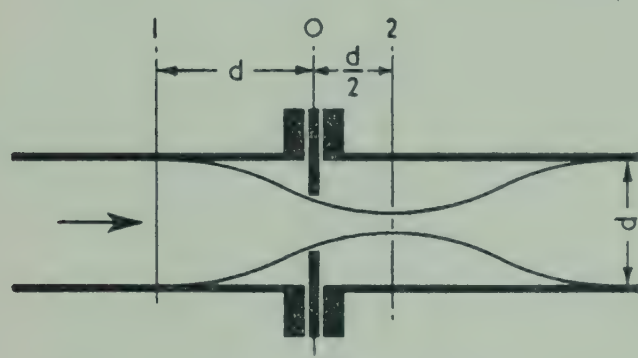


Fig. 4.10. Flow through orifice meter

i.e. $G = C_D \cdot A_0 \cdot \rho \sqrt{2gh_0} \dots (4.16b)$

where h_0 is the difference in head across the orifice, expressed in terms of the fluid in question.

For the isothermal flow of an ideal gas,

$$G = \frac{C_c A_0}{v_2} \sqrt{\frac{2\alpha_2 \left\{ g(z_1 - z_2) + P_1 v_1 \ln \frac{P_1}{P_2} \right\}}{1 - \frac{\alpha_2}{\alpha_1} \left(\frac{C_c v_1 A_0}{v_2 A_1} \right)^2}} \quad \text{(from 4.12)} \dots (4.17)$$

Again using a coefficient of discharge to account for frictional losses, we can write

$$G = \frac{C_D A_0}{v_2} \sqrt{\frac{2 \left\{ g(z_1 - z_2) + P_1 v_1 \ln \frac{P_1}{P_2} \right\}}{1 - \left(\frac{v_1 A_0}{v_2 A_1} \right)^2}} \dots (4.18)$$

For a horizontal meter in which the area of the orifice is small compared with that of the pipe,

$$G = \frac{C_D A_0}{v_2} \sqrt{2 P_1 v_1 \ln \frac{P_1}{P_2}} \dots (4.18a)$$

$$= C_D A_0 \sqrt{2 \frac{P_2}{v_2} \ln \frac{P_1}{P_2}}$$

$$= C_D A_0 \sqrt{2 \frac{P_2}{v_2} \ln \left(1 - \frac{\Delta P}{P_2} \right)} \quad \text{(where } \Delta P = P_2 - P_1 \text{)}$$

$$= C_D A_0 \sqrt{2 \cdot \frac{-\Delta P}{v_2}}, \text{ if } \Delta P \text{ is small compared with } P_2 \dots (4.18b)$$

For non-isothermal flow of an ideal gas,

$$G = \frac{A_0 C_c}{v_2} \sqrt{\frac{2\alpha_2 \left\{ g(z_1 - z_2) + P_1 v_1 \frac{k}{k-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right] \right\}}{1 - \left(\frac{C_c v_1 A_0}{v_2 A_1} \right)^2 \frac{\alpha_2}{\alpha_1}}} \quad \text{(from 4.13)} \dots (4.19)$$

Allowing for frictional losses by means of a coefficient of discharge,

$$G = \frac{A_0 C_D}{v_2} \sqrt{\frac{2 \left\{ g(z_1 - z_2) + P_1 v_1 \frac{k}{k-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right] \right\}}{1 - \left(\frac{v_1 A_0}{v_2 A_1} \right)^2}} \dots (4.20)$$

For a horizontal orifice in which $\frac{A_0}{A_1}$ is small,

$$G = \frac{A_0 C_D}{v_2} \sqrt{2P_1 v_1 \frac{k}{k-1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right\}} \quad \dots (4.20a)$$

$$\begin{aligned} &= \frac{A_0 C_D}{v_2} \sqrt{2P_1 v_1 \frac{k}{k-1} \left\{ 1 - \left(1 + \frac{\Delta P}{P_1} \right)^{(k-1)/k} \right\}} \\ &= A_0 C_D \sqrt{2 \frac{-\Delta P}{v_2}} \quad \dots (4.20b) \end{aligned}$$

if ΔP is small compared with P_2 .

From equation 4.20a,

$$G = A_0 C_D \cdot \left(\frac{P_2}{P_1} \right)^{1/k} \sqrt{2P_1 \cdot \frac{1}{v_1} \frac{k}{k-1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{(k-1)/k} \right\}}$$

$$\begin{aligned} \therefore G^2 &= A_0^2 \cdot C_D^2 \cdot 2P_1 \cdot \frac{1}{v_1} \frac{k}{k-1} \left(w^{\frac{2}{k}} - w^{\frac{k+1}{k}} \right) \\ &= k' \left(w^{\frac{2}{k}} - w^{\frac{k+1}{k}} \right) \quad \dots (4.21) \end{aligned}$$

where $w = \frac{P_2}{P_1}$ and k' is independent of P_2 .

High Pressure Ratios

It is clear that, for a given upstream pressure P_1 , the flow is zero when $w = 0$ or when $w = 1$. At some intermediate value of w the discharge is a maximum; this occurs when $\frac{dG}{dw} = 0$.

Differentiating equation 4.21 with respect to w ,

$$2G \frac{dG}{dw} = k' \left\{ \frac{2}{k} w^{\frac{2}{k}-1} - \frac{k+1}{k} w^{\frac{1}{k}} \right\}$$

When $\frac{dG}{dw} = 0$,

$$2w^{\frac{2}{k}-1} = (k+1)w^{\frac{1}{k}}$$

$$\begin{aligned} \text{i.e.} \quad w &= \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad \dots (4.22) \\ &= w_c \text{ (say)} \end{aligned}$$

w_c is known as the critical pressure ratio.

From equation 4.13, we have that the theoretical velocity u_2 at the vena contracta for turbulent flow ($\alpha = 1$) in a horizontal orifice in which A_2 is small compared with A_1 is given by

$$u_2^2 = 2P_1 v_1 \frac{k}{k-1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right\} \quad \dots (4.23)$$

Inserting the critical value of $\frac{P_2}{P_1}$ from equation 4.22,

$$\begin{aligned}
 u_2^2 &= 2P_1v_1 \frac{k}{k-1} \left(1 - \frac{2}{k+1} \right) \\
 &= \frac{2k}{k+1} P_1v_1 \quad \dots (4.24) \\
 &= \frac{2k}{k+1} \cdot \frac{P_2}{w_c} v_2 \cdot w_c^{1/k} \\
 \text{i.e. } u_2 &= \sqrt{kP_2v_2}
 \end{aligned}$$

This has already been shown (equation 3.62a) to be the velocity of propagation of a pressure wave.

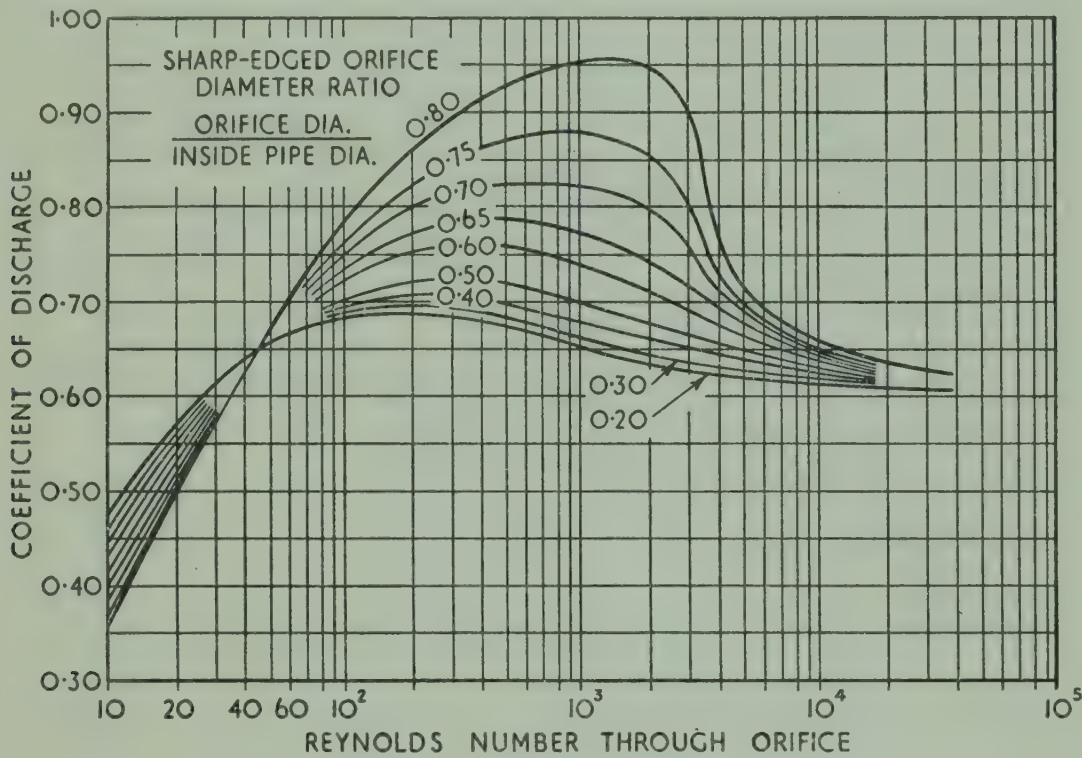


Fig. 4.11. Coefficient for orifice meter

The maximum discharge is given by

$$\begin{aligned}
 G &= \frac{C_D A_0}{v_2} u_2 \\
 &= C_D A_0 \sqrt{\frac{2k}{k+1} P_1 v_1 \frac{1}{v_1^2} \cdot \left(\frac{2}{k+1} \right)^{2/(k-1)}} \quad (\text{from equations 4.22 and 4.24}) \\
 &= C_D A_0 \sqrt{\frac{kP_1}{v_1} \left(\frac{2}{k+1} \right)^{(k+1)/(k-1)}} \quad \dots (4.25)
 \end{aligned}$$

It should be noted that frictional losses have been neglected in the calculation of the theoretical velocity at the vena contracta. For an isentropic process, we can put $k = \gamma$ in equation 4.22; for air and steam ($\gamma = 1.4$), the critical ratio is then about 0.5. Thus, if the downstream pressure is reduced below about one-half of the upstream pressure, the rate of flow is given by equation 4.25 and is independent of the downstream pressure P_2 . The pressure never falls below $w_c P_1$ at the vena contracta; it falls from $w_c P_1$ to P_2 beyond the vena contracta.

The coefficient of discharge C_D for the orifice meter is a function of the Reynolds Number at the orifice and the ratio of the orifice area to the pipe area; the variation can be seen from Fig. 4.11. At low Reynolds Numbers the

coefficient increases steeply with increase in velocity; it passes through a maximum value ranging between 0.7 and 0.9 and then falls to about 0.62 at Reynolds Numbers greater than about 10^4 . For high Reynolds Numbers the coefficient for the orifice meter is very nearly constant.

Example. Water flows through an orifice of 1 in. diameter situated in a 3 in. pipe, at the rate of 4 gal/min. What will be the difference in level on a water manometer connected across the meter? (Take the viscosity of water as 1 centipoise.)

Solution. Velocity of liquid through the orifice

$$= \frac{10 \times 4}{60} \times 454 \times \frac{1}{\frac{1}{4}\pi \cdot 2.54^2}$$

$$= 59.8 \text{ cm/sec}$$

$$\text{Reynolds Number at orifice} = \frac{59.8 \times 2.54 \times 1}{0.01}$$

$$= 15200$$

From Fig. 4.11, corresponding value of $C_D = 0.67$ (diameter ratio 0.33)

$$\left\{1 - \left(\frac{A_0}{A_1}\right)^2\right\}^{\frac{1}{2}} = \left(1 - \frac{1}{81}\right)^{\frac{1}{2}} = 0.997 \approx 1$$

Equation 4.16b can therefore be applied.

$$\text{i.e.} \quad \frac{40}{60} \text{ lb/sec} = G = 0.67 \times \frac{\pi}{4} \times \frac{1}{144} \times 62.3 \sqrt{2 \times 32.2 \times h_0}$$

$$\text{Hence} \quad \sqrt{h_0} = 0.365 \text{ ft of water}$$

$$h_0 = 0.133 \text{ ft of water}$$

$$= \underline{\underline{1.6 \text{ in. of water}}}$$

Example. A cylinder contains gas at a pressure of 60 atmospheres gauge and discharges to the atmosphere through a valve which may be taken as equivalent to a sharp edged orifice of $\frac{1}{4}$ in. diameter (coefficient of discharge = 0.6). Plot the rate of discharge of air from the cylinder against cylinder pressure. Assume that the expansion at the valve is approximately isentropic and that the temperature of the gas in the cylinder remains unchanged at 0°C .

Solution. Critical pressure ratio for discharge through the valve $= \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$. γ varies from 1.40 to 1.54 over the pressure range of 1 to 60 atmospheres. Critical conditions occur at relatively low cylinder pressure. Taking $\gamma = 1.40$,

$$\text{critical ratio} = \left(\frac{2}{2.40}\right)^{1.4/0.4} = 0.834^{3.5} = 0.53$$

Sonic velocity will occur until the pressure in the cylinder falls to $\frac{1}{0.53} = 1.89$ ats. absolute = 0.89 ats. gauge.

Rate of discharge for cylinder pressures greater than 0.89 ats. gauge is given by equation 4.25.

$$G = C_D A_0 \sqrt{k \frac{P_1}{v_1} \left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

$$= 0.6 \frac{\pi}{4} \left(\frac{1}{48}\right)^2 \sqrt{1.47 \frac{P_1}{v_1} \cdot 0.810^{5.26}} \quad \text{for isentropic conditions, taking mean value of } \gamma \text{ as } 1.47$$

$$= 0.000205 \times 0.693 \sqrt{\frac{P_1}{v_1}}$$

$$= 0.000143 \left(\frac{P_1}{v_1}\right)^{\frac{1}{2}}$$

Now if P_a and v_a are atmospheric pressure and the specific volume of air at that pressure,

$$\frac{1}{v_1} = \frac{1}{v_a} \frac{P_1}{P_a}$$

But $v_a = \frac{359}{28.5} = 12.6$ cu ft/lb, $P_a = 14.7 \times 144 \times 32.2 = 6.8 \times 10^4$ poundals/sq ft,

$$G = 1.544 \times 10^{-7} P_1 \text{ lb/sec}$$

$$= 0.0105 P_{1a} \text{ lb/sec}$$

where P_{1a} is pressure, P_1 , expressed in atmospheres.

When the pressure in the cylinder is less than 0.89 ats. gauge, the rate of flow is given by equation 4.20a.

Hence

$$G^2 = \left(\frac{A_0 C_D}{v_2} \right)^2 2P_1 v_1 \frac{k}{k-1} \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \right\}$$

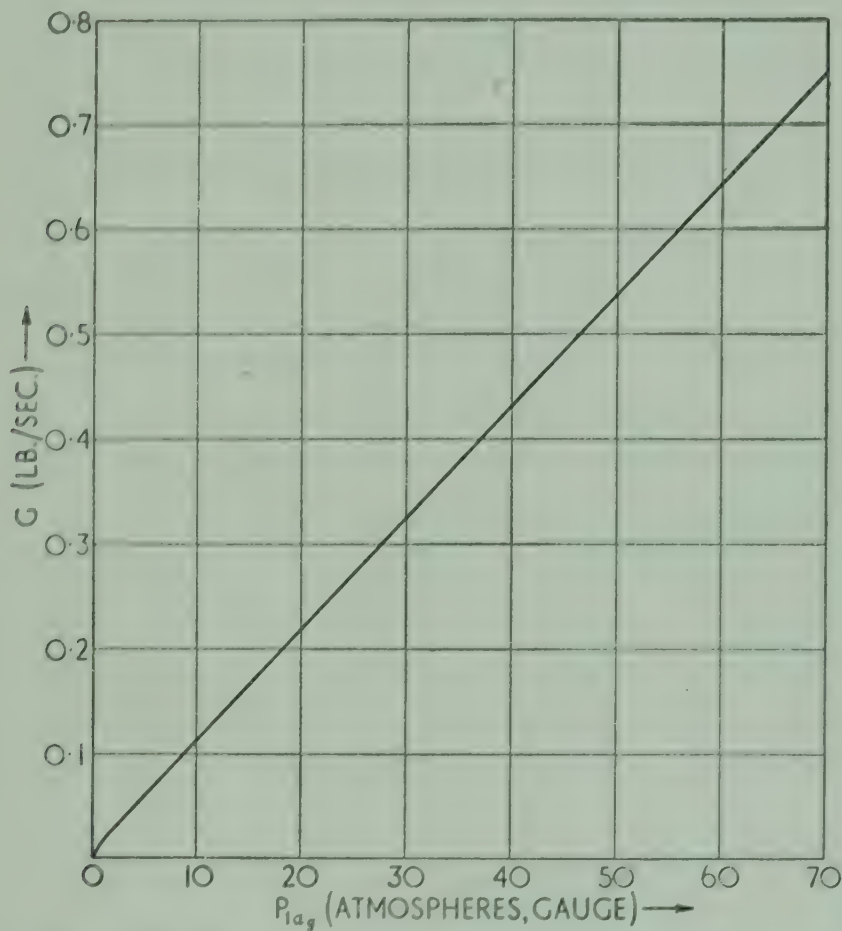


Fig. 4.12. Rate of discharge (G) vs. cylinder pressure

For isentropic flow through the valve, $k = \gamma$, and putting $\gamma = 1.40$,

$$\frac{P_1 v_1}{v_2^2} = \frac{P_1 v_1}{v_1^2} \cdot \left(\frac{P_2}{P_1} \right)^{2/\gamma} = \frac{P_1^2}{v_a P_a} \left(\frac{P_a}{P_1} \right)^{2/\gamma} = P_1^{2 - \frac{2}{\gamma}} \cdot P_a^{\frac{2}{\gamma} - 1} v_a^{-1}$$

Since $P_2 = P_a$ (atmospheric pressure).

Thus

$$G^2 = (A_0 C_D)^2 \cdot \frac{1}{v_a} P_a^{\frac{2}{\gamma} - 1} \frac{2\gamma}{\gamma - 1} \left\{ 1 - \left(\frac{P_a}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right\} P_1^{2 - \frac{2}{\gamma}}$$

$$= 0.000205^2 \cdot \frac{1}{12.6} (6.8 \times 10^4)^{0.43} \times \frac{2.80}{0.40} \left\{ 1 - P_{1a}^{-0.29} \right\} P_1^{0.58}$$

$$= 2.4 \times 10^{-6} \left\{ 1 - P_{1a}^{-0.29} \right\} P_{1a}^{0.58} \times (6.8 \times 10^4)^{0.58}$$

$$G = 0.04 P_{1a}^{0.29} \left\{ 1 - P_{1a}^{-0.29} \right\}^{\frac{1}{2}} \text{ lb/sec}$$

The Nozzle. Flow rates can be measured by means of the nozzle. This is similar to the orifice meter but has a converging tube in place of the orifice plate, as shown in Fig. 4.13. The velocity of the fluid is gradually increased and the contours are so designed that almost frictionless flow takes place in the converging portion; the outlet corresponds to the vena contracta on the orifice meter. The nozzle has a constant high coefficient of discharge (c. 0.99) over a

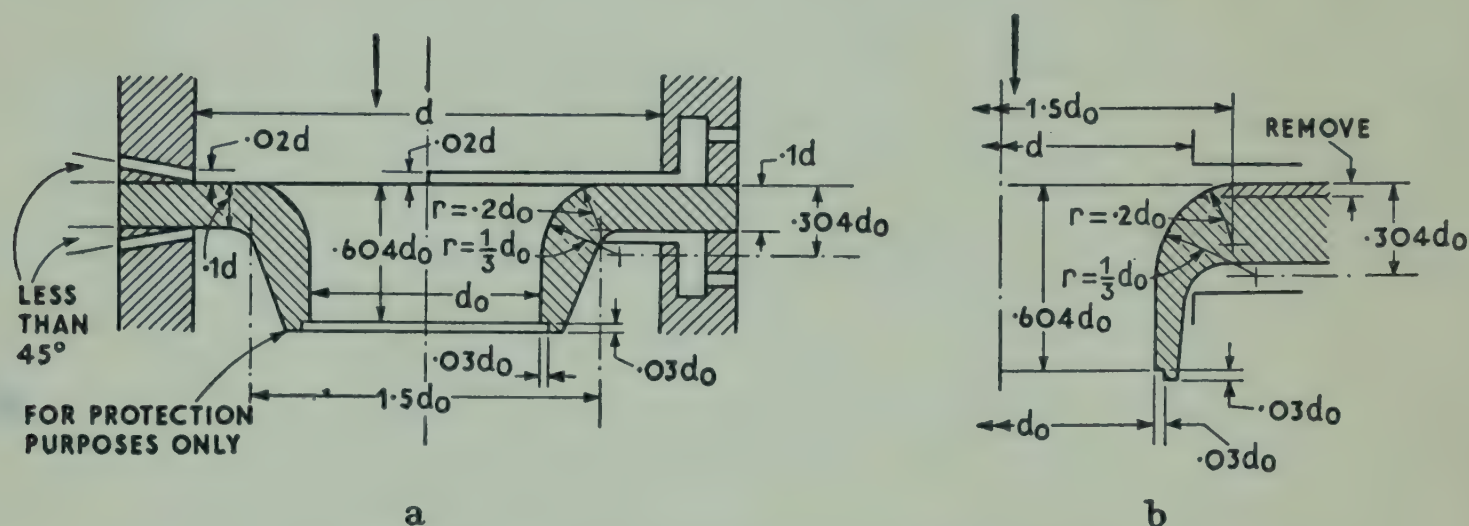


Fig. 4.13. (a) Standard nozzle $\frac{A_0}{A_1}$ is less than 0.45. Left half shows construction for corner tapplings. Right half, construction for piezometer ring. (b) Standard nozzle where $\frac{A_0}{A_1}$ is greater than 0.45

wide range of conditions because the coefficient of contraction is unity; it is, however, very much more costly than the orifice meter. It is extensively used for metering steam. When the ratio of the pressure at the nozzle exit to the upstream pressure is less than the critical pressure ratio w_c , the flow rate is independent of the downstream pressure and can be calculated from the upstream pressure alone. The simple nozzle is not usually fitted with a diverging section and therefore frictional losses are high. It is often used for calibrating other instruments.

The Venturi Meter. (Fig. 4.14). In the venturi meter, the fluid is accelerated by its passage through a converging cone of angle about 20° . The pressure difference between the upstream end of the cone (section 1) and the throat (section 2) is



Fig. 4.14. The venturi meter

measured. The fluid is then retarded in a cone of small angle (not more than 7°) and a large proportion of the excess kinetic energy is converted back into pressure energy. The total frictional loss over the meter can be determined by measuring the pressure at the downstream end of the diverging cone (section 3). The exact dimensions of the standard instrument are given in B.S. 1042⁽³²⁾.

The equations which have been derived for the discharge of fluid through the orifice meter can be applied to the venturi meter but, because of the gradual

reduction of the area of flow, there is no vena contracta and the flow area is a minimum at the throat: that is to say the coefficient of contraction is unity. The coefficient of discharge, C_D , therefore has a correspondingly high value. For turbulent flow it will vary between about 0.90 and 0.99, with 0.98 as a typical average value, but under streamline conditions very much lower coefficients are obtained because the correction factor for the kinetic energy, α , is then appreciably less than one. The venturi meter can be used for both compressible and incompressible fluids and the rates of flow can be calculated from equations 4.16, 4.18, and 4.20.

For an incompressible fluid, for example,

$$G = C_D \cdot A_2 \cdot \rho \sqrt{\frac{2\{(P_1 - P_2)v + g(z_1 - z_2)\}}{\left\{1 - \left(\frac{A_2}{A_1}\right)^2\right\}}} \text{ from (4.16)} \quad \dots(4.26a)$$

$$= C_D \cdot \rho \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2(P_1 - P_2) \cdot v} \text{ (for a horizontal meter)} \quad \dots(4.26b)$$

$$= C_D \cdot \rho \cdot C' \sqrt{2gh_v} \quad \dots(4.26c)$$

where C' is a constant for the meter and

h_v is the loss in head over the converging cone.

The head lost due to friction will now be calculated in terms of the discharge coefficient. If there were no losses, the coefficient of discharge of the meter would be unity and for a flow rate G the loss in head would be $h_v - h_F$ (say), where h_F is the head lost due to friction. We could then write

$$G = \rho \cdot C' \sqrt{2g(h_v - h_F)} \quad \dots(4.27)$$

Dividing equations 4.27 and 4.26c, and squaring,

$$1 - \frac{h_F}{h_v} = C_D^2$$

$$\text{i.e.} \quad h_F = h_v(1 - C_D^2) \quad \dots(4.28)$$

Now suppose that the head recovered over the diverging cone is h_v' and the coefficient of discharge for the diverging cone is C_D' , then the rate of flow G is given by the relation

$$G = C_D' \cdot \rho \cdot C' \sqrt{2gh_v'} \quad \dots(4.29)$$

If the whole of the excess kinetic energy is recovered as pressure energy, the coefficient of discharge C_D' will be unity and the mass rate of flow G will be obtained with a recovery of head equal to h_v' plus some quantity, h_F' .

$$\text{Thus} \quad G = \rho \cdot C' \sqrt{2g(h_v' + h_F')} \quad \dots(4.30)$$

Equating the values of G given by equations 4.30 and 4.29 and squaring,

$$C_D'^2 = 1 + \frac{h_F'}{h_v'}$$

$$h_F' = h_v'(C_D'^2 - 1) \quad \dots(4.31)$$

The coefficient for the diverging portion of the meter will be greater than one.

The total loss of head over the meter $= h_F + h_{F'}$.

This should not exceed 10 to 15% of the difference in head over the converging portion (h_v).

The pressure difference between the throat and the upstream section of the venturi meter is conveniently measured by means of a simple differential manometer.

The difference in level between the two measuring sections is $z_2 - z_1$. The vertical distance between the upstream measuring section and the meniscus of

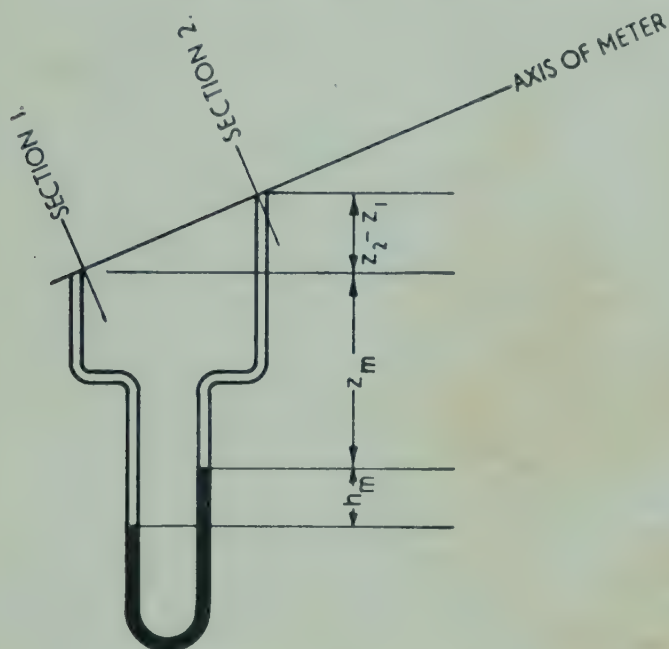


Fig. 4.15. Venturi meter with axis inclined to horizontal

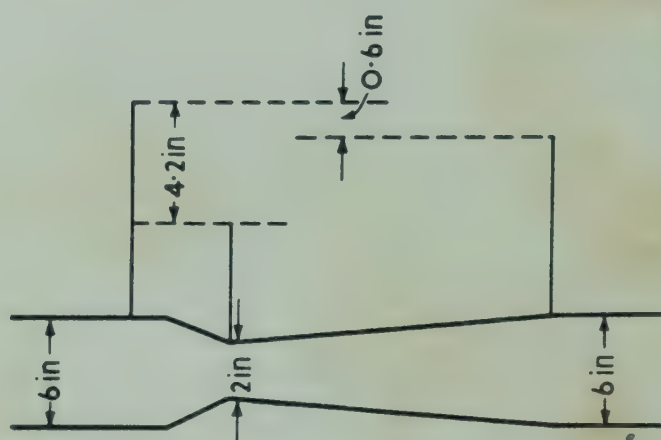


Fig. 4.16. Pressure drops over venturi meter

the manometer fluid (density ρ') in the second limb will be taken as z_m , as shown in Fig. 4.15. For a difference in reading in the manometer of h_m , we have

$$P_1 + \rho(z_m + h_m) \cdot g = P_2 + (z_2 - z_1) \cdot \rho \cdot g + z_m \cdot \rho \cdot g + h_m \cdot \rho' \cdot g$$

$$\therefore (P_1 - P_2) \cdot \frac{1}{\rho} + (z_1 - z_2)g = \frac{1}{\rho} \cdot h_m(\rho' - \rho)g \quad \dots (4.32)$$

The mass rate of flow through the venturi meter is therefore given by substituting from equation 4.32 into equation 4.26a,

$$\text{i.e.} \quad G = C_D \cdot A_2 \cdot \rho \sqrt{\frac{2h_m(\rho' - \rho)g}{\rho' \{1 - (A_2/A_1)^2\}}}$$

The manometer reading is therefore independent of the slope of the axis of the venturi meter for a given rate of flow.

Example. The rate of flow of water in a 6 in. diameter pipe is measured with a venturi meter with a 2 in. diameter throat. When the pressure drop over the converging section is 4.2 in. of water, the flow rate is 6.4 lb/sec. What is the coefficient for the converging cone of the meter at that flow rate, and what is the head lost due to friction? If the total loss of head over the meter is 0.6 in. water, what is the coefficient for the diverging cone?

Solution. Mass rate of flow $G = C_D \cdot \rho \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gh_v}$ (from equation 4.26b)

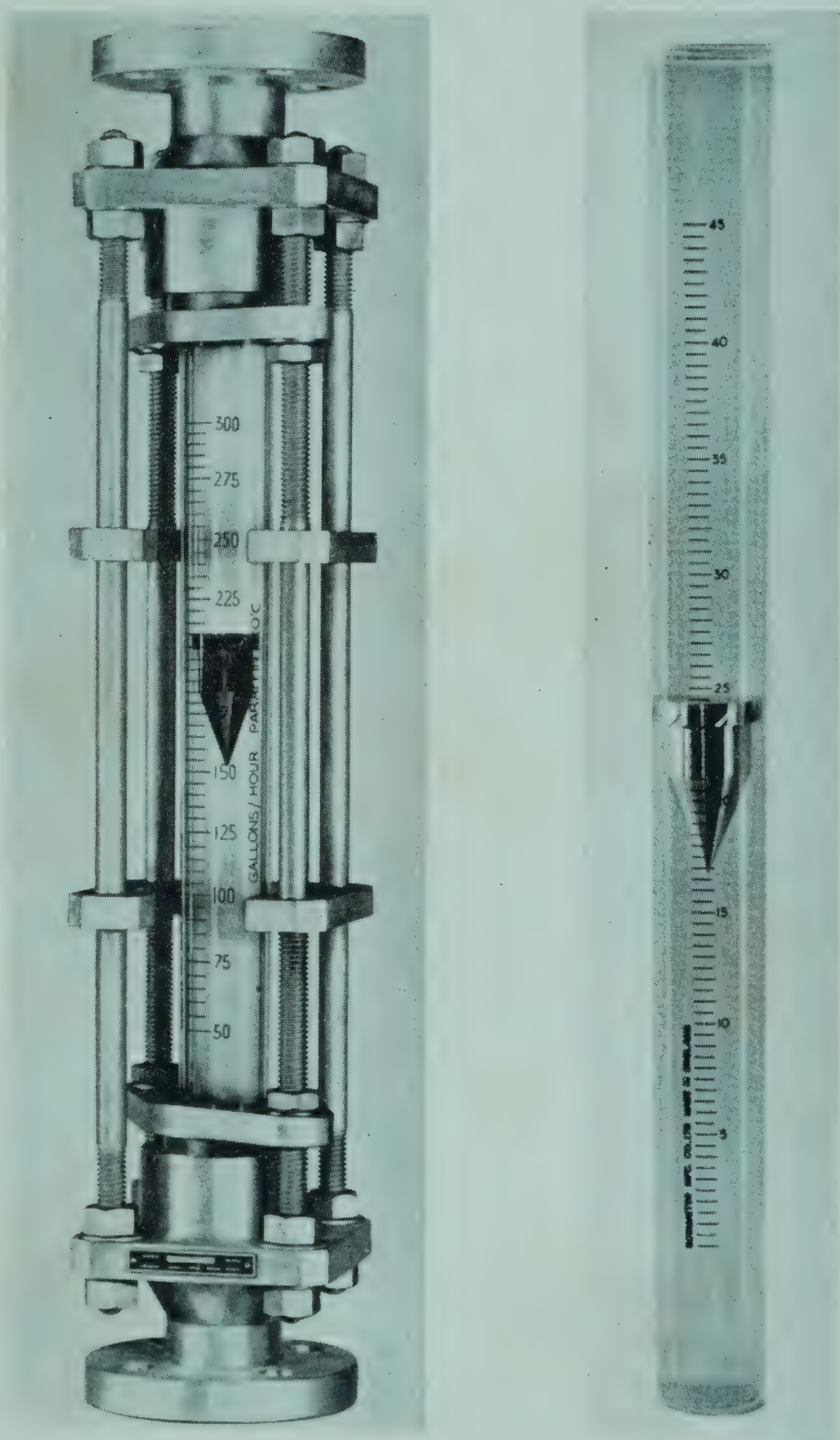


Fig. 4.17. Rotameter
(Courtesy of Rotameter Manufacturing Co. Ltd.)

The coefficient for the meter is therefore given by,

$$6.4 = C_D \cdot 62.5 \cdot \frac{(\frac{1}{4}\pi)^2(\frac{1}{2})^2(\frac{1}{6})^2}{\frac{1}{4}\pi\sqrt{(\frac{1}{2})^4 - (\frac{1}{6})^4}} \sqrt{64.4 \times \frac{4.2}{12}}$$

$$= C_D \cdot 62.5 \times 0.0220 \times 4.75$$

$$\therefore \quad \underline{\underline{C_D = 0.980}}$$

$$\begin{aligned} \text{Head lost due to friction, } h_F &= h_v(1 - C_D^2) \\ &= 4.2 \times 1.980 \times 0.020 \\ &= 0.17 \text{ in. water} \end{aligned}$$

Head lost due to friction over diverging cone = $0.60 - 0.17 = 0.43$ in. water.

Coefficient C_D' of diverging cone is given by,

$$6.4 = C_D' \cdot 62.5 \times 0.220 \times \sqrt{64.4 \times \frac{3.6}{12}} \quad (\text{from equation 4.29})$$

$$\therefore \quad \underline{\underline{C_D' = 1.06}}$$

Variable Area Meters—Rotameters

In the meters which have been described so far the area of the constriction or orifice is constant and the pressure drop is dependent on the rate of flow. In the variable area meter, the pressure drop is constant and the flow rate is a function of the area of the constriction.

One of the earlier forms of variable area meter is the Ewing ball and tube flowmeter⁽¹⁵⁾. This consists of a vertical glass tube with a very small taper towards the lower end. The fluid passes upwards and the flow rate is indicated by the position of a ball of slightly smaller diameter than the tube. This meter suffers from the disadvantage that the ball tends to be unstable and to chatter in the tube. In the rotameter the ball is replaced by a float which spins slowly; this system is far more stable (Fig. 4.17).

The float takes up an equilibrium position in the tube, so that the total upward force acting on it is equal to its weight. The pressure difference across the float is equal to its weight divided by its maximum cross-sectional area in a horizontal plane. The area available for flow is the annulus formed between the float and the wall of the tube; the higher the float in the tube the greater is this area. The rotameter can be considered as an orifice meter with variable aperture and the formulae already derived are therefore applicable. Both in the ordinary orifice meter and in the rotameter the pressure drop arises from two causes; namely, the conversion of pressure energy into kinetic energy and the frictional drag on the walls of the orifice or the float. The frictional force on the float can be accounted for in the coefficient of discharge. The pressure difference — ΔP over the float is given by

$$-\Delta P = \frac{V_f(\rho_f - \rho) \cdot g}{A_f} \quad \dots(4.33)$$

where V_f is the volume of the float,

ρ_f is the density of the material of the float, and

A_f is the maximum cross-sectional area of the float in a horizontal plane.

If the area of the annulus between the float and the tube is A_2 and the cross-sectional area of the tube on the downstream side of the float is A_1 , we have from equation 4.16,

$$G = C_D \cdot A_2 \sqrt{\frac{2\rho(-\Delta P)}{\left\{1 - \left(\frac{A_2}{A_1}\right)^2\right\}}} \quad \dots(4.34)$$

(neglecting the difference in vertical height over the float).

Substituting the value of $-\Delta P$ from equation 4.33,

$$G = C_D \cdot A_2 \sqrt{\frac{2g \cdot V_f(\rho_f - \rho) \cdot \rho}{A_f \left\{1 - \left(\frac{A_2}{A_1}\right)^2\right\}}} \quad \dots(4.35)$$

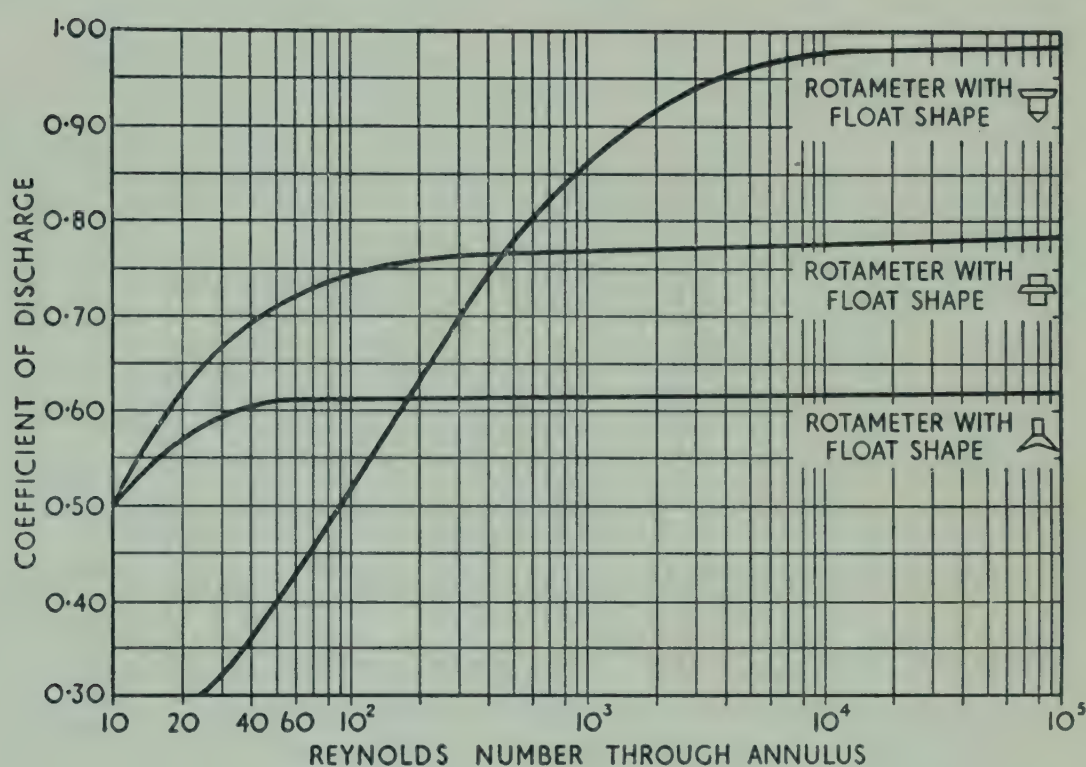


Fig. 4.18. Coefficient for rotameter

The range of a particular rotameter can be greatly increased by the use of floats of different densities. Rotameter tubes are made in sizes from about $\frac{1}{2}$ in. diameter upwards. For very large rates of flow, a central hole is drilled through the float so that it can ride on a vertical rod fixed at the axis of the tube. The float and tube are thereby protected against damage resulting from sudden changes in the flow rate. In high pressure work the transparent tube is replaced by a metal tube and a remote control method is used to measure the position of the float. Rotameters are extensively used with liquids containing solid particles in suspension.

The coefficient of discharge of the rotameter depends on the shape of the float and the Reynolds Number (based on the velocity in annulus and mean hydraulic diameter of annulus) for the flow through the annular space between the float and the tube. In general, those floats which give the most constant coefficient are of such a shape as to set up eddy currents in the annulus; in consequence the coefficient is comparatively low^(21, 28). The variation of the coefficient with

Reynolds Number is largely due to the variation of the viscous drag of the fluid on the float and therefore, if turbulence is artificially increased, the drag force reaches a limiting, though high, value at a comparatively low Reynolds Number. The variation of discharge coefficient with Reynolds Number is shown in Fig. 4.18 for floats of three different shapes⁽⁷⁾. Float *A* does not promote turbulence in the fluid stream; its coefficient increases rapidly with increase in Reynolds Number and attains an approximately constant value of about 0.98 for Reynolds Numbers greater than about 10^4 . At the other extreme, float *C* is a good turbulence promoter; the coefficient is low but becomes constant at a Reynolds Number a little over 100. For accurate work, the meter must be calibrated for each liquid used.

The Notch

The flow of a liquid presenting a free surface can be measured by means of a notch. The pressure energy of the liquid is converted into kinetic energy as it flows through the notch. The velocity with which the liquid leaves depends on its initial depth below the surface. For unit mass of liquid, initially at a depth h below the free surface, discharging through a notch, the energy balance equation, 2.34a, gives,

$$\Delta \frac{u^2}{2} + v \cdot \Delta P = 0 \quad \dots (4.36)$$

for a frictionless flow under turbulent conditions. If the velocity upstream from the notch is small, and the flow upstream is assumed to be unaffected by the change of area at the notch,

$$\begin{aligned} \frac{1}{2}u_2^2 &= -v \cdot \Delta P = g \cdot h \\ \text{i.e.} \quad u_2 &= \sqrt{2gh} \end{aligned} \quad \dots (4.37)$$

The total rate of flow from a notch can then be calculated by considering the discharge through a small element of depth dh , at a distance h from the free surface: the total flow is obtained by integrating between $h = 0$ and $h = D$, where D is the total depth of liquid above the bottom of the notch. The relation between the volumetric rate of flow Q , and the depth D , will depend on the width of the notch at various distances above the bottom. The discharge will, in practice, be rather less than the value calculated by this means because the flow area will decrease to a vena contracta and the velocity will be reduced as a result of friction.

For a rectangular notch (Fig. 4.19), the rate of discharge of fluid at a depth h through an element of cross section of depth dh will be given by

$$dQ = C_D \cdot B \cdot dh \sqrt{2g \cdot h}$$

where C_D is the coefficient of discharge (usually about 0.6) and B is the width of the notch.

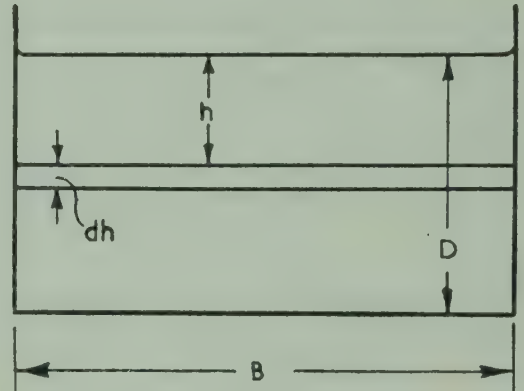


Fig. 4.19. Rectangular notch

Thus the total rate of flow,

$$Q = C_D \cdot B \cdot \sqrt{2g} \cdot \frac{2}{3} D^{\frac{3}{2}} \quad \dots(4.38)$$

where D is the total depth of liquid above the bottom of the notch.

The empirical Francis formula,

$$Q = 3.33(B - 0.01\sigma D) D^{\frac{3}{2}} \quad \dots(4.38a)$$

gives the flow rate in cubic feet per second if the dimensions of the notch are expressed in feet. $\sigma = 0$ if notch is the full width of the channel; $\sigma = 1$ if the notch is narrower than the channel but is arranged with one edge coincident with the edge of the channel; $\sigma = 2$ if the notch is narrower than the channel and is situated symmetrically. σ is known as the number of end contractions. Equation (4.38a) suggests that the coefficient of discharge C_D is a function of the depth of liquid in the notch.

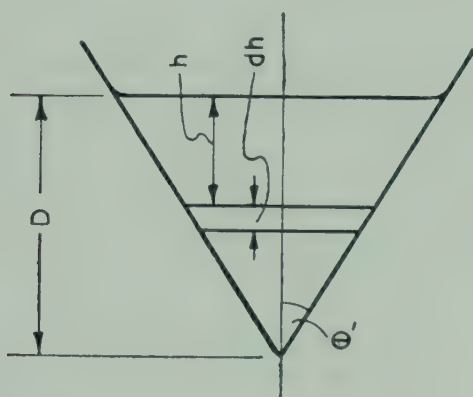


Fig. 4.20. Triangular notch

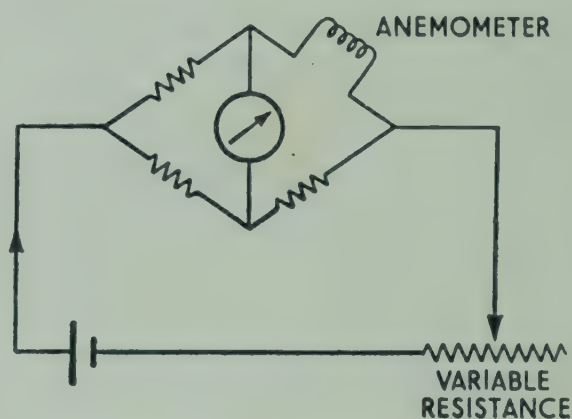


Fig. 4.21. Typical circuit for a hot wire anemometer

For a triangular notch of angle $2\theta'$, the flow dQ through the thin element of cross section is given by

$$dQ = C_D(D - h)2 \tan \theta' dh \cdot \sqrt{2gh}$$

The total rate of flow,

$$\begin{aligned} Q &= 2C_D \tan \theta' \cdot \sqrt{2g} \cdot \int_0^D (Dh^{\frac{1}{2}} - h^{\frac{3}{2}}) dh \\ &= 2C_D \tan \theta' \cdot \sqrt{2g} \cdot \left(\frac{2}{3} D^{\frac{5}{2}} - \frac{2}{5} D^{\frac{5}{2}} \right) \\ &= \frac{8}{15} C_D \tan \theta' \cdot \sqrt{2g} \cdot D^{\frac{5}{2}} \quad \dots(4.39) \\ &\propto D^{\frac{5}{2}} \quad \dots(4.39a) \end{aligned}$$

Thus for a rectangular notch the rate of discharge is proportional to the liquid depth raised to a power of $\frac{3}{2}$, and for a triangular notch to a power of $\frac{5}{2}$. A triangular notch will, therefore, handle a wider range of flow rates. In general the index of D is a function of the contours of the walls of the notch and any desired relation between flow rate and depth can be obtained by a suitable choice of contour. It is sometimes convenient to employ a notch in which the rate of discharge is directly proportional to the depth of liquid above the bottom of the notch. It can be shown that the notch must have curved walls giving a

large width to the bottom of the notch and a comparatively small width towards the top.

Other Methods of Measuring Flow Rates

The meters which have been described so far depend for their operation on the conversion of some of the kinetic energy of the fluid into pressure energy or vice versa. They form by far the largest class of flowmeters. Other meters are used for special purposes and brief reference will now be made to a few of these.

Hot Wire Anemometer. If a heated wire is immersed in a fluid, the rate of loss of heat will be a function of the flow rate. In the hot wire anemometer a fine wire whose electrical resistance has a high temperature coefficient is heated electrically: under equilibrium conditions the rate of loss of heat is then proportional to $I^2\omega$, where ω is the resistance of the wire and I is the current flowing.

Either the current or the resistance (and hence the temperature) of the wire is maintained constant. The following is an example of a method in which the resistance is maintained constant. The wire is incorporated as one of the resistances of a Wheatstone network (Fig. 4.21) in which the other three resistances have low temperature coefficients. The circuit is balanced when the wire is immersed in the stationary fluid but, when the fluid is set in motion, the rate of loss of heat increases and the temperature of the wire falls. Its resistance therefore changes and the bridge is thrown out of balance. The balance can be restored by increasing the current so that the temperature and resistance of the wire are brought back to their original values; the other three resistances will be unaffected by the change in current because of their low temperature coefficients. The current flowing in the wire is then measured using either an ammeter or a voltmeter. The rate of loss of heat is found to be approximately proportional to $\sqrt{u\rho + b'}$, so that

$$a'\sqrt{u\rho + b'} = I^2\omega \text{ under equilibrium conditions}$$

where u is the velocity of the fluid,

ρ is its density, and

a' and b' are constant for a given meter.

Since the resistance of the wire is maintained constant,

$$u \cdot \rho = I^4 \frac{\omega^2}{a'^2} - b' = a''I^4 - b' \quad \dots (4.40)$$

where $a'' = \frac{\omega^2}{a'^2}$ remains constant, i.e. the mass rate of flow per unit area is a function of the fourth power of the current, which must therefore be accurately measured.

The hot wire anemometer is very accurate even for very low rates of flow. It is one of the most convenient instruments for the measurement of the flow of

gases at low velocities; accurate readings are obtained for velocities down to about 0.1 f.p.s. If the ammeter has a high natural frequency, pulsating flows can be measured. Platinum wire is commonly used.

The Thomas Meter.⁽¹³⁾ In this meter the rise in temperature resulting from the addition of heat at a controlled rate to the fluid is measured. The temperature change is obtained by means of two thermometers, one upstream and the other downstream from the heater. If the specific heat of the fluid is constant, the temperature rise will be inversely proportional to the rate of flow. The meter is used for measuring low rates of flow of gases and the heat is usually supplied electrically.

Laboratory Flowgauge. The flow rate of a fluid can be determined by measuring the pressure drop across a capillary tube or constriction by means of a simple manometer. The meter must be calibrated against a standard instrument; its range can be extended by the use of tubes of varying sizes.

Quantity Meters

The meters which have been described so far give an indication of the rate of flow of fluid; the total amount passing in a given time must be obtained by integration. A number of instruments are available, however, for measuring directly the total quantity of fluid which has passed. An average rate of flow can then be obtained by dividing the quantity by the time of passage.

All measuring devices must eventually be standardized against a method in which the volume or mass of fluid passing in a given time is measured. In most cases, however, it is sufficient to calibrate against a standard meter which has itself been calibrated in this manner.

Gases

A simple quantity meter which is used for the measurement of the flow of gas in an accessible duct is the anemometer (Fig. 4.22). A shaft carrying radial vanes or cups supported in low friction bearings is caused to rotate by the passage of the gas; the relative velocity between the gas stream and the surface of the vanes is low because the frictional resistance of the shaft is small. The number of revolutions of the spindle is counted automatically, using a gear train connected to a series of dials. The meter must be calibrated and should be checked at frequent intervals because the friction of the bearings will not necessarily remain constant. The anemometer is useful for gas velocities down to about 0.5 f.p.s.

Quantity meters, suitable for the measurement of the flow of gas through a pipe, include the standard wet and dry meters. In the wet meter, the gas fills a rotating segment and an equal volume of gas is expelled from another segment (Fig. 4.23). The dry gas meter employs a pair of bellows. Gas enters one of the bellows and automatically expels gas from the other; the number of cycles is counted and recorded on a series of dials. Both of these are positive displacement meters and therefore do not need frequent calibration. Gas meters usually

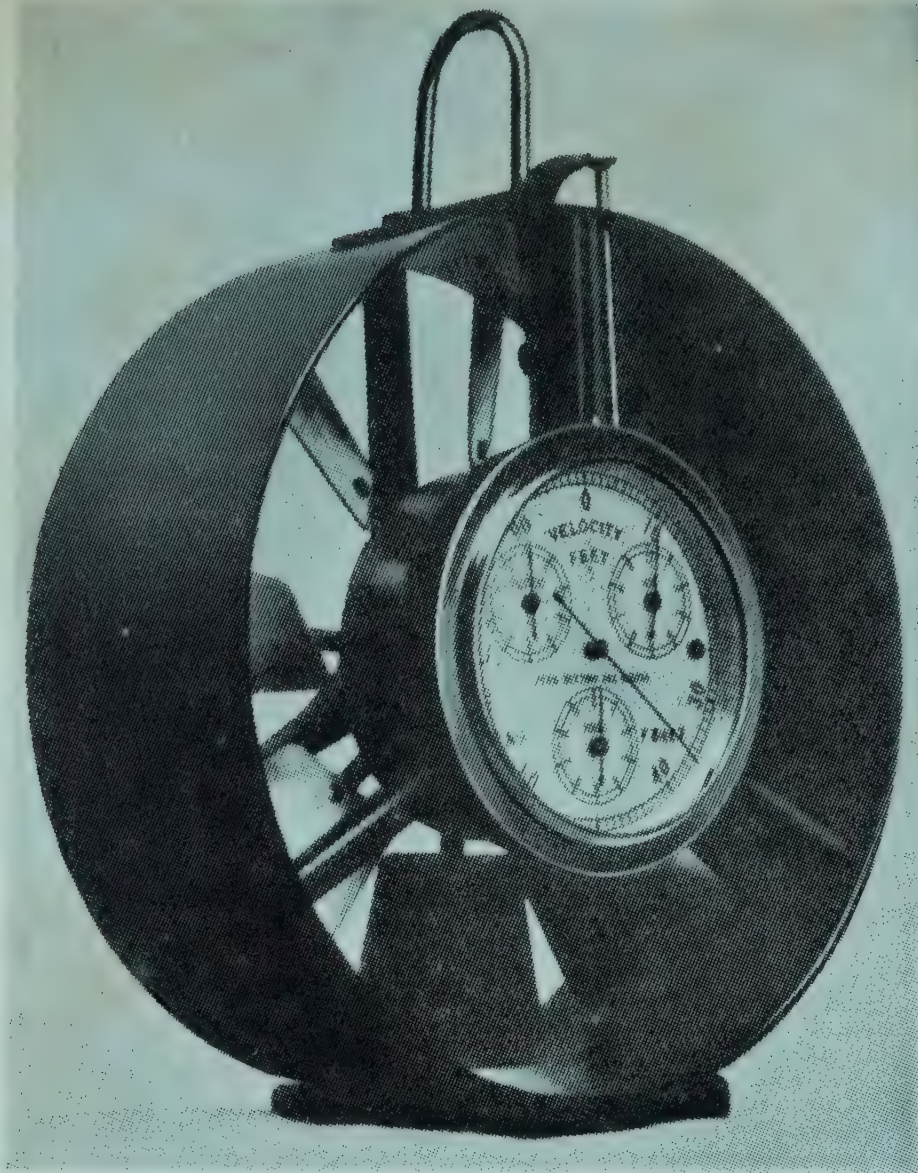


Fig. 4.22. Vane anemometer

(Reproduced by permission from *Measurement of Air Flow*, by E. OWER;
Chapman & Hall, 1939)

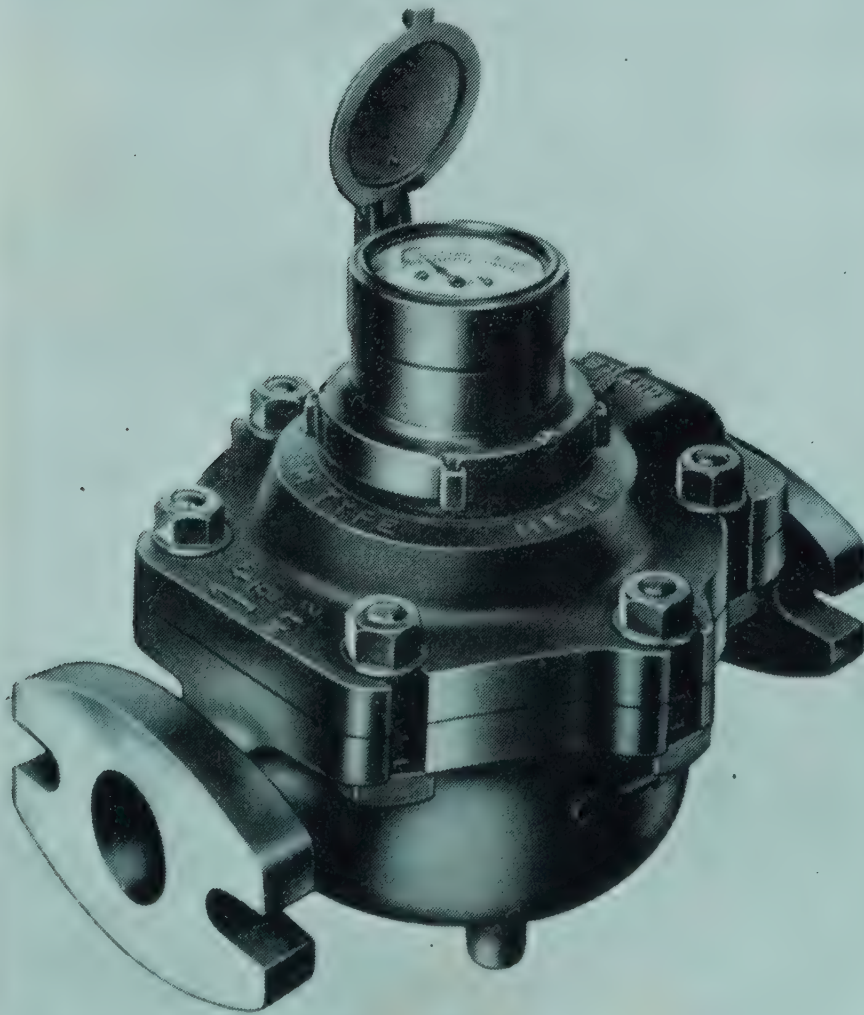


Fig. 4.24. Water meter

(Courtesy of George Kent Ltd.)

appear very bulky for the quantities they are measuring. This is because the linear velocity of a gas in a pipe is normally very high compared with that of a liquid and the large volume is needed so that the speed of the moving parts can be reduced and wear minimized.

Liquids

A variety of liquid meters are on the market. These can be divided into two classes; the positive and the non-positive types. In most cases the positive meters are like small pumps which are driven by the passage of the fluid; they operate at a volumetric efficiency of approximately 100%. One meter employs a single reciprocating piston; another a series of pistons; other models incorporate an oscillating disc or a rotating cylindrical piston. The non-positive

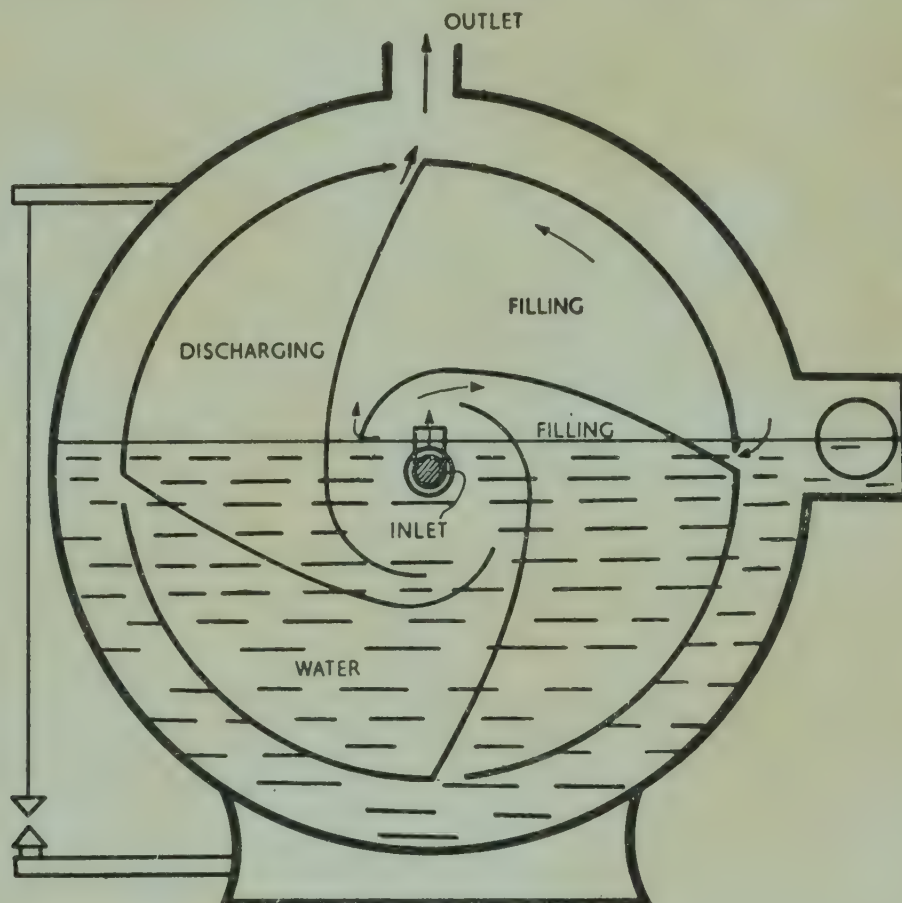


Fig. 4.23. Wet gas meter

meters are similar to the anemometer and depend on the impingement of fluid on a series of rotating vanes. Positive meters are capable of giving a higher accuracy but they suffer from the disadvantage that they cause an interruption in the flow if they become jammed. Mechanical meters will handle rates of flow of between about 4 and 10,000 gal./hr.

Further details of these and other flow measuring equipment are given in the books by OWER⁽⁸⁾ and LINFORD⁽⁷⁾. In many cases the simple measuring device is arranged to record the flow rate and sometimes it is combined with a controller which maintains a constant rate.

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LIST OF SYMBOLS FOR CHAPTERS 2, 3, 4

A	Area perpendicular to direction of flow	L^2
A_f	Maximum cross-sectional area of float in direction perpendicular to flow	L^2
a	Coefficient in Van der Waals' equation	$M \cdot L^5 \cdot T^{-1}$
a'	Constant for a hot wire anemometer	$M^{\frac{1}{2}}L^3T^{-\frac{5}{2}}$
a''	Constant for a hot wire anemometer	$\mu^2M^{-1}L^{-4}T^3$
B	Width of rectangular channel or notch	L
b	Coefficient in Van der Waals' equation	L^3
b'	Constant for a hot wire anemometer	$ML^{-2}T^{-1}$
C	Compressibility factor for non-ideal gas or Chezy coefficient	— or $L^{\frac{1}{2}}T^{-1}$
C'	Constant for venturi meter	L^2
C_D	Coefficient of discharge	—
C_D'	Coefficient of discharge for diverging cone of venturi meter	—
C_c	Coefficient of contraction	—
C_f	Coefficient for flow over a bank of tubes	—
C_p	Specific heat at constant pressure per unit mass	$L^2T^{-2}\theta^{-1}$
C_v	Specific heat at constant volume per unit mass	$L^2T^{-2}\theta^{-1}$
c	Root mean square velocity of molecules	LT^{-1}
c_a	Velocity of a group of molecules	LT^{-1}
D	Depth of liquid in channel or above bottom of notch	L
d	Diameter of pipe	L
d_a	Dimension of rectangular duct	L
d_b	Dimension of rectangular duct	L
d_m	Hydraulic mean diameter	L
E	Eddy kinematic viscosity	L^2T^{-1}
e	Pipe roughness	L
F	Energy per unit mass degraded because of irreversibility of process	L^2T^{-2}
f	Distance between two parallel plates	L
G	Mass rate of flow	MT^{-1}
G_w	Maximum mass rate of flow	MT^{-1}
g	Acceleration due to gravity	LT^{-2}
H	Enthalpy per unit mass	L^2T^{-2}
h	Depth below surface measured perpendicular to bottom of channel or notch	L
h_f	Head lost due to friction	L
h_F	Head lost due to friction in converging cone of venturi meter	L
h_v	Fall in head over converging cone of venturi meter	L
h_F'	Head lost due to friction in diverging cone of venturi meter	L
h_v'	Increase in head over diverging cone of venturi meter	L
h_m	Reading on manometer	L
h_i	Difference between impact and static heads on pitot tube	L
h_o	Fall in head over orifice meter	L
I	Electric current	$\mu^{-\frac{1}{2}}M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
i	Hydraulic gradient (h_f/l)	—
J	Specific energy of fluid in open channel	L^2T^{-2}
j	Number of banks of pipes in direction of flow	—
K	Total energy per unit mass for flow of ideal gas under adiabatic conditions in pipe	L^2T^{-2}
k	Numerical constant used as index	—
k_1	Quantity independent of downstream pressure at orifice	M^2T^{-2}
L	Characteristic linear dimension	L

l	Length of pipe or channel	L
l_m	Reading on inclined manometer	L
M	Molecular weight	—
m	Mass of molecule	M
N	Numerical concentration of molecules	L^{-3}
N_a	Numerical concentration of molecules with velocity u_a in given direction	L^{-3}
N_1	Numerical concentration of molecules with velocity u_1 in given direction	L^{-3}
N_2	Numerical concentration of molecules with velocity u_2 in given direction	L^{-3}
n	Number of moles of fluid	M
P	Pressure	$ML^{-1}T^{-2}$
P'	Dimensionless derivative of P	—
P_0	Pressure at condition at which entropy is zero	$ML^{-1}T^{-2}$
P_w	Pressure at downstream end of pipe for maximum flow conditions	$ML^{-1}T^{-2}$
P_R	Reduced pressure	—
P_a	Atmospheric pressure	$ML^{-1}T^{-2}$
P_f	Pressure due to friction	$ML^{-1}T^{-2}$
P_2^+, P_2^-	Upper and lower design outlet pressures for nozzle	$ML^{-1}T^{-2}$
p	Wetted perimeter	L
Q	Volumetric rate of flow	L^3T^{-1}
q	Net heat flow into system from surroundings	L^2T^{-2}
R	Shear stress in fluid at boundary surface	$ML^{-1}T^{-2}$
R_m	Mean value of shear stress at surface of channel	$ML^{-1}T^{-2}$
R_y	Shear stress at some point in fluid	$ML^{-1}T^{-2}$
R	Universal gas constant	$L^2T^{-2}\theta^{-1}$
r	Radius of pipe, or outer pipe in case of annulus	L
r_i	Radius of inner pipe of annulus	L
S	Entropy per unit mass	$L^2T^{-2}\theta^{-1}$
s	Distance from axis of pipe or from centre plane	L
T	Absolute temperature	θ
T_0	Temperature at condition where entropy is zero	θ
T_R	Reduced temperature	—
t	Time	T
U	Internal energy per unit mass	L^2T^{-2}
u	Mean velocity	LT^{-1}
u'	Dimensionless derivative of velocity	—
u_m	Velocity at mean of pressures P_1 and P_2	LT^{-1}
u_w	Velocity of propagation of pressure wave	LT^{-1}
u_c	Integration constant	LT^{-1}
u_0	Reference velocity	LT^{-1}
u_s	Maximum velocity in pipe	LT^{-1}
u_y	Velocity at right angles to main direction of flow	LT^{-1}
u_t	Velocity at narrowest cross section of bank of tubes	LT^{-1}
u_x	Velocity in X -direction at a point	LT^{-1}
u_a	Component of velocity of molecules in given direction	LT^{-1}
u_1	Component of velocity of molecules in given direction	LT^{-1}
u_2	Component of velocity of molecules in given direction	LT^{-1}
V	Volume of fluid	L^3
V_R	Reduced volume	—
V_f	Volume of rotameter float	L^3
v	Volume per unit mass of fluid	L^3M^{-1}
v_m	Volume per unit mass at mean of pressures P_1 and P_2	L^3M^{-1}
v_w	Volume per unit mass at P_w	L^3M^{-1}
W	Net work per unit mass done by system on surroundings	L^2T^{-2}
w	Pressure ratio P_2/P_1	—
w_c	Critical pressure ratio	—

X	Principal direction of motion	—
x	Distance in X -direction or in direction of motion	L
x'	Dimensionless derivative of x	—
Y	Principal direction of motion	—
y	Distance in Y -direction or distance from surface	L
y'	Dimensionless derivative of y	—
Z	Principal direction of motion	—
z	Distance in z -direction or in vertical direction	L
z'	Dimensionless derivative of z	—
z_m	Vertical distance between level of manometer liquid and axis of venturi meter	L
α	Constant in expression for kinetic energy of fluid	—
β	Angle of inclination of pipe to horizontal	—
γ	Ratio of specific heats at constant pressure and constant volume	—
ε	Bulk modulus of elasticity of fluid	$ML^{-1}T^{-2}$
λ	Shear force acting on inner surface of annulus	MLT^{-2}
μ	Viscosity of fluid	$ML^{-1}T^{-1}$
ρ	Density of fluid	ML^{-3}
ρ_m	Density of fluid at mean of pressures P_1 and P_2	ML^{-3}
ρ_f	Density of rotameter float	ML^{-3}
ρ'	Density of manometer liquid	ML^{-3}
σ	Number of end contractions	—
θ	Angle of slope of channel	—
θ'	Half angle of triangular notch	—
ϕ	Angle between inclined manometer and vertical	—
ω	Electrical resistance	μLT^{-1}
Δ	Finite change in a quantity	—
$Re.$	Reynolds Number with respect to pipe diameter	—

Suffixes

- 1 Refers to section 1, generally upstream.
- 2 Refers to section 2, generally downstream or at constriction.
- 3 Refers to section 3, either at the discharge end of a pump or at the end of the diverging cone of a venturi meter.

Pumps for Chemical Works

FOR pumping liquids or gases from one vessel to another or through long pipes, some form of mechanical pump is usually employed. The energy required by the pump will depend on the height through which the fluid is raised, the length and diameter of the pipe and the rate of flow, together with physical properties of the fluid, particularly its viscosity and density. The pumping of liquids such as sulphuric acid, from the bulk store to the process building, or the pumping of fluids round the actual reaction units, are typical illustrations of the use of pumps in the chemical industry. In the first case the length of pipe will generally be high, whilst in the second case the pipe may be short but the pressure relatively high. A wide variety of pumps have been developed for these purposes, but the majority are of three general types, the reciprocating piston, the gear pump, and the centrifugal pump. When handling chemicals it is necessary to consider their corrosive or erosive nature and whether they contain any suspended particles.

The main function of the pump is to raise the pressure of the fluid to the required value. In general pumps used for circulating gases work at higher speeds than those used for liquids and lighter valves are used. Again the clearances between moving parts are smaller on gas pumps because of the much lower viscosity of gases and increased tendency for leakage to occur. When a pump is used to provide a vacuum it is even more important to guard against leakage.

The work done by the pump is found by setting up an energy balance equation. If W is the work done by unit mass of fluid on the surroundings then $-W$ is the work done on the fluid by the pump. From equations 2.32 and 2.32 (a) developed in the previous chapter we have,

$$-W = \Delta \frac{u^2}{2\alpha} + g \cdot \Delta z + \int_1^2 v \cdot dP + F \quad \dots(2.32)$$

and
$$-W = \Delta \frac{u^2}{2\alpha} + g \cdot \Delta z + \Delta H - q \quad \dots(2.32a)$$

In any practical system the pump will not be perfectly efficient and more energy must be supplied by the engine driving the pump than is given by $-W$. Considering liquids as incompressible there is no change in specific volume from the inlet to the delivery side of the pump. The physical properties of gases are, however, considerably influenced by the pressure, and the work done in raising the pressure of a gas is determined by the rate of heat flow between the gas and the surroundings. Thus if the process is carried out adiabatically all the energy added to the system appears in the gas and its temperature rises. If an ideal

gas is compressed and then cooled to its initial temperature, its enthalpy will be unchanged and the whole of energy supplied by the compressor is dissipated to the surroundings. However, if the compressed gas is allowed to expand it will absorb heat and is therefore capable of doing work at the expense of heat energy from the surroundings.

PUMPING EQUIPMENT FOR LIQUIDS

The liquids used in the chemical industries differ considerably in physical and chemical properties and it has been necessary to develop a wide variety of pumping equipment. We can consider these either as positive displacement or centrifugal pumps. In the former the volume of liquid delivered is directly related to the displacement of the piston element and therefore increases directly with speed and is not appreciably influenced by the pressure. In this group we have the reciprocating piston pump and the rotary gear pump, both of which are commonly used for delivery against high pressures and where nearly constant delivery rates are required. The centrifugal type depends on giving the liquid a high kinetic energy which is then converted as efficiently as possible into pressure energy. For some conditions, as when handling particularly corrosive liquids or those with abrasive suspensions, compressed air is used as the motive force instead of a mechanical pump. An illustration of the use of this form of equipment is the blowing of the contents of a reaction mixture from one vessel to another.

The following factors influence the choice of pump for a particular operation.

(1) The quantity of liquid to be handled. This primarily affects the size of the pump and determines whether it is desirable to use a number of pumps in parallel.

(2) The head against which the liquid is to be pumped. This will be determined by the pressure and vertical height of the downstream reservoir and by the frictional losses which occur in the delivery line. The suitability or otherwise of a centrifugal pump and the number of stages required will largely be determined by this factor.

(3) The nature of the liquid to be pumped. For a given throughput the viscosity largely determines the friction losses and hence the power required. The corrosive nature will determine the material of construction both for the pump and the packing. With suspensions, the clearances in the pump must be large compared with the size of the particles.

(4) The nature of the power supply. If the pump is driven by an electric motor or internal combustion engine, a high-speed centrifugal or rotary pump will be preferred as it can be coupled directly to the motor. Simple reciprocating pumps can be connected to steam or gas engines.

(5) If the pump is only used intermittently corrosion troubles are more likely than with continuous working.

The cost and mechanical efficiency of the pump must always be considered and it may be advantageous to select a cheap pump and pay higher replacement or maintenance costs rather than to instal a very expensive pump of high efficiency.

The Reciprocating Pump

The Piston Pump. The piston pump consists essentially of a cylinder with a reciprocating piston connected to a rod which passes through a gland at the end of the cylinder as indicated in Fig. 5.1. To prevent leakage past the gland and the piston, packing material such as graphited asbestos, lead, aluminium, leather, rubber, plastic, or fibre is used: in some cases the piston is fitted with piston rings. The liquid enters from the suction line through a suction valve and is discharged through a delivery valve. The pump may be single acting, with the liquid admitted only to the portion of the cylinder in front of the piston or it

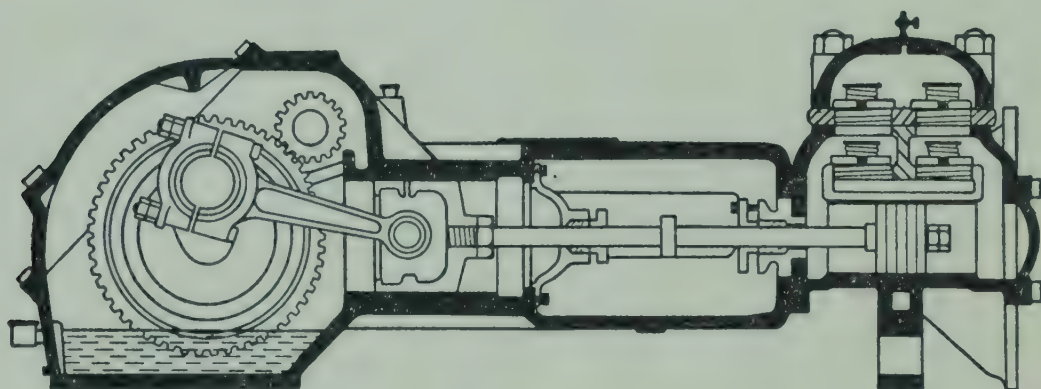


Fig. 5.1. Piston pump

may be double acting, in which case use is made of the volume on each side of the piston.

The velocity of the piston varies in an approximately sinusoidal manner and the volumetric rate of discharge of the liquid shows corresponding fluctuations. In a single cylinder pump the delivery will rise from zero as the piston begins to move forward, to a maximum when the piston is fully accelerated at approximately the mid point of its stroke; the delivery will then gradually fall off to zero. If the pump is single acting there will be an interval during the return stroke when the cylinder will fill with liquid and the delivery will remain zero. On the other hand, in a double acting pump the delivery will be similar in the forward and return strokes. In many cases, however, the cross sectional area of the piston rod may be significant compared with that of the piston and the volume delivered during the return stroke will therefore be less than that during the forward stroke. A more even delivery is obtained if a number of cylinders are suitably compounded. If two double acting cylinders are used there will be a lag between the deliveries of the two cylinders, and the total delivery will then be the sum of the deliveries from the individual cylinders. Typical curves of delivery rate for a single cylinder (simplex) pump are shown in Fig. 5.2a. The delivery from a two cylinder (duplex) pump in which both the cylinders are double acting is shown in Fig. 5.2b: the broken lines indicate the deliveries from the individual cylinders and the unbroken line indicates the total delivery. It will be seen that the delivery is much smoother than that obtained with the

simplex pump, the minimum delivery being equal to the maximum obtained from a single cylinder.)

The theoretical delivery of a piston pump is equal to the total swept volume of the cylinders. The actual delivery may be less than the theoretical value because of leakage past the piston and the valves or because of inertia of the valves. In some cases, however, the actual discharge is greater than theoretical value because the momentum of the liquid in the delivery line and sluggishness in the operation of the delivery valve may result in continued delivery during a portion of the suction stroke. The volumetric efficiency, which is defined as the ratio of the actual discharge to the swept volume, is normally greater than 90%.

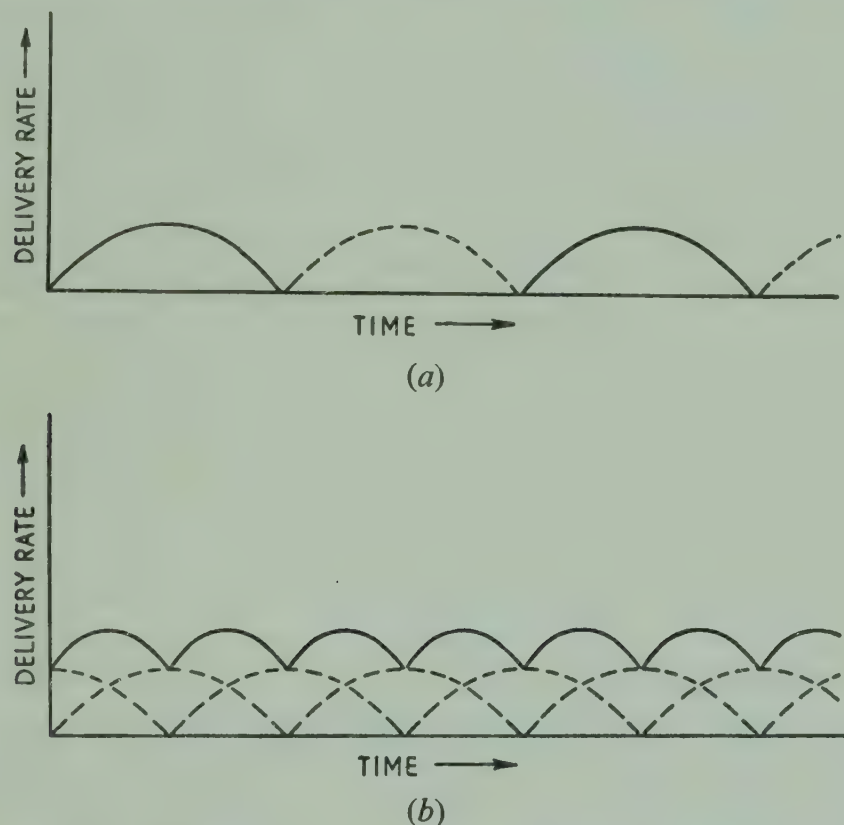


Fig. 5.2. Delivery from (a) simplex and (b) duplex pumps

The size of the suction and delivery valves is determined by the throughput of the pump. Where the rate of flow is high, two or more valves may be used in parallel.

The piston pump can be directly driven by steam, in which case the piston rod is common to both the pump and the steam engine (Fig. 5.3). Alternatively, an electric motor or an internal combustion engine may supply the motive power through a crankshaft; because the load is very uneven, a heavy flywheel should then be fitted.

The pressure at the delivery of the pump is made up of the following four components.

- (1) The static pressure at the delivery point.
- (2) Pressure attributable to the kinetic energy imparted to the fluid.
- (3) Pressure required to overcome the frictional losses in the delivery pipe.
- (4) The pressure for the acceleration of the fluid at the commencement of the delivery stroke.

The liquid in the delivery line is accelerated and retarded in phase with the motion of the piston and therefore the whole of the liquid must be accelerated at the commencement of the delivery stroke and retarded at the end of it. Every time the fluid is accelerated, work has to be done on it and therefore in a long delivery line the expenditure of energy is very large since the excess kinetic energy of the fluid is not usefully recovered during the suction stroke. Due to the momentum of the fluid, the pressure at the pump may fall sufficiently low for separation to occur. The pump is then said to “knock.” The flow in the delivery line can be evened out and the energy at the beginning of each stroke reduced, by the introduction of an air vessel at the pump discharge. This consists of a sealed vessel which contains air at the top and liquid at the bottom. When the delivery stroke commences, liquid is pumped into the air vessel and the air is compressed. When the discharge from the pump decreases towards the end of the stroke, the pressure in the air vessel is sufficiently high for some of the liquid to be expelled into the delivery line. If the air vessel is sufficiently large and is fitted close to the pump, the velocity of the liquid in the delivery line can be maintained approximately constant. The frictional losses are also reduced by the incorporation of an air vessel because the friction loss under turbulent conditions is approximately proportional to the 1.8 power of the linear velocity in the pipe; i.e. the reduced friction losses during the period of minimum discharge do not compensate for the greatly increased losses when the pump is delivering at maximum rate. Further the maximum stresses set up in the pump are reduced by the use of an air vessel.

Air vessels are also incorporated in the suction line for a similar reason. Here they may be of even greater importance because the pressure drop along the suction line is necessarily limited to rather less than one atmosphere if the suction tank is at atmospheric pressure. The flow rate may be limited if part of the pressure drop available must be utilized in accelerating the fluid in the suction line; the air vessel should therefore be sufficiently large for the flow rate to be maintained approximately constant.

The body of the piston pump is frequently made of cast iron. Bronze is commonly used for the valves and the cylinder liner which is normally replaceable. The piston and piston rod are usually of steel.

The piston type of pump is widely used, since it is comparatively simple in construction and operates with a high efficiency over a range of operating conditions. It is a positive displacement pump and consequently will operate against a high head and does not require priming. The delivery from the pump is uneven and consequently it gives an uneven load to the driving mechanism and causes high pressures to be developed at certain stages of the delivery cycle; the pump is therefore rather bulky. The piston pump will not handle corrosive liquids and slurries containing abrasive solids since damage is done to the valves and the machined surfaces of the cylinder and piston. It is also relatively expensive.

The Plunger or Ram Pump. The plunger type, Fig. 5.4, although similar

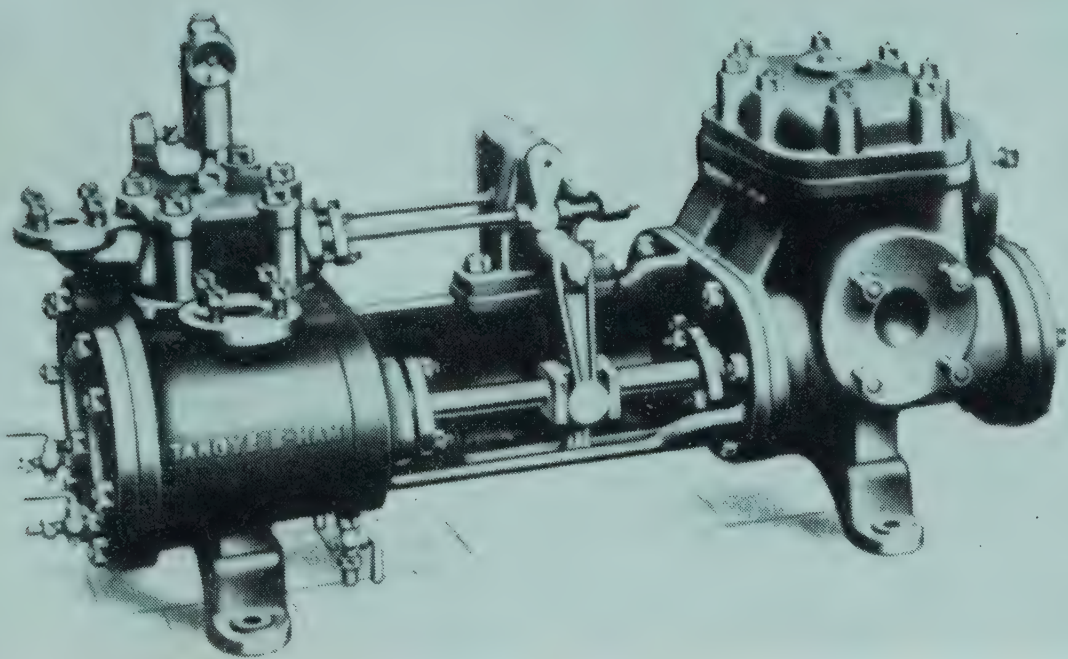


Fig. 5.3. Piston pump directly coupled to steam cylinder
(Courtesy of *Tangyes Ltd.*)

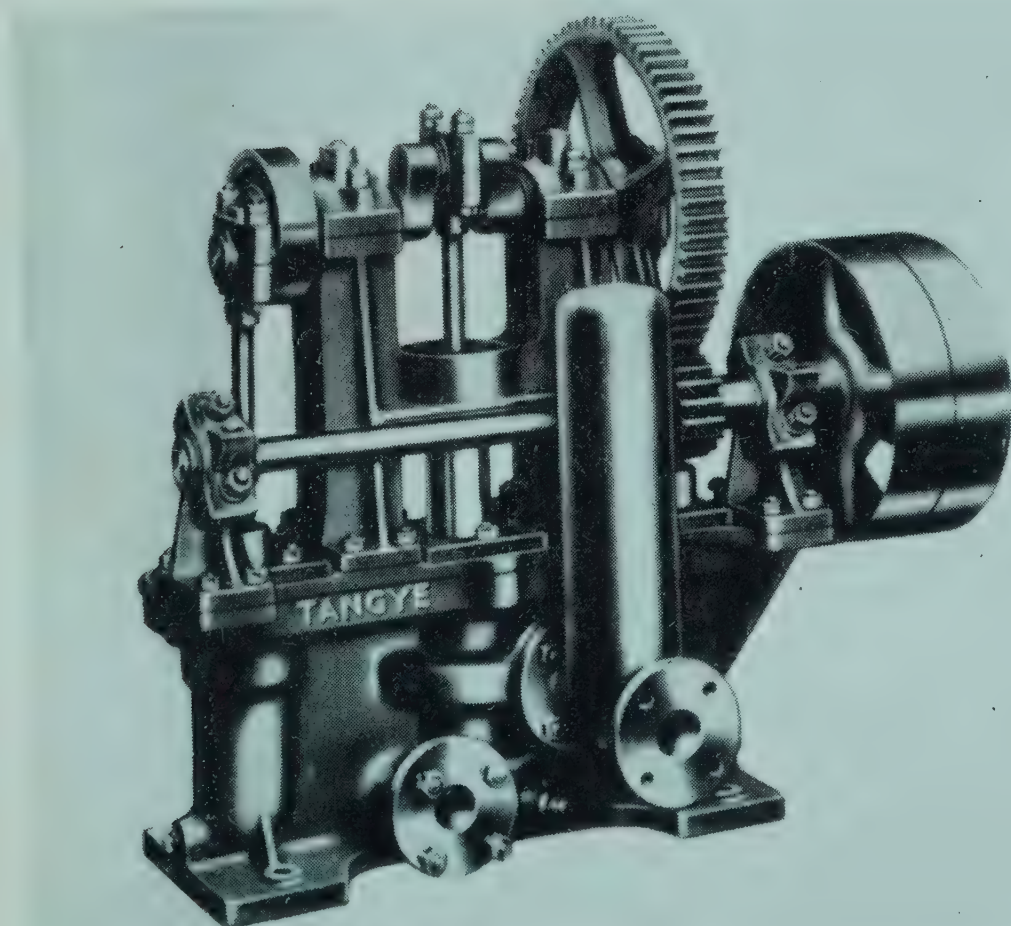


Fig. 5.4. Ram pump
(Courtesy of *Tangyes Ltd*)

in construction differs from the piston pump in that the plunger passes through a gland attached to the end of the cylinder; this ensures a more rapid removal and replacement of the packing material and leakage is immediately visible. The plunger pump will give a higher delivery pressure than a piston pump of the same size but the initial cost is higher.

The Diaphragm Pump. The diaphragm pump has been developed for handling corrosive liquids and those containing suspensions of abrasive solids. It is in two sections separated by a diaphragm of rubber, leather, or plastic

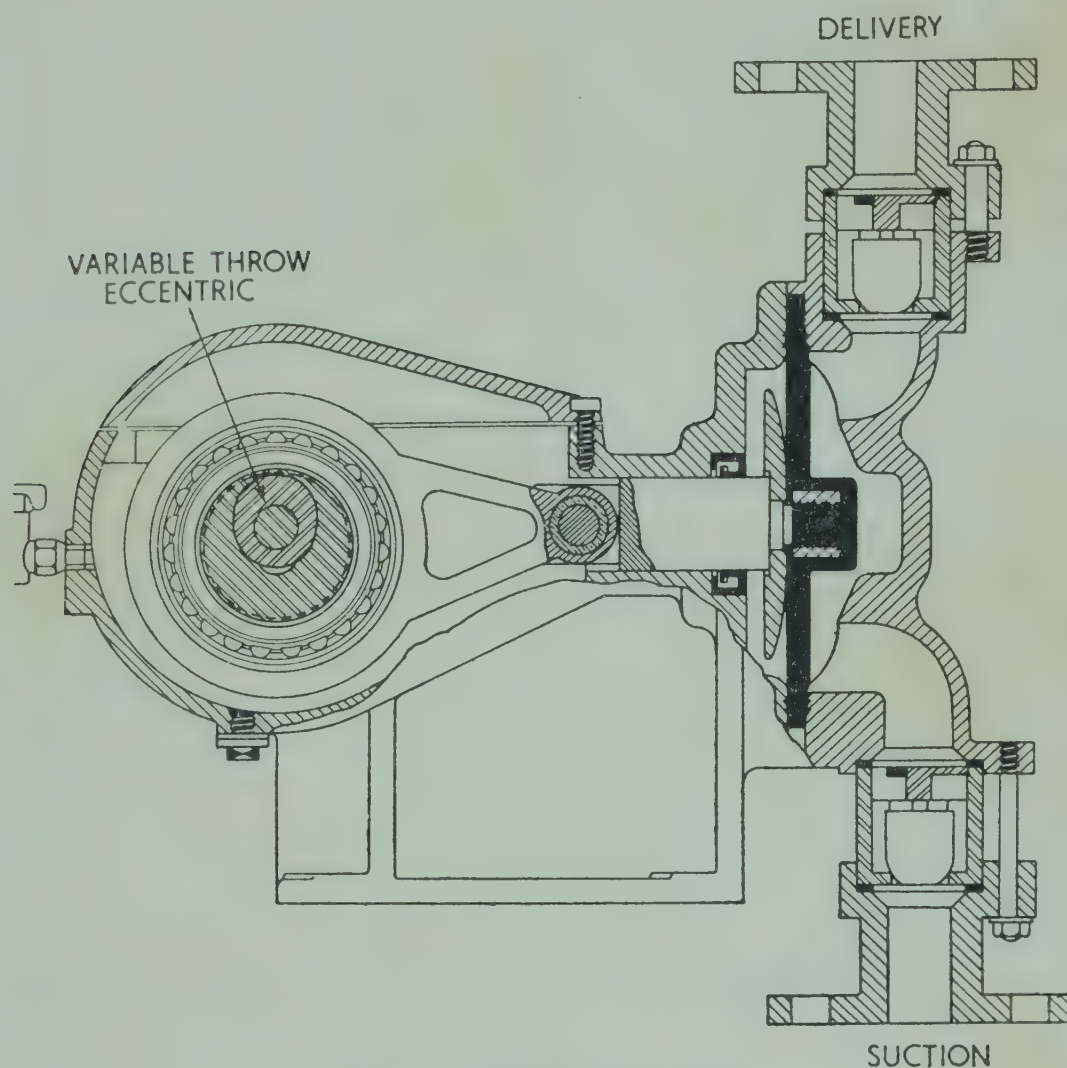


Fig. 5.5. Diaphragm pump
(Courtesy of E.C.D. Ltd.)

material. In one section a plunger or piston operates in a cylinder in which a non-corrosive fluid is displaced. The movement of the fluid is transmitted by means of the flexible diaphragm to the liquid to be pumped. The only moving parts of the pump that are in contact with the liquid are the valves and these can be specially designed to handle the material. In some cases the movement of the diaphragm is produced by direct mechanical action, as shown in Fig. 5.5. The pump illustrated is a metering pump, referred to on page 123.

The Ferrari's Acid Pump is similar in principle to the diaphragm pump. The piston or plunger operates in a cylinder in contact with oil. The motion of the oil is transmitted to the acid in a relatively large vessel, the upper portion of which contains oil and the lower part acid. Again the only moving parts in contact with the acid are the valves which can be made of a material resistant to the acid.

Kestner Glandless Pump. The Kestner Glandless Pump (Fig. 5.6) is a vertical ram pump constructed throughout of high silicon iron. The clearance between the ram and the cylinder is about $1/2000$ in. The ram is not fitted with a gland, and a maximum specified leak is allowed to take place in the space between the ram and the cylinder; the liquid that leaks past is fed back to the suction side of the pump. The leakage is reduced to a minimum by arranging that, in any position of the ram, the fluid has to flow a considerable distance in the narrow annulus between the ram and the piston before it can escape. In

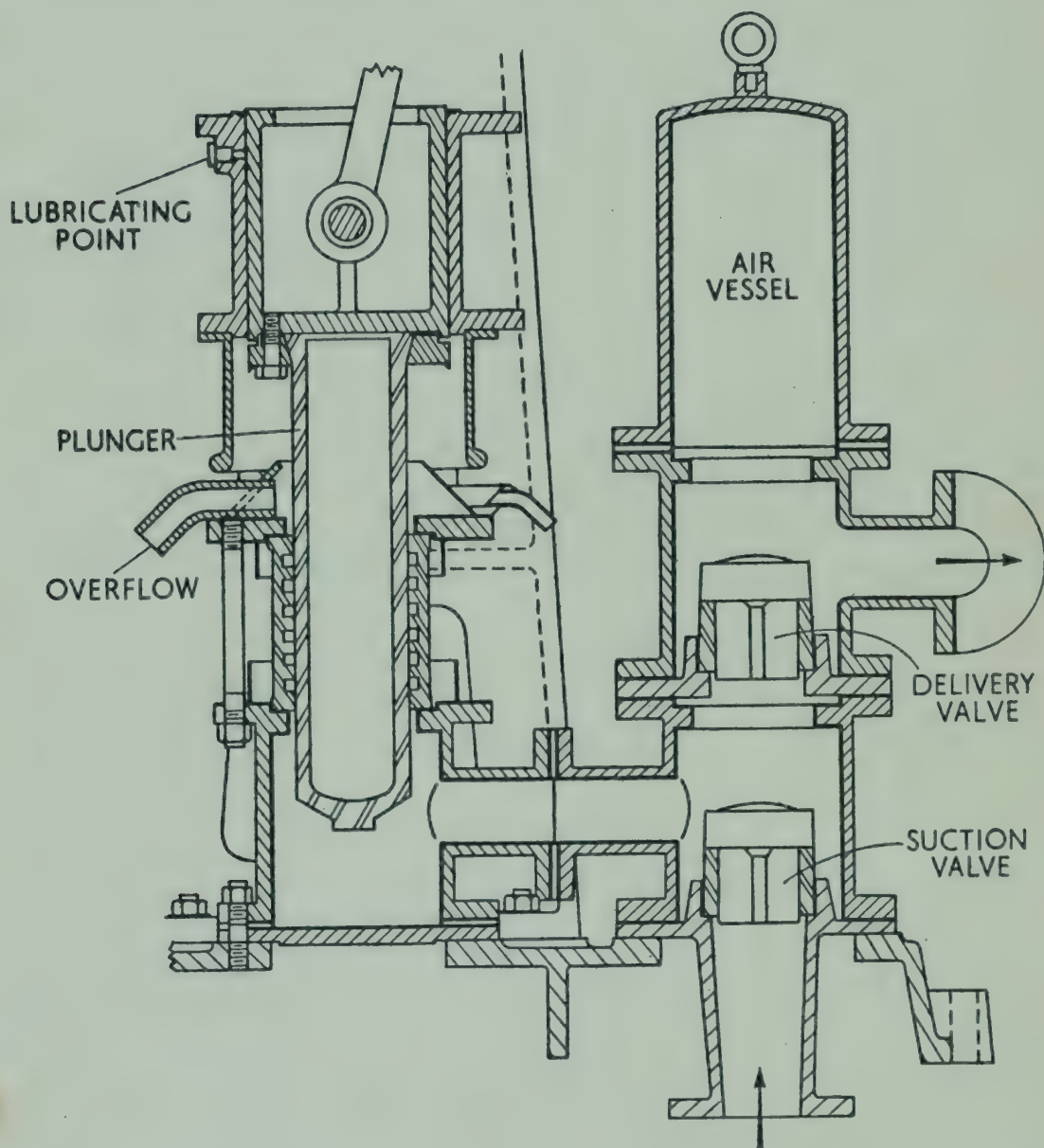


Fig. 5.6. Kestner double-plunger glandless acid pump
(Courtesy of Kestner Evaporator & Engineering Co. Ltd.)

other words, the stroke occupies only a small proportion of the total length of the plunger. This pump has been developed to handle 98% sulphuric acid against heads of up to 500 ft; the liquid itself acts as the lubricant. Because of the absence of friction from glands, the pump operates at a very high efficiency. There are commonly two cylinders of $4\frac{1}{2}$ -in. diameter and 6-in. stroke; piston speeds of up to 50 ft/sec are used.

Example. A single acting reciprocating pump has a cylinder diameter of $4\frac{1}{2}$ in. and a stroke of 9 in. The suction line is 20 ft long and 2 in. in diameter and the level of the water in the suction tank is 10 ft below the cylinder of the pump. What is the maximum speed at which the pump can run without an air vessel if separation is not to occur in the suction line? The piston undergoes approximately simple harmonic motion. Atmospheric pressure is equivalent

to a head of 34 ft of water and separation occurs at an absolute pressure corresponding to a head of 4 ft of water.

Solution. The tendency for separation to occur will be greatest (a) at the inlet to the cylinder, because the static pressure is a minimum there and the head required to accelerate the fluid in the suction line is a maximum; (b) at the commencement of the suction stroke, because the acceleration of the piston is then a maximum.

Let N r.p.m. be the maximum permissible speed of the pump.

Angular velocity of driving mechanism

$$= \frac{N}{60} \cdot 2\pi \text{ radians/sec}$$

Acceleration of piston

$$= \frac{1}{2} \cdot \frac{9}{12} \cdot \left(\frac{N}{60} \cdot 2\pi \right)^2 \cos \left(\frac{N}{60} 2\pi \right) \cdot t$$

Maximum acceleration ($t = 0$)

$$= 0.00411N^2 \text{ ft/sec}^2$$

Maximum acceleration of liquid in suction pipe

$$= \left(\frac{4.5}{2} \right)^2 \times 0.00411N^2 \text{ ft/sec}^2$$

$$= 0.0208N^2 \text{ ft/sec}^2$$

Accelerating force acting on liquid

$$= 0.0208N^2 \times \frac{\pi}{4} \left(\frac{2}{12} \right)^2 \times 20 \times 62.4 \text{ poundals}$$

Pressure drop in suction line due to acceleration

$$= 0.0208N^2 \times 20 \times 62.4 \text{ poundals/sq ft}$$

$$\text{Acceleration head} = 0.0208N^2 \times \frac{20}{32}$$

$$= 0.013N^2 \text{ ft of water}$$

Pressure head at cylinder when separation is about to occur

$$= 4 = 34 - 10 - 0.013N^2 \text{ feet}$$

$$\therefore N^2 = \frac{20}{0.013}$$

$$\text{i.e. } N = \underline{\underline{39 \text{ r.p.m.}}}$$

Positive Displacement Rotary Pumps

The Gear Pump. The gear pump (Fig. 5.7) is the most widely used of the positive action rotary pumps. (Two gear wheels operate inside a casing, with a small clearance between the tips of the gear teeth and the casing. One of the gear wheels is driven and the other rotates in mesh with it. The liquid is carried round in the space between consecutive teeth and the casing and is then ejected as the teeth come into mesh.) The pump has no valves and depends for its seal on the small clearance between the gear wheels and the case. It is a positive

displacement pump and will deliver against high pressures; such as 5000 lb/sq. in. The delivery is almost independent of pressure and priming is unnecessary.

The main advantage of the gear pump over the reciprocating pump is that it gives an even delivery and can be directly connected to an electric motor drive. It will handle liquids of very high viscosities and is extensively used in the oil industry for pumping viscous residual oils from distillation plant. Because the

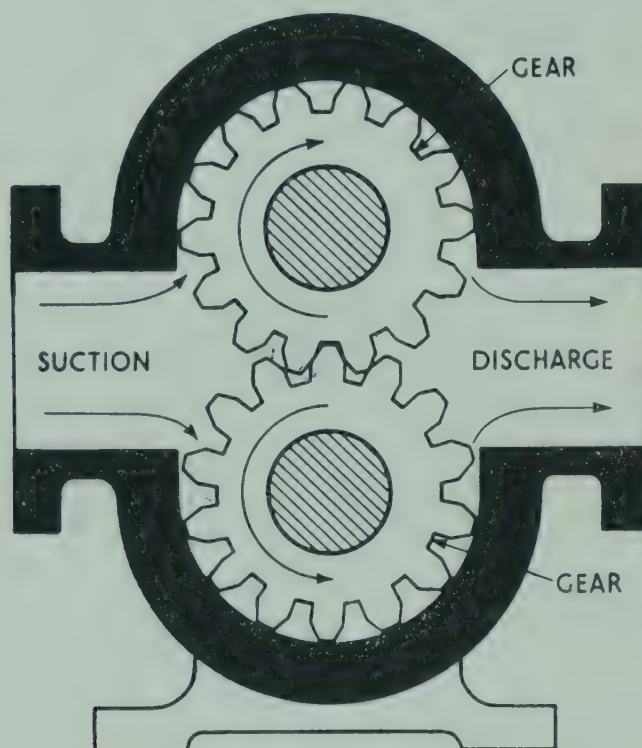


Fig. 5.7. Gear pump

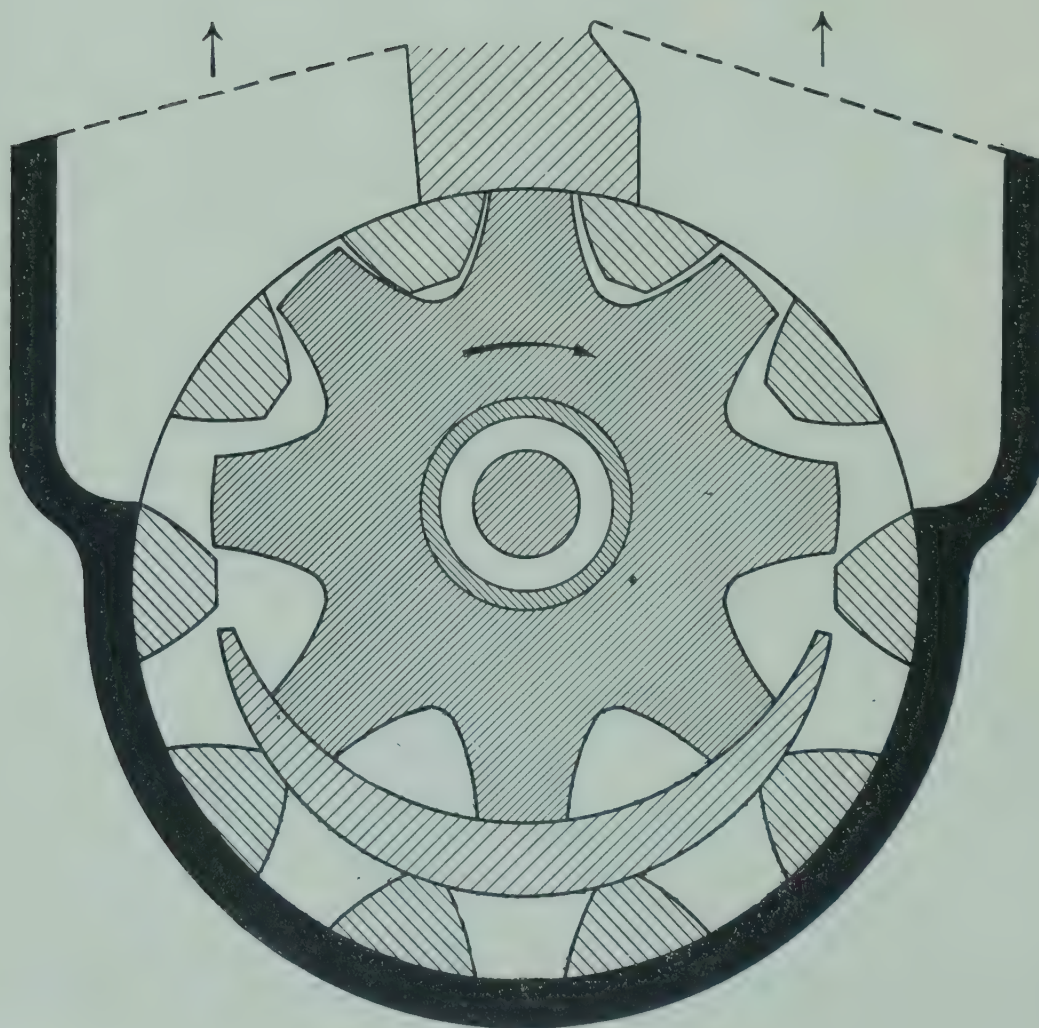


Fig. 5.8. Internal gear pump
(Courtesy of Varley Pumps & Engineering Ltd.)

spaces between the gear teeth are comparatively small, the pump cannot be used for suspensions. It is, however, suitable for a wide range of corrosive liquids, since it can readily be made in a variety of materials. In some pumps, very small holes are drilled at the base of the teeth in the idler gear wheel and serve to relieve the excess pressure if small quantities of liquid get trapped as the teeth come into mesh.

A form of gear pump which is less frequently used (Fig. 5.8) consists of a relatively large gear wheel with internal teeth, which meshes with a smaller gear wheel with external teeth. The small gear wheel is set off centre from the large one. The liquid is carried round in the spaces between the teeth of the two gear wheels.

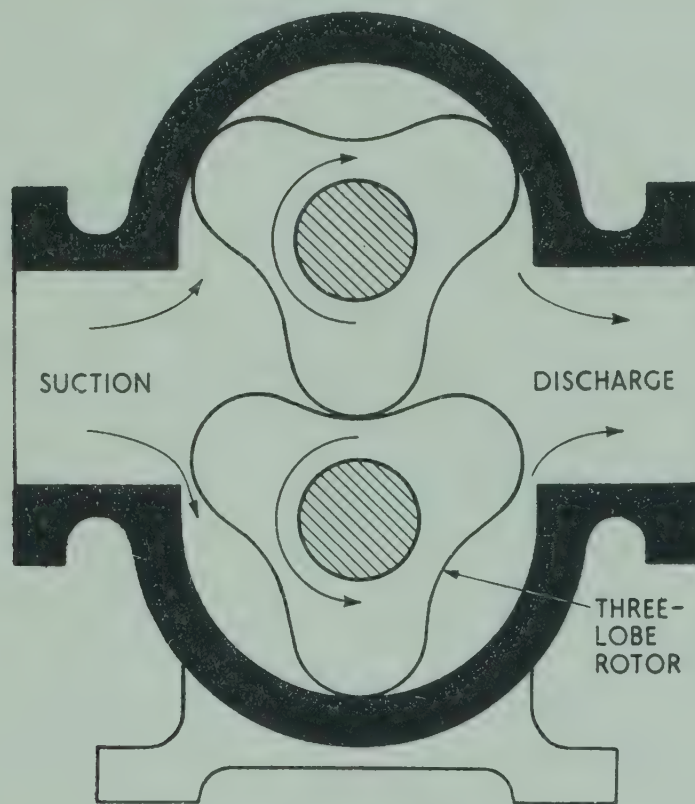


Fig. 5.9. Lobe pump

The Lobe Pump. The lobe pump (Fig. 5.9) is similar to the gear pump but the gear wheels are replaced by two impellers, each of which carries either two or three lobes. The impellers are driven independently of one another and wear can, therefore, be reduced by maintaining a small clearance between them. The characteristics of the lobe pump are generally similar to those of the gear pump.

The Cam Pump. A rotating cam is mounted eccentrically at the centre of a cylindrical casing and a very small clearance is maintained between the outer edge of the cam and the casing. A diagram of this type of pump is given in Fig. 5.10. As the cam rotates it expels liquid from the space ahead of it and sucks in liquid behind it. The delivery and suction sides of the pump are separated by a sliding valve which rides on the cam. The characteristics again are similar to those of the gear pump.

The Vane Pump. The rotor of the vane pump is mounted off centre in a cylindrical casing (Fig. 5.11). It carries rectangular vanes in a series of slots arranged at intervals round the curved surface of the rotor. The vanes are

thrown outwards by centrifugal action and the fluid is carried in the spaces bounded by adjacent vanes, the rotor, and the casing. Most of the wear is on the vanes and these can readily be replaced.

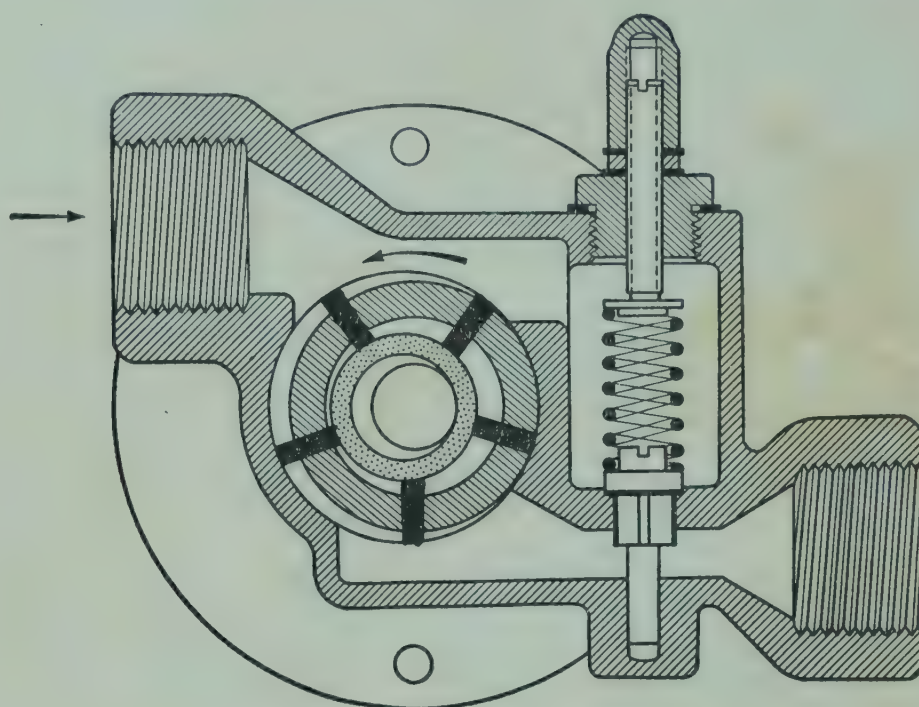


Fig. 5.11. Vane pump

(Courtesy of Varley Pumps & Engineering Ltd.)

The Mono Pump. Another example of a positive acting rotary pump is the Mono Pump (Fig. 5.12) in which a specially shaped helical metal worm rotates in a stator made of rubber or other similar material, the liquid being forced through the space between the stator and the rotor. The Mono Pump gives a uniform

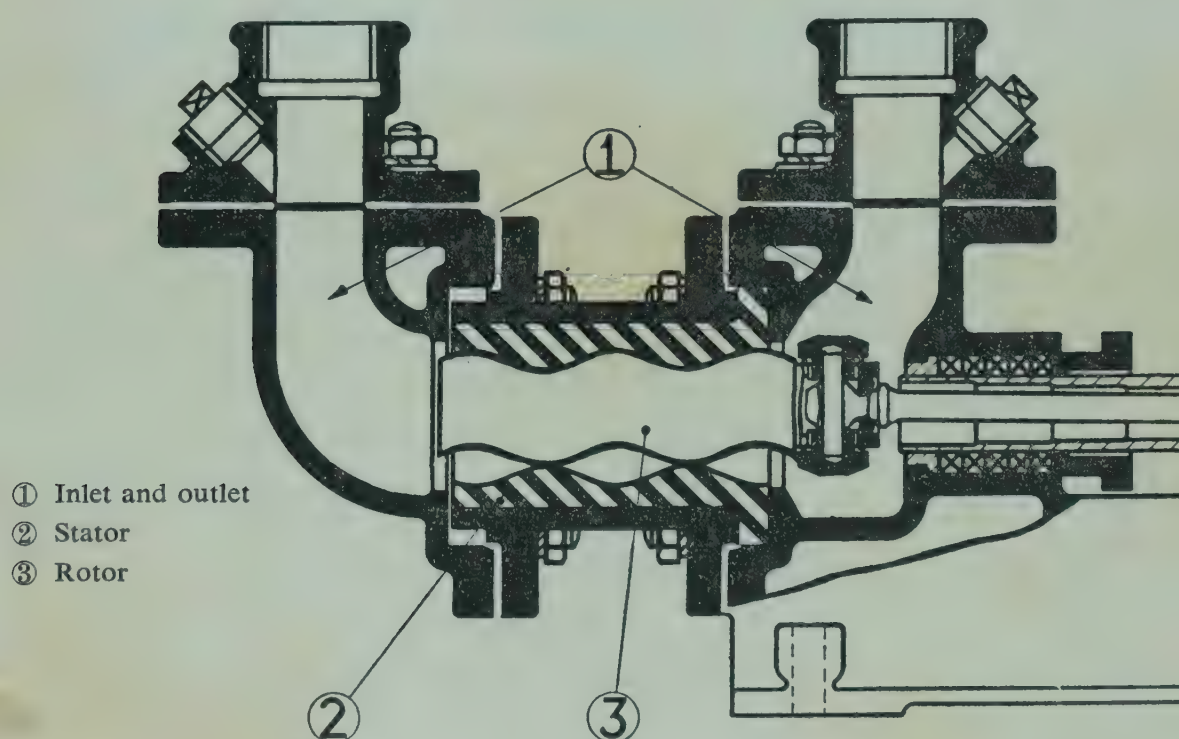


Fig. 5.12. Mono pump

(Courtesy of Mono Pumps Ltd.)

flow and is quiet in operation. It will pump against high pressures; the higher the required pressure, the longer are the stator and the rotor and the greater the number of turns. The pump can handle corrosive and gritty liquids and is extensively used for feeding slurries to filter presses. It must never be run dry.

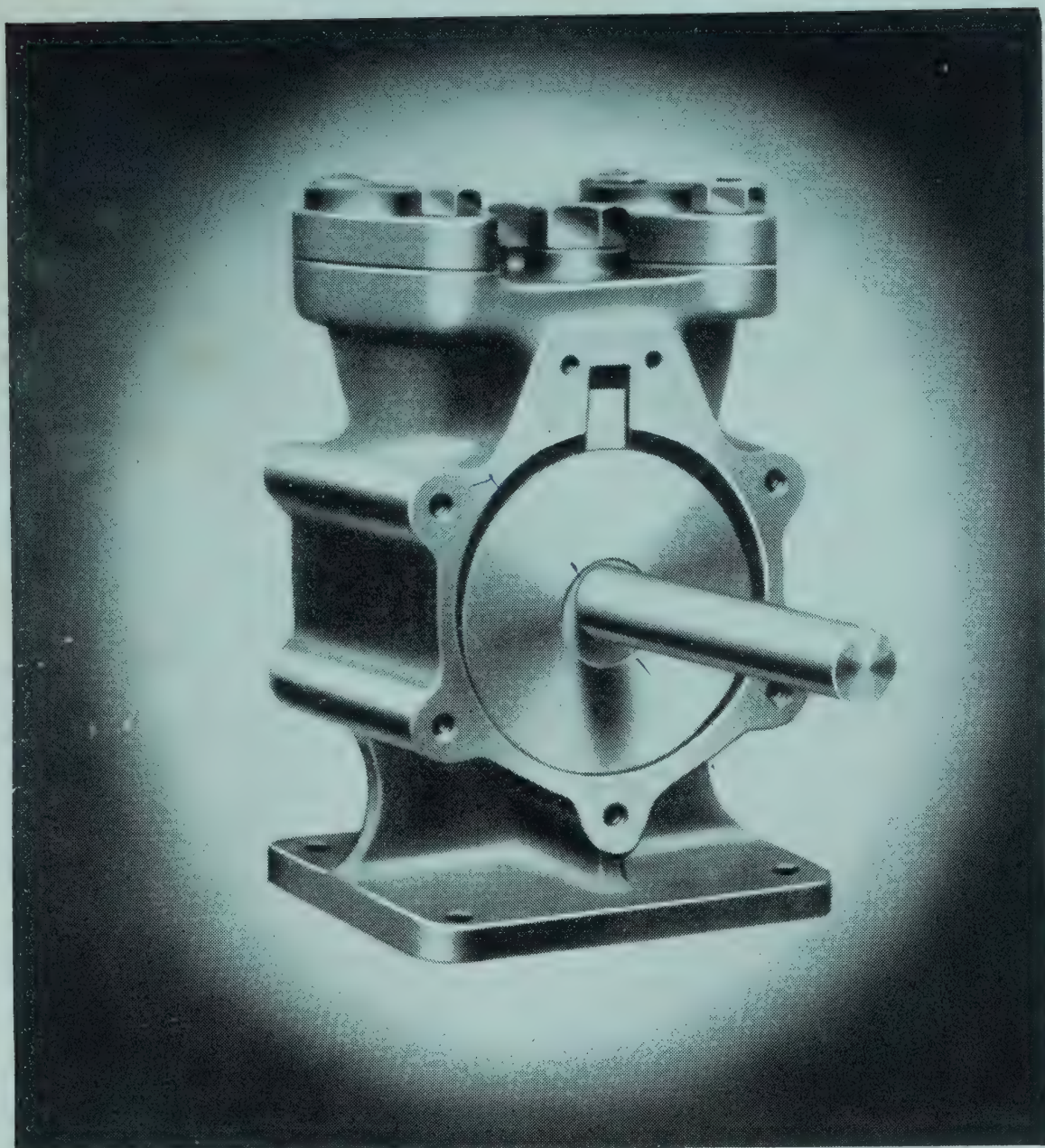


Fig. 5.10. Cam pump
(Courtesy of *Joseph Evans & Sons Ltd.*)

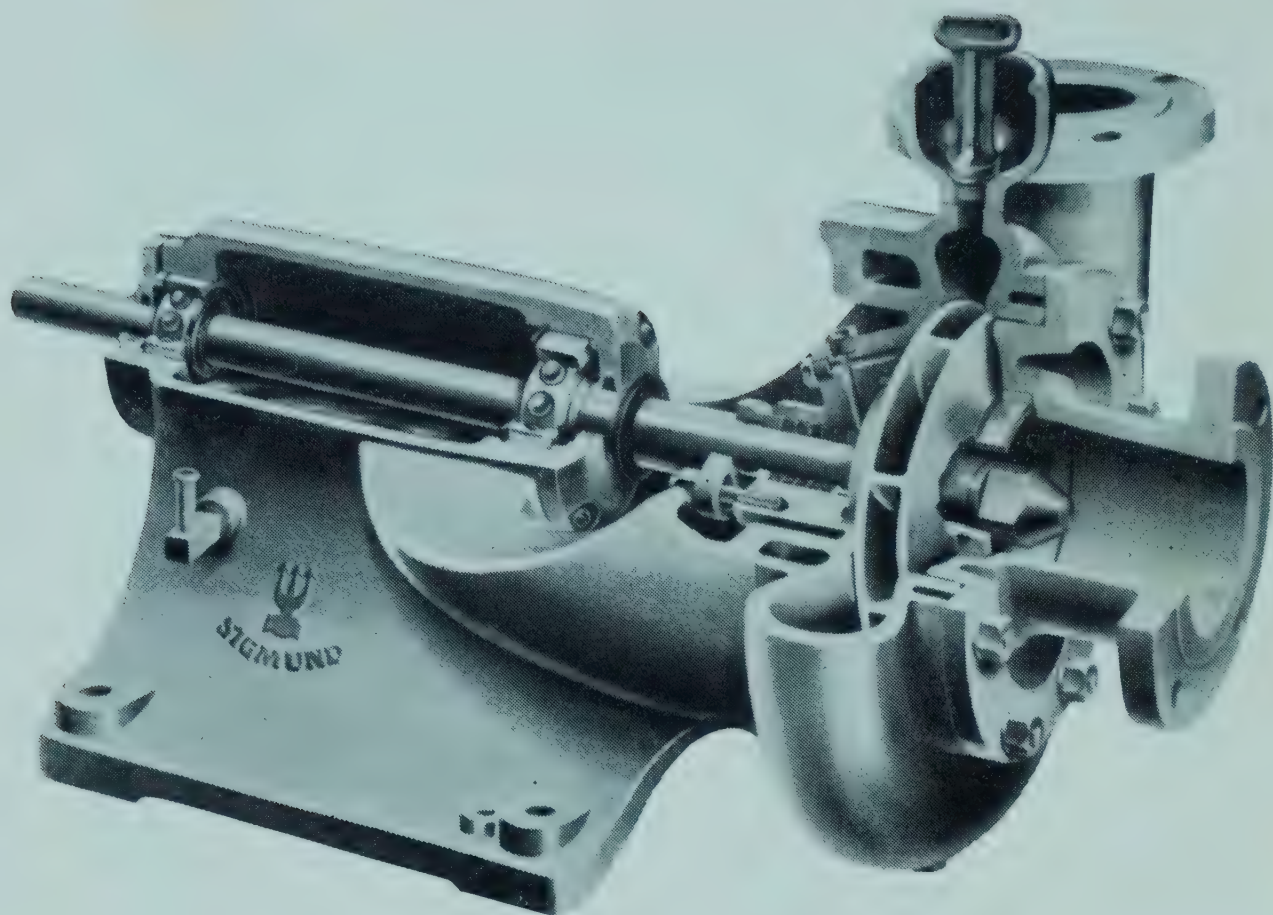


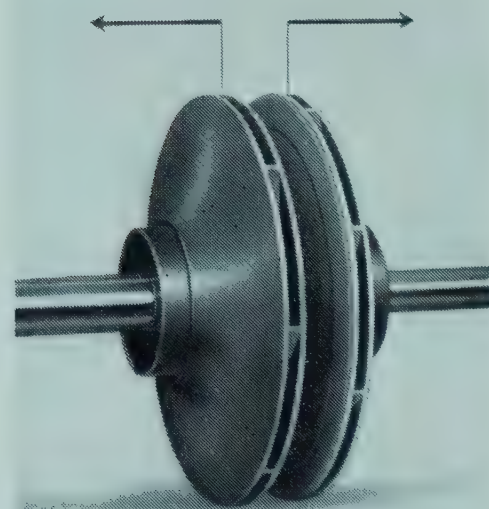
Fig. 5.13. Section of centrifugal pump
(Courtesy of Sigmund Pumps Ltd.)



(a)



(b)



(c)

Fig. 5.14. Types of impeller
(Courtesy of Sigmund Pumps Ltd.)

- (a) As used for pumping suspensions.
- (b) Standard closed impeller.
- (c) Double impeller.

The Centrifugal Pump

In the centrifugal pump (Fig. 5.13) the liquid is fed to the centre of a rotating impeller and is thrown outwards by centrifugal action. As a result of the high speed of rotation of the impeller the liquid acquires a large kinetic energy and a pressure difference between the suction and delivery sides of the pump is produced by the conversion of kinetic energy into pressure energy. The suction and delivery pressures of the pump depend on the efficiency with which this conversion of energy from one form to another takes place.

The impeller (Fig. 5.14) consists of a series of curved vanes so shaped that the flow within the pump is as smooth as possible. The greater the number of vanes on the impeller, the greater is the control over the direction of motion of the liquid and hence the smaller are the losses due to turbulence and circulation between the vanes. In the open impeller the vanes are fixed to a central hub; whereas in the closed type the vanes are held between two supporting plates and leakage across the impeller is reduced. As will be seen later, the angle of the tips of the blades very largely determines the operating characteristics of the pump.

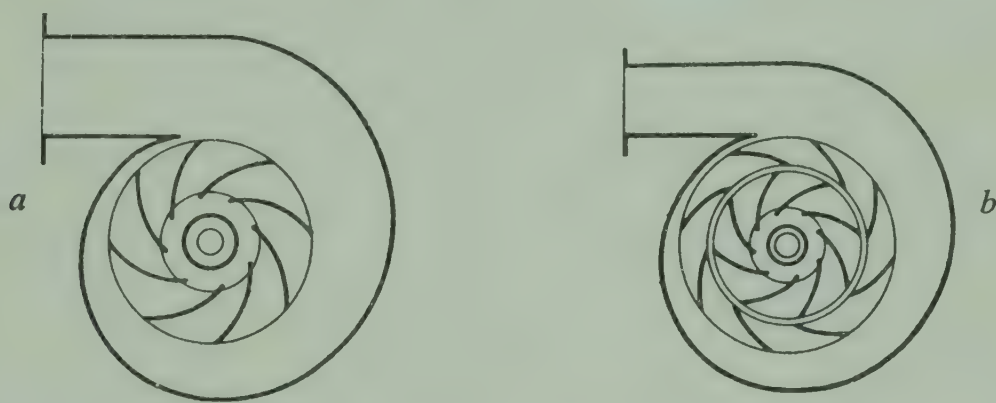


Fig. 5.15. (a) Volute and (b) diffuser pumps

The liquid enters the casing of the pump, normally in an axial direction, and is picked up by the vanes of the impeller. In the simple type of centrifugal pump, the liquid discharges into a volute, a chamber of gradually increasing cross section with a tangential outlet. A volute type of pump is shown in Fig. 5.15a. In the turbine pump (Fig. 5.15b), the liquid flows from the moving vanes of the impeller through a series of fixed vanes forming a diffusion ring. This gives a more gradual change in direction to the fluid and more efficient conversion of kinetic energy into pressure energy than is obtained with the volute type. The angle of the leading edge of the fixed vanes should be such that the fluid is received without shock. The liquid flows along the surface of the impeller vane with a certain velocity whilst the tip of the vane is moving relative to the casing of the pump. The direction of motion of the liquid relative to the pump casing—and the required angle of the fixed vanes—is found by compounding these two velocities. In Fig. 5.16(a), u_r is the velocity of the liquid relative to the vane and u_t is the tangential velocity of the tip of the vane; compounding these two velocities gives the resultant velocity u_2 of the liquid. It is apparent therefore that the required vane angle in the diffuser is dependent on the throughput,

the speed of rotation, and the angle of the impeller blades. The pump will therefore operate at maximum efficiency only over a narrow range of conditions.

Virtual Head of a Centrifugal Pump. The maximum pressure is developed when the whole of the excess kinetic energy of the fluid is converted into pressure energy. As indicated below, the head is proportional to the

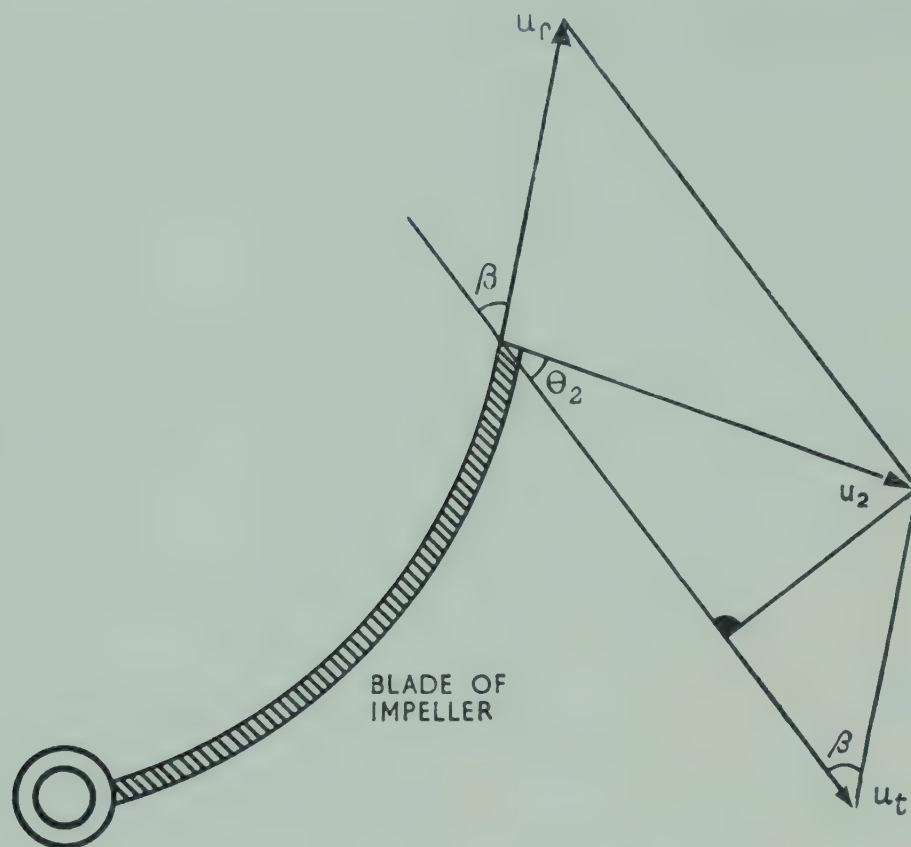


Fig. 5.16 (a). Velocity diagram

square of the radius and to the speed, and is of the order of 100 feet for a single stage centrifugal pump; for higher pressures multistage pumps must be used. Consider the liquid which is rotating at a distance of between r and $r + dr$ from the centre of the pump. Mass of this element of fluid

$$= 2\pi r dr b \rho$$

where ρ is the density of the fluid, and b is the width of the element of fluid.

If the fluid is travelling with a velocity u and at an angle θ to the tangential direction, the angular momentum of this mass of fluid

$$= (2\pi r dr b \rho)(u \cdot r \cdot \cos \theta)$$

The torque acting on the fluid $d\tau$ is equal to the rate of change of angular momentum with time, therefore

$$d\tau = (2\pi r dr b \rho) \frac{\partial}{\partial t} (u \cdot r \cos \theta)$$

The volumetric rate of flow of liquid through the pump,

$$Q = 2\pi r b \frac{\partial r}{\partial t}$$

\therefore

$$d\tau = Q \rho d(u \cdot r \cos \theta)$$

The total torque acting on the liquid in the pump is therefore obtained by integrating $d\tau$ between the limits denoted by suffix 1 and suffix 2, where suffix 1 refers to the conditions at the inlet to the pump and suffix 2 refers to the conditions at the discharge. Thus

$$\tau = Q\rho(u_2r_2 \cos \theta_2 - u_1r_1 \cos \theta_1) \quad \dots(5.1)$$

The power p developed by the pump is equal to the product of the torque and the angular velocity ω ; therefore

$$p = Q\rho\omega(u_2r_2 \cos \theta_2 - u_1r_1 \cos \theta_1) \quad \dots(5.2)$$

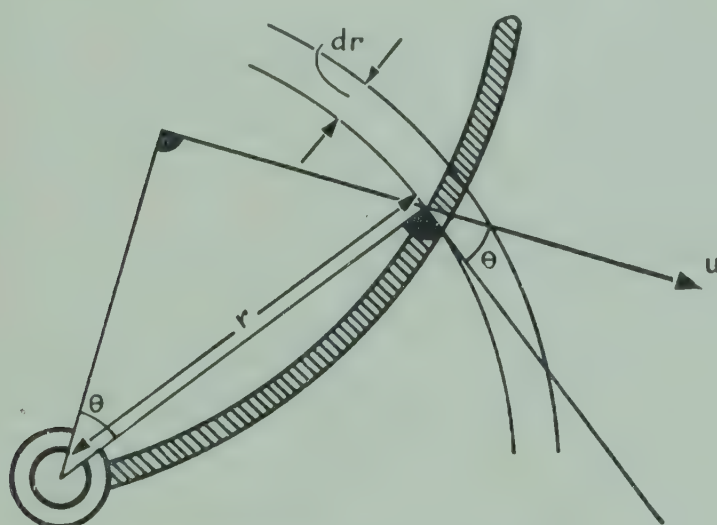


Fig. 5.16 (b). Virtual head

The power can also be expressed as the product $G \cdot h \cdot g$, where G is the mass rate of flow of liquid through the pump, g is the acceleration due to gravity, and h is termed the virtual head developed by the pump.

Thus
$$G \cdot h \cdot g = Q\rho\omega(u_2r_2 \cos \theta_2 - u_1r_1 \cos \theta_1)$$

i.e.
$$h = \frac{\omega}{g} (u_2r_2 \cos \theta_2 - u_1r_1 \cos \theta_1) \quad \dots(5.3)$$

Since u_1 will be approximately zero, the virtual head

$$= \frac{\omega}{g} \cdot u_2r_2 \cos \theta_2 \quad \dots(5.4)$$

where g , ω and r_2 are known in any given instance, and u_2 and θ_2 are to be expressed in terms of known quantities.

From the geometry of Fig. 5.16 (b) we have

$$u_r \sin \beta = u_2 \sin \theta_2 \quad \dots(5.5)$$

and
$$u_t = u_r \cos \beta + u_2 \cos \theta_2 \quad \dots(5.6)$$

(where β is the angle between the tip of the blade of the impeller and the tangent to the direction of its motion. If the blade curves backwards, β lies between 0 and 90° and if it curves forwards, β lies between 90° and 180° .)

The volumetric rate of flow through the pump Q is equal to the product of

the area available for flow at the outlet of the impeller and the radial component of the velocity, i.e.

$$\begin{aligned} Q &= 2\pi r_2 b u_2 \sin \theta_2 \\ &= 2\pi r_2 b u_r \sin \beta \quad (\text{from 5.5}) \end{aligned} \quad \dots(5.7)$$

$$\therefore u_r = \frac{Q}{2\pi r_2 b \sin \beta} \quad \dots(5.8)$$

Thus

$$\begin{aligned} h &= \frac{\omega}{g} r_2 (u_t - u_r \cos \beta) \quad (\text{from 5.4 and 5.6}) \\ &= \frac{\omega}{g} r_2 \left\{ r_2 \omega - \frac{Q}{2\pi r_2 b \tan \beta} \right\} \quad (\text{since } u_t = r_2 \omega) \\ &= \frac{r_2^2 \omega^2}{g} - \frac{Q \omega}{2\pi b g \tan \beta} \end{aligned} \quad \dots(5.9)$$

The virtual head developed by the pump is therefore independent of the density of the fluid, and the pressure will thus be directly proportional to the density. For this reason a centrifugal pump needs priming. If the pump is initially full of air, the pressure developed is reduced by a factor equal to the ratio of the density of air to that of the liquid, and is insufficient to drive the liquid through the delivery pipe.

For a given speed of rotation there is a linear relation between the head developed and the rate of flow. If the tips of the blades of the impeller are inclined backwards, β is less than 90° , $\tan \beta$ is positive and therefore the head decreases as the throughput increases. If β is greater than 90° (i.e. the tips of the blades are inclined forwards), the head increases as the delivery increases. The angle of the blade tips therefore profoundly affects the performance and characteristics of the pump. For radial blades, the head should be independent of the throughput.

If θ_2 remains approximately constant u_r , u_t , and u_2 will all be directly proportional to one another and to $r_2 \cdot \omega$, since $u_t = r_2 \cdot \omega$.

The output from a centrifugal pump is a function of its linear dimensions, the shape, number and arrangement of the impellers, the speed of rotation, and the head against which it is operating. From equations 5.7 and 5.9 we then have

$$Q \propto r_2^2 \cdot b \cdot \omega \quad \dots(5.10)$$

and

$$h \propto r_2^2 \cdot \omega^2 \quad \dots(5.11)$$

Thus the larger the pump, the greater is the head which it will develop. The power p developed by the pump is proportional to the product $Q \cdot h$.

$$\therefore p \propto r_2^4 \cdot b \cdot \omega^3 \quad \dots(5.12)$$

For a given pump, therefore,

$$Q \propto \omega, h \propto \omega^2, \text{ and } p \propto \omega^3$$

Again for a series of geometrically similar pumps in which $b \propto r_2$, for a given speed of rotation,

$$Q \propto r_2^3, h \propto r_2^2, \text{ and } p \propto r_2^5$$

If r_2 is eliminated using equations 5.10 and 5.11, we have

$$\frac{\omega Q^{\frac{1}{2}}}{h^{\frac{3}{4}}} = \text{constant, for geometrically similar pumps}$$

Since ω is proportional to N , the number of revolutions per unit time, the dimensionless group $\frac{N \cdot Q^{\frac{1}{2}}}{(gh)^{\frac{3}{4}}}$ is a constant. This is referred to as the "specific speed" of the pump, and is useful in correlating the performances of geometrically similar pumps. It is usually quoted for the conditions at which the pump is operating at maximum efficiency and varies from 0.03 to 0.9. The specific speed is often quoted as the speed of rotation (r.p.m.) required to give unit rate of discharge (gallons per minute) when the pump is delivering water against a head of 1 ft.

Pump Characteristics. The actual head is always less than the virtual head; the main reasons for this are as follows:

(1) The fluid tends to circulate and form eddies in the spaces between the vanes. This effect can be minimized by using a large number of vanes on the impeller.

(2) Frictional losses occur in the suction port, the impeller and the delivery from the pump. These increase with increase in the speed of the pump and with increase in the viscosity of the liquid.

(3) Losses occur as the fluid is discharged from the impeller. The vane angles are correct only for a limited range of conditions. Deviation from these conditions in either direction causes an increase in the losses due to turbulence.

(4) Leakage reduces the head developed, especially at low discharge rates, because the leakage then accounts for a higher proportion of the total throughput.

The operating characteristics of a particular centrifugal pump are most conveniently given in the form of curves of head developed against delivery for various running speeds. If lines of constant efficiency are then plotted they take the form of closed loops, the curves denoting the higher efficiencies lying inside those of low efficiency. Typical curves are shown in Fig. 5.17.

If the delivery valve of the pump is closed, all the kinetic energy of the fluid will be converted into heat as a result of the turbulence set up. This will not damage the pump unless it is run under these conditions sufficiently long for overheating to occur; this is most likely to take place with a large pump.

In some pumps the necessity for priming is obviated by arranging that the liquid remaining in the casing is emulsified with the air which is then vented

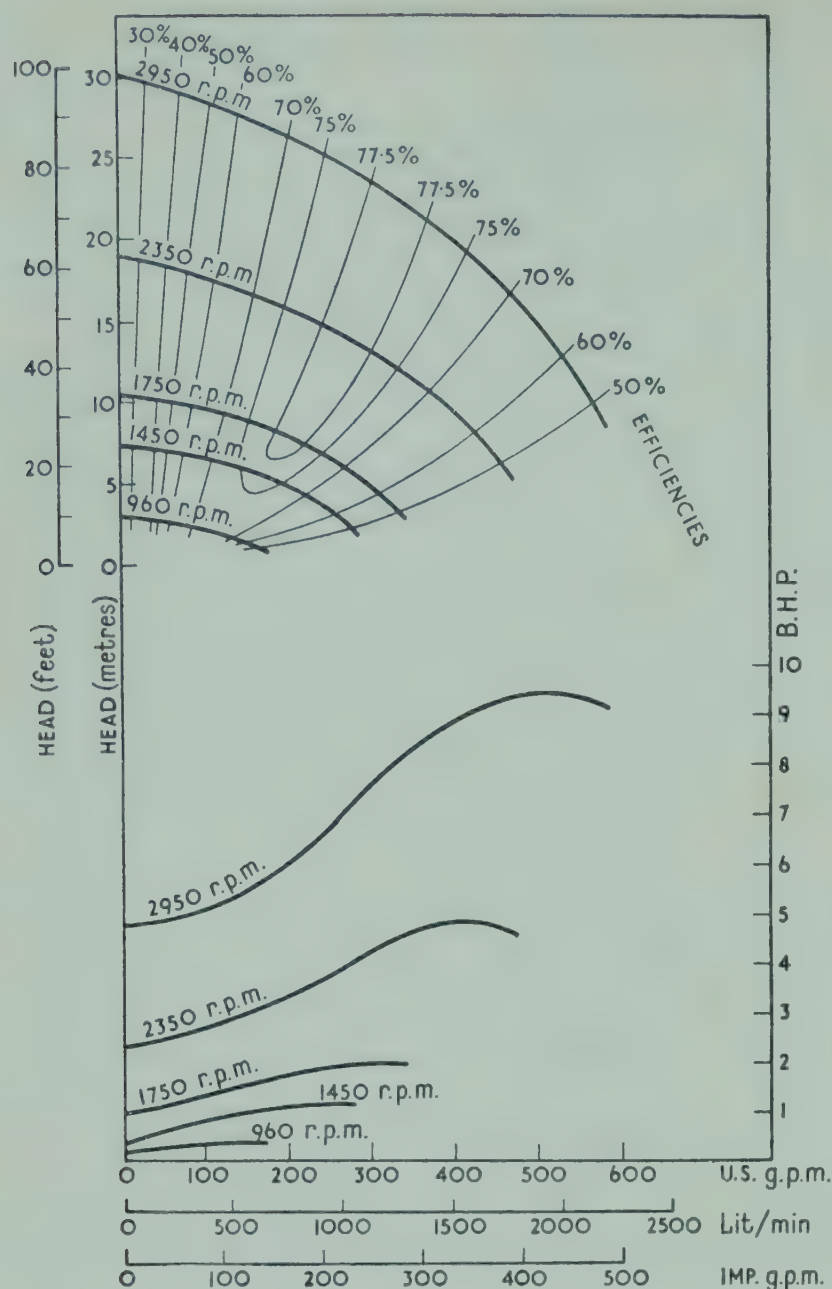


Fig. 5.17. Characteristic curves for centrifugal pump
(Courtesy of Sigmund Pumps Ltd.)

from the system. These pumps tend to be expensive and their efficiencies are not always high. A self-priming pump is shown in Fig. 5.19.

The flow relative to the impeller must be as smooth as possible so that local variations of pressure are avoided. If the pressure at some point in the pump falls to a very low value, "cavitation" may occur. This involves vapour formation and damage to the moving parts is caused subsequently by the sudden collapsing of the vapour bubbles. It is a phenomenon similar to knocking in the reciprocating pump. The occurrence of cavitation indicates either that the impeller is badly designed for the particular conditions under which the pump is operating, or that the pump speed is too high. It is sometimes possible to eliminate cavitation by artificially increasing the pressure at the suction side of the pump, possibly by opening the valve on the suction line more widely.

Centrifugal pumps must be fitted with good bearings, since there is a tendency for an axial thrust to be produced if the suction is taken only on one side of the impeller. This thrust can be balanced by feeding back a small quantity of the high pressure liquid to a specially designed thrust bearing. By this method the risk of air leaking into the pump at the gland and reducing the pressure developed is minimized. The glandless centrifugal pump, which is used extensively for corrosive liquids, works on a similar principle and a small

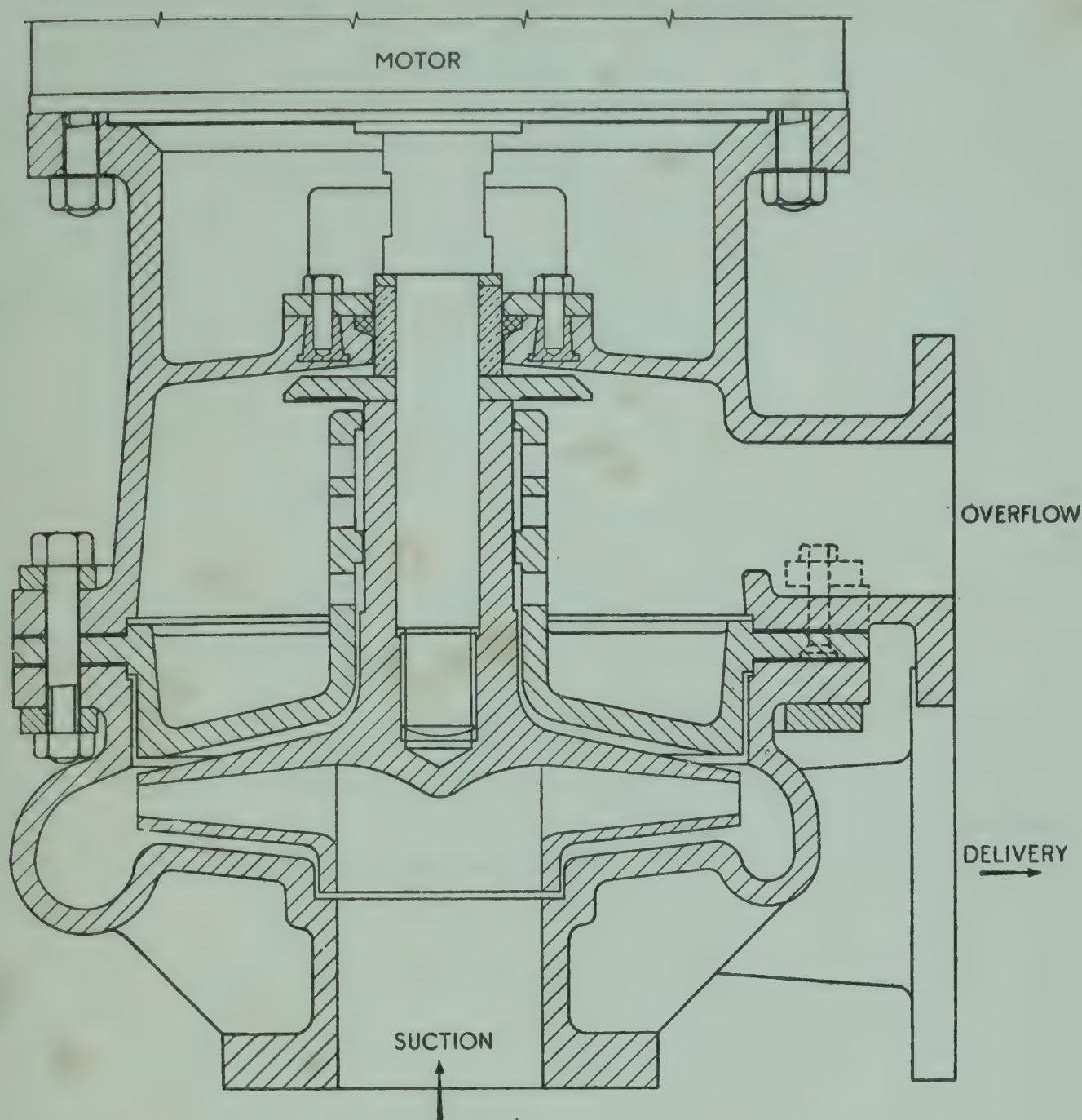


Fig. 5.18. Kestner patent glandless centrifugal pump
(Courtesy of Kestner Evaporator & Engineering Co. Ltd.)

flow of liquid past the pump shaft is permitted. An end thrust is sometimes produced as a result of the high pressure liquid circulating behind closed impellers. This can be reduced by drilling small holes in the impeller but the efficiency of the pump is somewhat reduced by this means because circulating currents are set up. Pumps with double impellers (Fig. 5.14c) take suction on both sides and difficulties arising from asymmetry are thereby avoided.

Centrifugal pumps are widely used in the chemical industry because they can be constructed to handle corrosive liquids and suspensions. A glandless centrifugal pump is shown in Fig. 5.18, and a special impeller for use with suspensions is illustrated in Fig. 5.14(a). These pumps can be made of a wide range of materials and in many cases the impeller and the casing are covered with resistant material. Thus high silicon iron, nickel, rubber, stoneware, and carbon are all used. When the pump is used with suspensions the ports and spaces between the vanes must be made sufficiently large to eliminate the risk of blockage. This does mean, however, that the efficiency of the pump is reduced. The "Vacseal Pump," developed by the

International Combustion Company for pumping slurries, will handle suspensions containing up to 50% by volume of solids. The whole impeller may be rubber covered and has three small vanes, as shown in Fig. 5.20. The back of the plate of the impeller has a second set of vanes of larger diameter. The pressure at the gland is thereby reduced below atmospheric pressure and below the pressure in the suction line; there is, therefore, no risk of the gritty suspension leaking into the gland and bearings. If leakage does occur, air will enter the pump. As mentioned previously this may reduce the pressure which the pump can deliver, but this is preferable to damaging the bearings by allowing them to become contaminated with grit. This is another example of the necessity for tolerating rather low efficiencies in pumps that handle difficult materials.

The Advantages and Disadvantages of the Centrifugal Pump. The main advantages are:

1. It is simple in construction and cheap. It can, therefore, be made in a wide range of materials.
2. There is a complete absence of valves.
3. It operates at high speed (up to 4000 r.p.m.) and, therefore, can be coupled directly to an electric motor. In general, the higher the speed the smaller the pump and motor for a given duty.
4. It gives a steady delivery.
5. Maintenance costs are lower than for any other type of pump.
6. No damage is done to the pump if the delivery line becomes blocked, provided it is not run in this condition for a prolonged period.
7. It is much smaller than other pumps of equal capacity. It can, therefore, be made into a sealed unit with the driving motor and immersed in the suction tank.
8. Liquids containing high proportions of suspended solids are readily handled.

The main disadvantages are:

1. The single stage pump will not develop a high pressure. Multistage pumps will develop greater heads but they are very much more expensive and cannot readily be made in corrosion resistant material because of their greater complexity. It is generally better to use very high speeds in order to reduce the number of stages required.
2. It operates at a high efficiency over only a limited range of conditions: this applies especially to turbine pumps.
3. It is not usually self priming.
4. If a non-return valve is not incorporated in the delivery or suction line, the liquid will run back into the suction tank as soon as the pump stops.
5. Very viscous liquids cannot be handled efficiently.

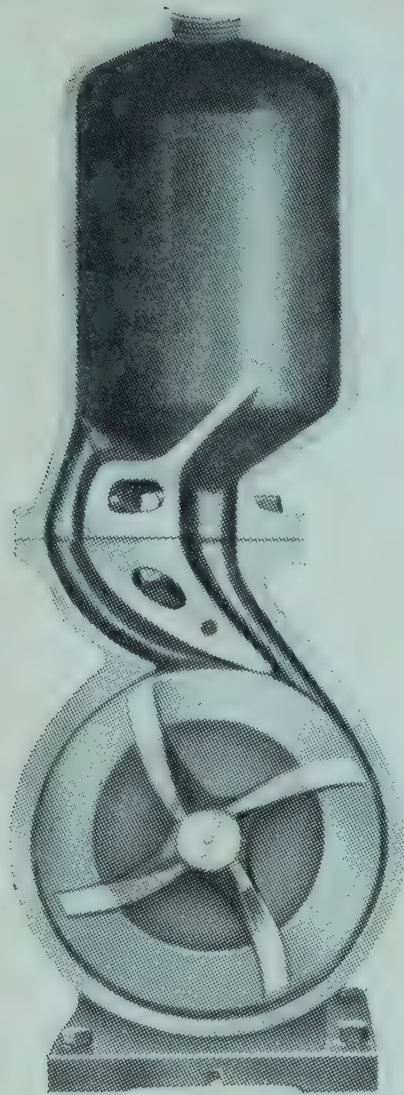


Fig. 5.19. Self-priming centrifugal pump
(Courtesy of *British LaBour Pump Co. Ltd.*)

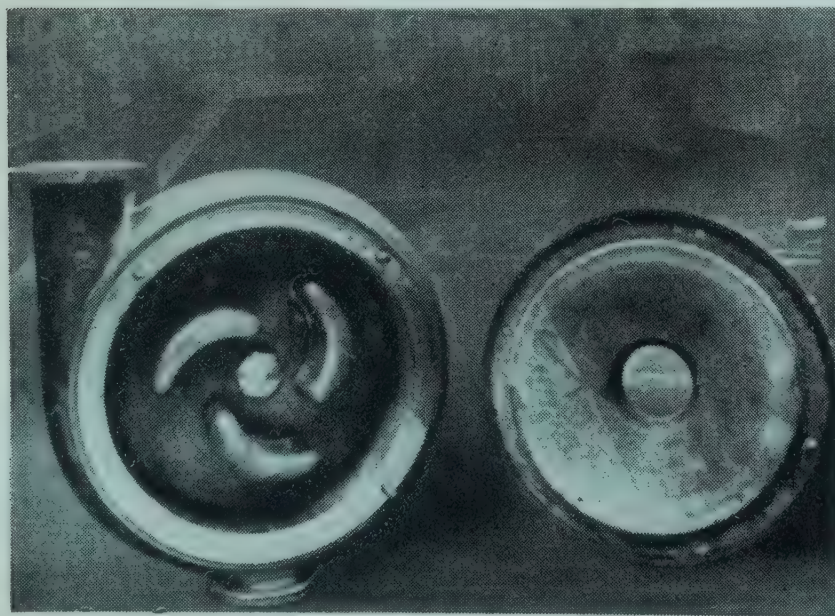


Fig. 5.20. Vacseal grit pump
(Courtesy of *International Combustion Ltd.*)

Metering Pumps

Metering pumps are positive displacement pumps, driven by constant speed electric motors. They are used where a constant rate of supply of liquid is required, irrespective of the pressure. The motor, therefore, should be of such a power that it is not appreciably retarded as the load increases. The delivery is varied by an adjustment on the pump itself. The metering pump is usually a plunger type pump (Fig. 5.21) incorporating one or more plungers and the delivery is varied by an adjustment of the length of the stroke. In some cases, the plungers are replaced by a flexible diaphragm (Fig. 5.5), whose movement can be regulated. These pumps are being increasingly used in chemical works

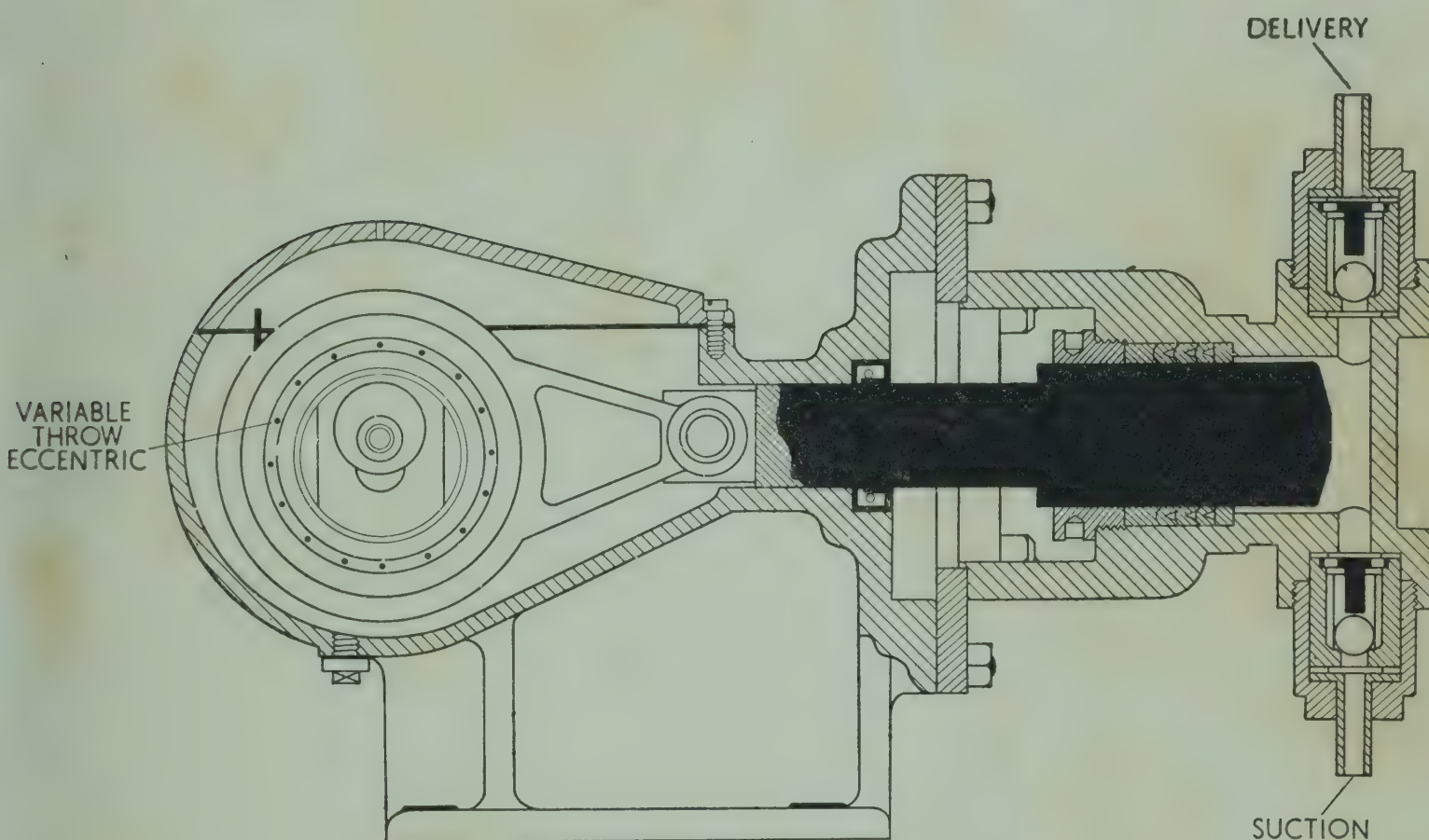


Fig. 5.21. Metering pump
(Courtesy of E.C.D. Ltd.)

since they offer a very convenient method of giving a uniform and at the same time easily varied rate of delivery. This is particularly useful for feeding reaction vessels, for instance in organic nitrations or sulphonations. Two or three pumps are frequently driven from the same motor thus giving a neat unit which enables not only the absolute rate but also the ratio of two or more rates to be adjusted.

The Use of Compressed Air for Pumping

One form of apparatus, as made by the Kestner Company⁽⁷⁾, for pumping liquids in which compressed air is the motive power is illustrated in Fig. 5.22. The liquid to be pumped flows by gravity through a non-return valve into the main vessel or egg. The float gradually rises and when the egg is full shuts off the feed, and at the same time operates a valve mechanism (Fig. 5.23) which admits compressed air to the egg; the air exhaust valve is closed by the pressure of the compressed air. The part of the top cover where the float moves is made

narrow to give a sharp cut off of fluid and quick action. The air pressure forces the liquid out of the egg up the delivery pipe and when the liquid level falls the float drops and shuts off the compressed air supply. The liquid and compressed air are discharged through the delivery line and the cycle repeated. The valve mechanism is situated above the surface of the liquid in the feed tank so as to eliminate risk of flooding.

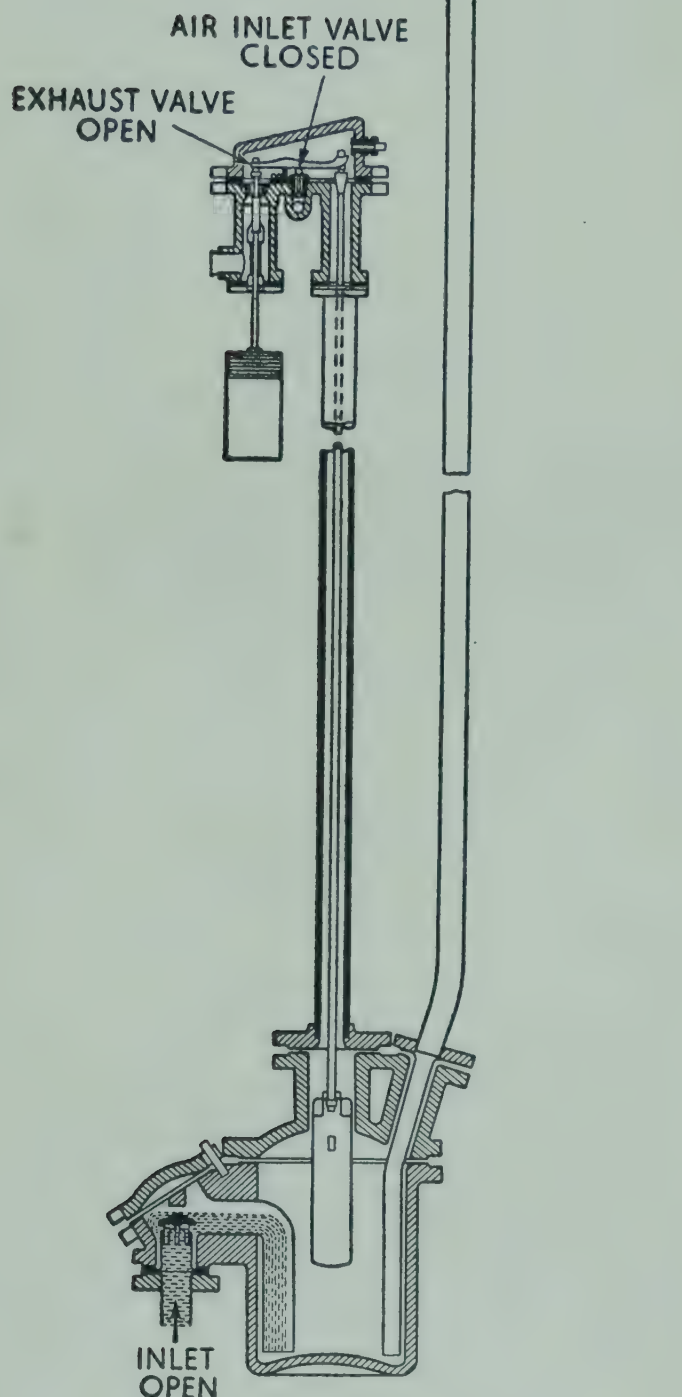


Fig. 5.22. Acid elevator

The efficiency of these pumps is low, since it is difficult to make use of the air under pressure in the vessel, and it is important that the automatic valve mechanism is correctly adjusted as otherwise there will be excessive waste of air. Liquids, particularly corrosive ones, may be blown from storage tanks or from reaction vessels by introducing compressed air but these are illustrations of batch processes where convenience and not efficiency is the main consideration. Since the only moving parts are the valves, and these are well away from the liquor, these pumps are used mainly for acids and other corrosive liquors, where their low efficiency must be accepted. They can conveniently be made from acid-resisting material such as tantiron (high silicon iron), since there is very little machining required.

Consider one cycle of an automatic elevator. If M_e is the mass of liquid held by the egg and h_r is the net increase in height, the useful work done per cycle is $M_e g h_r$.

If V_a is the volume of free air used per cycle and P is the pressure of the air supply, the total available energy in the air, assuming an isothermal expansion, is $P_a V_a \ln \frac{P}{P_a}$, where P_a is atmospheric pressure. Thus the overall efficiency of the pump itself

$$= \frac{M_e g h_r}{P_a V_a \ln \frac{P}{P_a}} \quad \dots (5.13)$$

When considering the overall efficiency of the plant, the energy losses in the compressor and air lines must also be taken into account.

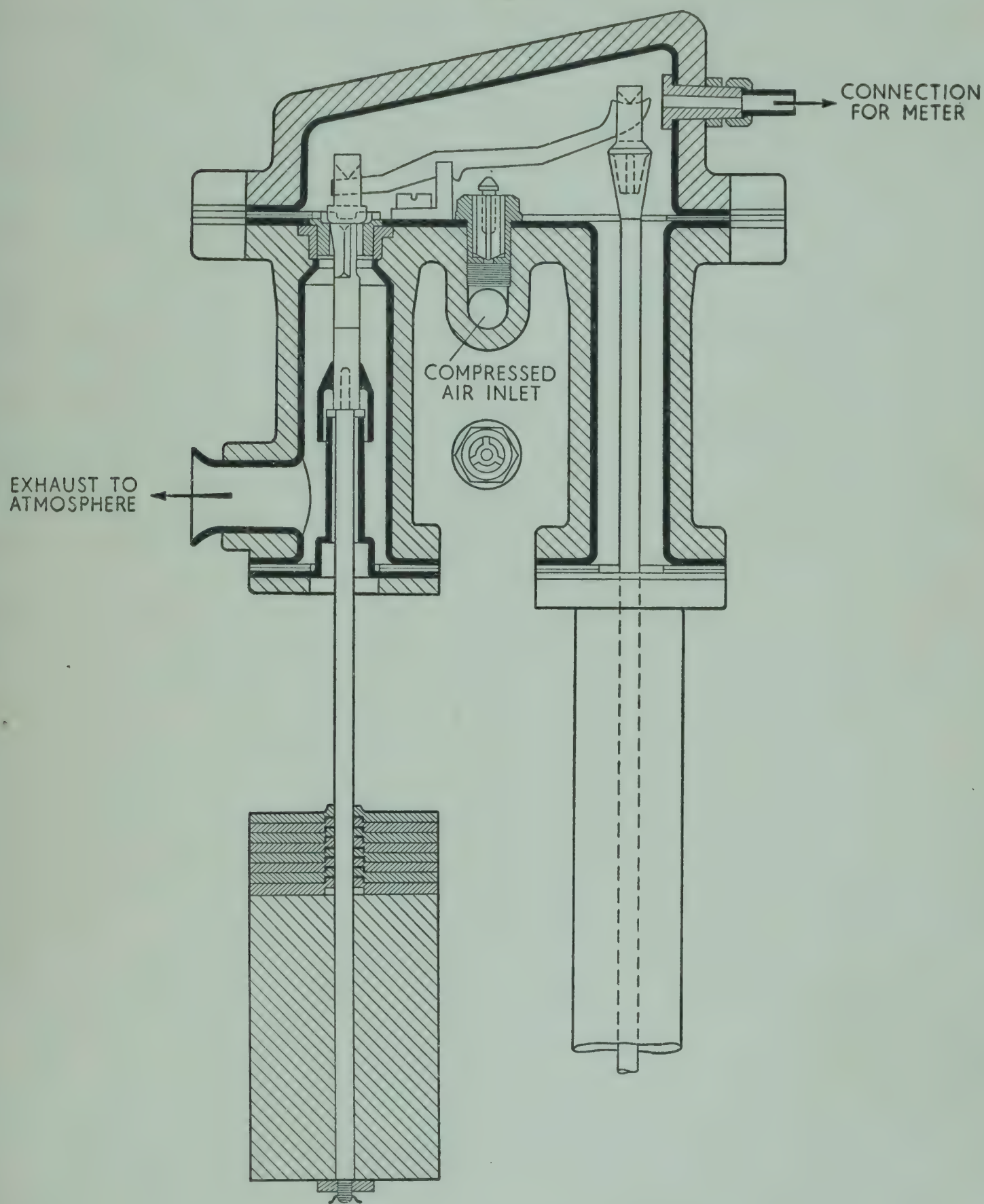


Fig. 5.23. Automatic valve of acid elevator

The main energy losses which account for the low efficiency of the acid elevator are as follows:

(1) The potential energy of the liquid in the feed tank is converted partly into kinetic energy, which is subsequently dissipated, and the remainder is utilized in overcoming friction losses in the feed line. If the feed tank is lowered,

however, the egg will take longer to fill and the capacity of the plant will be reduced.

(2) The kinetic energy of the fluid as it emerges from the delivery line is not, in general, recoverable unless a bell mouth is fitted to the end of the pipe.

(3) Energy is dissipated as the result of friction losses in the delivery line.

(4) The pressure energy of the air remaining in the egg after the expulsion of the liquid is not available, since the air expands to atmospheric pressure without doing useful work.

The elevator operates most efficiently at low air pressures, since the losses under headings (2), (3), and (4) are then a minimum, whilst that under

heading (1) is independent of the operating pressure. A lower limit is, however, set to the operating pressure since the pressure in the egg, after the ejection of the liquid, must be greater than that due to the static pressure of the liquid in the delivery line. In practice, however, the rate of delivery that is required will set a lower limit to the pressure employed, because the delivery rate rises with increase of pressure. Air pressures up to about 100 lb/sq in. are frequently used.

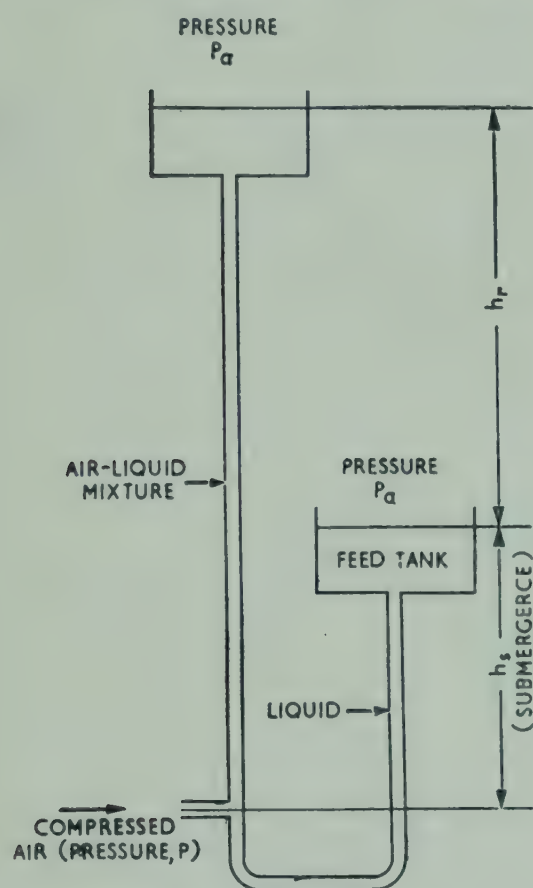


Fig. 5.24. Air lift pump

The Air Lift Pump. In the air lift pump (Fig. 5.24) a higher efficiency is obtained by using the air expansively so that it expands to atmospheric pressure in contact with the liquid. It can be regarded as a U-tube in a state of dynamic equilibrium. One limb containing only liquid is relatively short and is connected to the feed tank, whilst air is injected near the bottom

of the longer limb which therefore contains a mixture of liquid and air of lower density. If the air is introduced sufficiently rapidly, liquid will flow from the short to the long limb and be discharged into the delivery tank. The rate of flow will depend on the difference in density and will, therefore, rise as the air rate is increased, but will reach a maximum because the frictional resistance increases with the volumetric rate of flow.

If a mass M of liquid is raised through a net height h_r by a mass m of air, the net work done on the liquid is $M \cdot g \cdot h_r$. If the pressure of the entering air is P , the work done by the air in expanding isothermally to atmospheric pressure,

P_a , is $P_a v_a m \ln \frac{P}{P_a}$, where v_a is the specific volume of air at atmospheric pressure.

The expansion will be almost exactly isothermal because of the intimate contact between the liquid and the air.

The efficiency of the pump E is therefore equal to

$$\frac{M \cdot g \cdot h_r}{m P_a v_a \ln \frac{P}{P_a}} \quad \dots (5.14)$$

The mass of air required to pump unit mass of liquid is, therefore, given by

$$\frac{m}{M} = \frac{g \cdot h_r}{E P_a v_a \ln \frac{P}{P_a}} \quad \dots (5.15)$$

If the weight of fluid in the long limb is only infinitesimally less than that in the short limb, the pump will deliver infinitely slowly at an efficiency of unity. The pressure at the point of introduction of the compressed air will then be equal to atmospheric pressure together with the pressure due to the column of liquid of height h_s , the vertical distance between the liquid level in the suction tank and the air inlet point. Losses arising from the relative motion between the air and liquid, and the turbulence resulting therefrom, are neglected for the moment. We can therefore write

$$P_a = h_a \rho g \text{ (say)}$$

and

$$P = (h_a + h_s) \rho g$$

where ρ is the density of the liquid.

Thus from equation 5.15 the mass of air required to pump unit mass of liquid $\frac{m}{M}$ is equal to

$$\frac{h_r g}{P_a v_a \ln \frac{h_s + h_a}{h_a}} \quad \dots (5.16)$$

for the perfectly efficient pump. This is therefore the minimum air requirement for the pump if all losses are neglected. It will be seen that $\frac{m}{M}$ decreases as h_s increases; if h_s is zero $\frac{m}{M}$ is infinite and therefore the pump will not work. A high submergence (h_s) is therefore desirable.

The Ingersoll-Rand Company has developed an entirely empirical formula for the volume of free air required to raise 1 gal of water.

$$V_{\text{air}} = 0.67 \frac{h_r}{C \log_{10} \left(\frac{h_s + 34}{34} \right)} \quad \dots (5.17)$$

where V_{air} is the required volume of free air in cubic feet and C has the following values,

h_r (feet)	C
10-60	245
61-200	233
201-500	216
501-650	185
651-750	156

h_r and h_s are both expressed in feet.

It will be seen that this empirical equation has the same form as equation 5.16.

We will now consider the pressure distribution in the limb containing the air-liquid mixture. The fluid consists of liquid of fixed density, and of air whose density varies with height in the limb. Consider a pipe of cross-sectional area A , and a point in it above which the volume of liquid is $A \cdot s$. The total pressure at that point is then $(h_a + s)\rho \cdot g$. In an infinitesimal length of pipe at this depth, we have a volume $A \cdot ds$ of liquid. The mass of liquid is therefore $\rho A ds$ and the mass of air associated with it is $\frac{m}{M} \rho A ds$ if the two components are uniformly distributed. The corresponding volume of air is then $\frac{m}{M} \rho A ds \cdot v_a \frac{h_a}{h_a + s}$.

The total volume of liquid and air in the element

$$dV_b = A ds \left(1 + \frac{m}{M} \rho v_a \frac{h_a}{h_a + s} \right)$$

The total volume of liquid + air is obtained by integrating between the limits of $s = 0$ and $s = s$.

$$\text{Thus} \quad V_b = A \left(s + \frac{m}{M} \rho v_a h_a \ln \frac{s + h_a}{h_a} \right) \quad \dots (5.18)$$

This volume of liquid is contained in a length l of pipe where

$$l = s + \frac{m}{M} \rho v_a h_a \ln \frac{s + h_a}{h_a} \quad \dots (5.19)$$

This expression gives the fluid head s at a distance l from the top of the pipe. If we consider now the whole of the limb,

$$l = h_r + h_s$$

and

$$s = h_s$$

Therefore

$$h_r = \frac{m}{M} \rho v_a h_a \ln \frac{h_s + h_a}{h_a}$$

i.e.

$$\frac{m}{M} = \frac{h_r g}{P_a v_a \ln \frac{h_s + h_a}{h_a}} \quad (\text{since } P_a = h_a \rho g)$$

which is identical with equation 5.16.

Operation of the Air Lift Pump. Air is admitted at the bottom of the larger limb through a series of orifices in a "footpiece," with as little shock as possible so that losses due to turbulence in the gas-liquid mixture are minimized. Two typical footpieces are shown in Fig. 5.25. The behaviour of the mixture will depend to some extent on the size of the individual air bubbles because this influences their velocity of rise relative to the liquid. If the air is in the form of

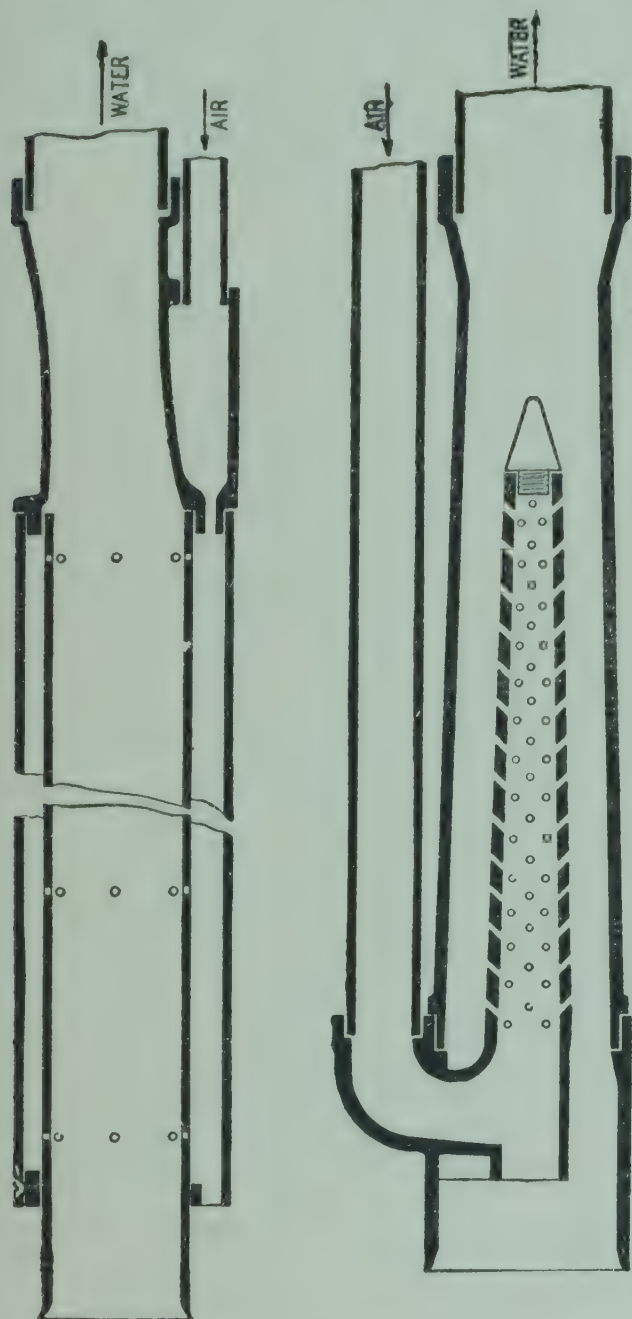


Fig. 5.25. Footpieces for air lift pump

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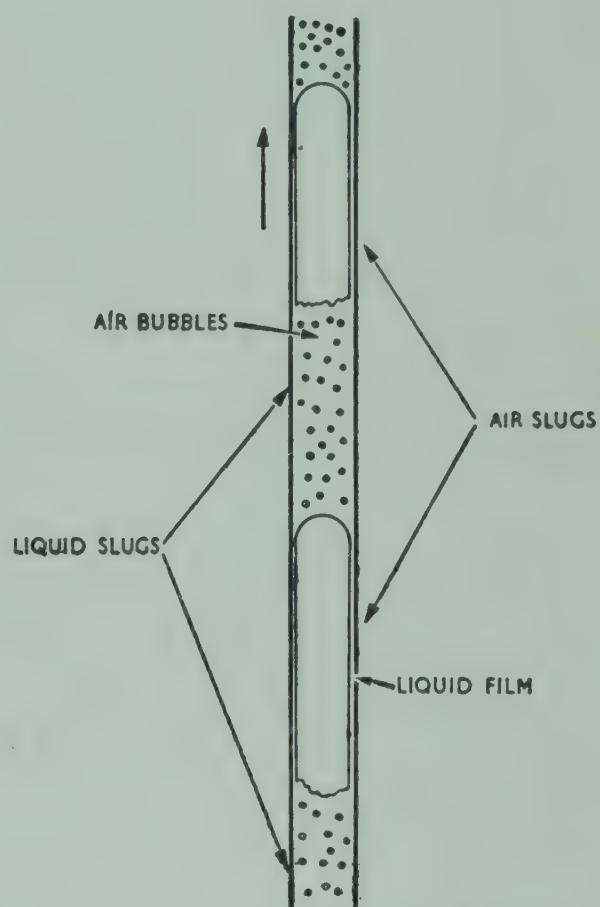


Fig. 5.26. Air-liquid slugs in rising liquid

small bubbles the relative velocity should be reduced to a minimum. Great difficulty is experienced, however, in forming very small bubbles at the rates required for the operation of air lift pumps, because coalescence occurs at the footpiece and subsequently in the limb of the pump. It has been found, as a result of experiments on a small laboratory air lift pump of 1 in. diameter, that the conditions about 6 ft above the footpiece are almost independent of the manner in which the air is introduced. If the air is introduced as small bubbles, coalescence is almost complete by this stage, and the dispersion is similar to that obtained by admitting large air bubbles. The behaviour of the mixture has been observed in a glass section of the pump, at a

height of about 10 ft from the footpiece. The first impression obtained is that alternate slugs of liquid and air are passing up the pipe. Further examination reveals that the liquid slugs contain suspended bubbles of air; the air slug occupies only the central core of the cross section and a film of liquid flows downwards at the walls. Thus each liquid slug is continuously draining liquid to the slug below, and the apparent length of each remains constant. If the rate of drainage is equal to the rate at which liquid is carried upwards in the slugs, there will be no discharge from the pump. If the drainage rate is less than the upward rate of transfer, a delivery is obtained. A diagram showing the slugs formed in a 1 in. pipe is given in Fig. 5.26, for the plant operating with water.

The actual operation is thus more complex than that suggested by the simple picture of a well-dispersed air-liquid mixture moving uniformly upwards in the pipe. The behaviour is further complicated by oscillations in the flow rate. When the pump is started the liquid has to be accelerated and therefore rather more air is admitted than is required to maintain uniform motion. As a consequence, the fluid becomes over-accelerated and excess liquid enters the limb, with the result that it becomes retarded and has subsequently to be accelerated again. During the period of retardation some liquid may actually run backwards; this can readily be avoided by including a non-return valve on the suction line.

The losses which occur in the air lift pump can be grouped as under⁽⁸⁾:

1. *In the footpiece*: The air enters the liquid stream through a series of orifices, and energy losses occur because of the sudden changes in the area available for flow and as a result of friction at the walls of the orifices. Further, the liquid is accelerated as it passes through the footpiece, because the total volumetric rate of flow of fluid is increased by the addition of the air. In order to accelerate the liquid as smoothly as possible, the air should be admitted in the form of relatively small bubbles over a considerable length of pipe, as in the Frizell⁽⁵⁾ type of footpiece. In the Pohlé⁽⁶⁾ footpiece air is introduced as relatively large bubbles so that alternate slugs of air and liquid form immediately in the rising pipe.

2. *Losses in the lines*: Friction losses in the rising limb will be large because of the very high degree of turbulence and the oscillations in the flow rate. The complex nature of the dispersion renders even an approximate estimate of the friction losses impossible. The upward velocity of the air relative to the liquid promotes turbulence and causes an increase in the rate of flow of air necessary to maintain the desired fluid density in the column. Efforts should therefore be directed at producing as even a distribution of air as possible. It would be expected that loss of air due to its relative velocity would be a minimum in liquids of high viscosity.

3. *Losses at the discharge from the pump*: The kinetic energy of the fluid cannot effectively be converted into pressure energy at the discharge from the pump because of the rapid oscillations in the flow rate.

4. *Losses in the air supply lines*.

Advantages of the Air Lift Pump. The air lift pump is more efficient than other pumps using compressed air directly, since the air is allowed to expand almost to atmospheric pressure in contact with the liquid. There are no valves other than those in the air lines and the non-return valve in the suction line. The pump is therefore readily constructed of corrosion resisting materials and will handle liquids containing high concentrations of suspended solids. Both the pipe and the footpiece can be made of metal, stoneware, or glass according to the material to be handled and the duty required of the pump. These pumps have been used extensively in the oil industry and for circulating nitric acid in absorption equipment.

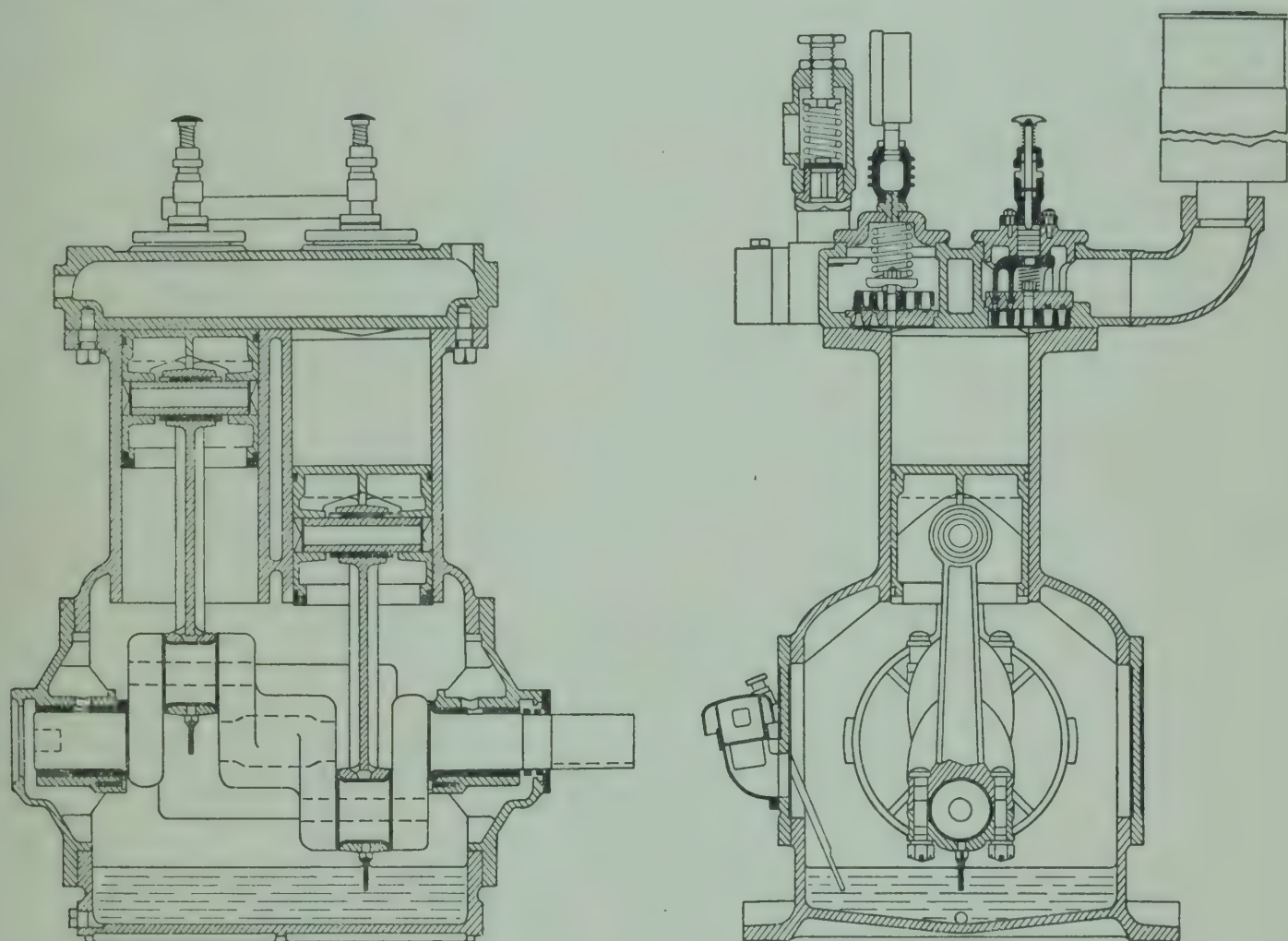


Fig. 5.27. Reciprocating compressor

(Courtesy of Reavell & Co. Ltd.)

The main disadvantage is that, in order to obtain a high efficiency, the air has to be introduced at a considerable depth below the feed point. Owing to lack of information concerning the magnitude of a number of the energy losses, it is difficult to design the pump for the greatest possible efficiency in operation and the size of a pump required for a given operation is determined largely as a result of experience.

PUMPING EQUIPMENT FOR GASES

Essentially the same basic types of mechanical equipment are used for handling gases and liquids, though the construction may be very different in the two cases. Under the normal range of operating pressures, the density of a gas is considerably less than that of a liquid so that higher speeds of operation can be employed

and lighter valves fitted to the delivery and suction lines. Because of the lower viscosity of a gas there is a greater tendency for leakage to occur and therefore gas compressors are designed with smaller clearances between the moving parts.

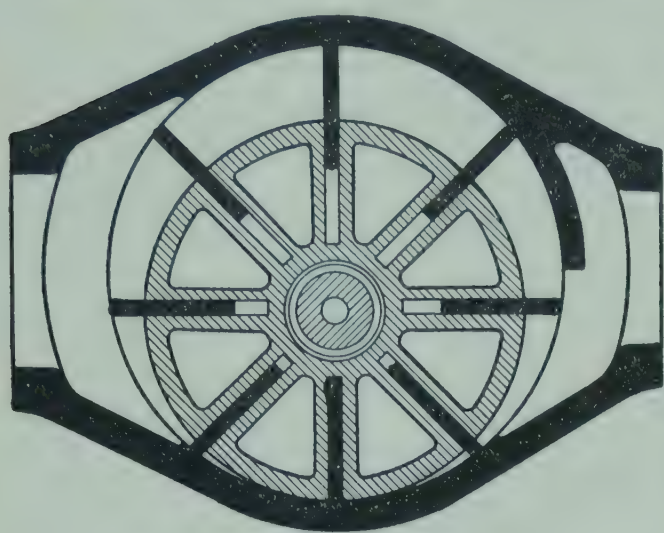


Fig. 5.28. Vane type blower
(Courtesy of Reavell & Co. Ltd.)

Further differences in construction are necessitated by the decrease in volume of gas as it is compressed and this must be allowed for in the design. Since a large proportion of the energy of compression appears as heat in the gas, there will normally be a considerable increase in temperature which may limit the operation of the compressor unless suitable cooling can be effected. For this reason gas compression is often carried out in a number of stages and the gas is cooled between each stage. Any gas which is

not expelled from the cylinder at the end of compression (the clearance volume) must be expanded again to the inlet pressure before a fresh charge can be admitted. This continual compression and expansion of the residual gas results in loss of efficiency because neither the compression nor the expansion can be carried out completely reversibly. With liquids this factor has no effect on the efficiency because the residual fluid is not compressed.

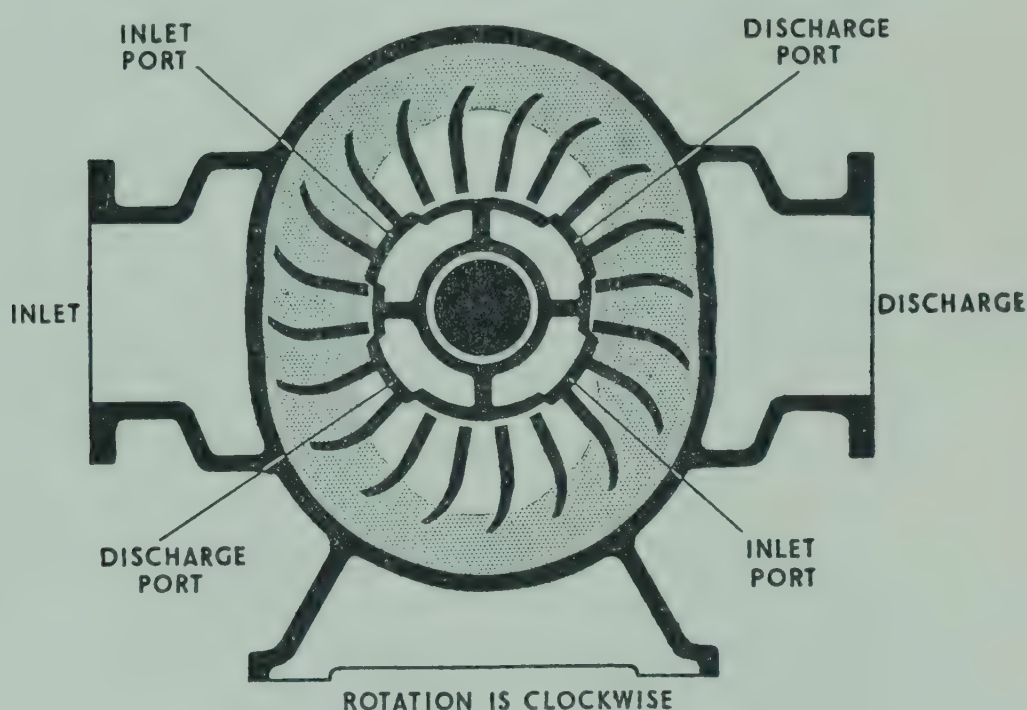


Fig. 5.29. Hytor pump
(Courtesy of the Nash Engineering Co. (Great Britain) Ltd.)

The principal types of compressors for gases are:

1. *The Reciprocating Piston Type.* This is illustrated in Fig. 5.27 and can consist of one or more stages. It is the only type which will develop pressures appreciably above 150 lb/sq in.

2. *Rotary Blowers and Compressors.* These can be divided into two classes, those which develop a high compression ratio and those which have very low

ratios. The former include the vane type in which the compression ratio is achieved by eccentric mounting of the rotor (Fig. 5.28) and the Nash Hytor Pump (Fig. 5.29) in which the compression ratio is achieved by means of a specially shaped casing and a liquid seal which rotates with the impeller. Because of the varying clearance between the vanes and the casing, the gas contained between adjacent vanes is compressed and expanded, and the pressure of the gas can therefore be raised by suitable arrangement of the inlet and outlet. The second type is represented by the Cycloidal or Rootes Blower (Fig. 5.30) which is similar in operation to the lobe pump.

3. *Centrifugal Blowers and Compressors, including Turbocompressors.* These depend on the conversion of kinetic energy into pressure energy. Fans are used

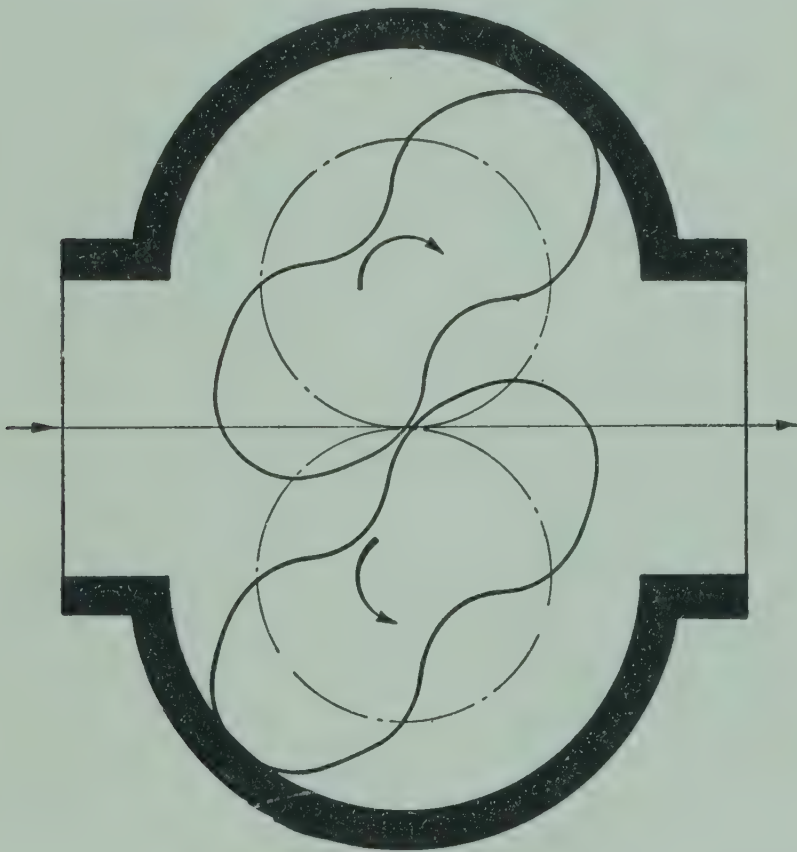


Fig. 5.30. Rootes blower
(Courtesy of Reavell & Co. Ltd.)

for low pressures, give axial flow and can be made to handle very large quantities of gases. For higher pressure ratios, radial flow machines are employed. Water cooling is normally adopted with multistage machines (Fig. 5.31) but only rarely with single stage units.

TABLE 5.1. *Characteristics of various types of compressors*

	Maximum delivery pressure	Maximum throughput (air at S.T.P.) ft ³ /min	Isothermal efficiency %
Reciprocating compressors	4000 atmospheres	5000	50–60
Rotary compressors:			
High compression ratio type	50–75 lb/sq in.	4000	50
Low compression ratio type	15 lb/sq in.	5000	45–50
Centrifugal types:			
Fans	12 in. water gauge	100,000	—
Single stage blowers	5 lb/sq in.	12,000	50
Multistage water-cooled compressors .	150 lb/sq in. (in about 10 stages)	50,000	50

In Table 5.1 the chief characteristics of the various types of compressors are given. These all refer to the compression of air from atmospheric pressure. The isothermal efficiency (referred to later) is the ratio of the work required for a perfectly efficient isothermal compression to that actually required.

The choice of compressor will, in the first instance, depend on the pressure to be developed. In order of cost, fans are the cheapest type, followed by the Rootes blower and the single-stage centrifugal. For higher pressure operation, it is necessary to balance the lower cost of the rotary compressor against the higher



Fig. 5.32. Rotary vane pump

(By permission from *A Manual of Vacuum Practice* by MARTIN & HILL, Melbourne University Press)

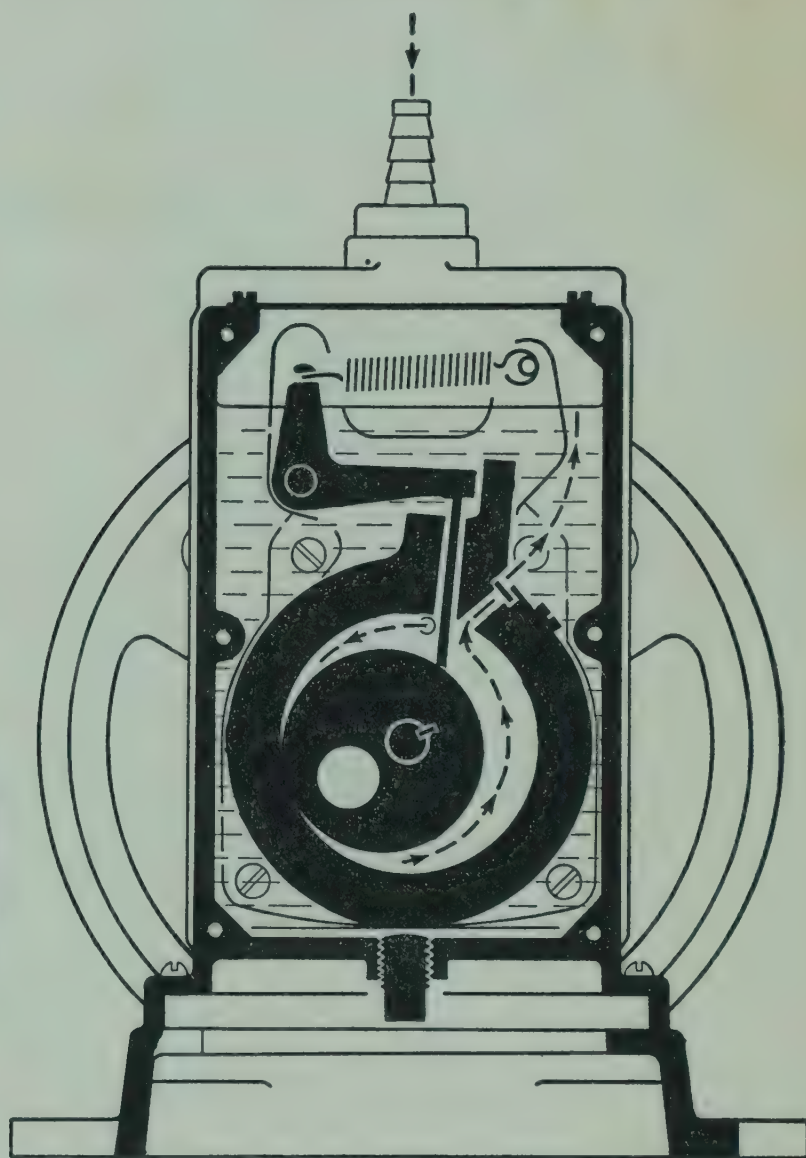


Fig. 5.33. Hyvac pump

(By permission from *A Manual of Vacuum Practice* by MARTIN & HILL, Melbourne University Press)

efficiency of the reciprocating compressor. The overall cost of operation will thus generally determine the type of machine used for pressures between about 15 lb/sq in. and 150 lb/sq in. The reciprocating compressor has two inherent disadvantages: it is comparatively bulky and operates at a lower speed, so that direct coupling to an electric motor is more difficult. An additional advantage of the centrifugal and Hytor types is that they deliver gas free of oil.

Vacuum pumps take in gas at a low pressure and discharge at atmospheric pressure. They must have a high compression ratio and therefore reciprocating piston, rotary vane (Fig. 5.32), Hyvac (Fig. 5.33) and Hytor pumps are frequently used. Vacuum pumps must be capable of handling large volumes of gases of low density and are therefore relatively bulky for the mass of gas

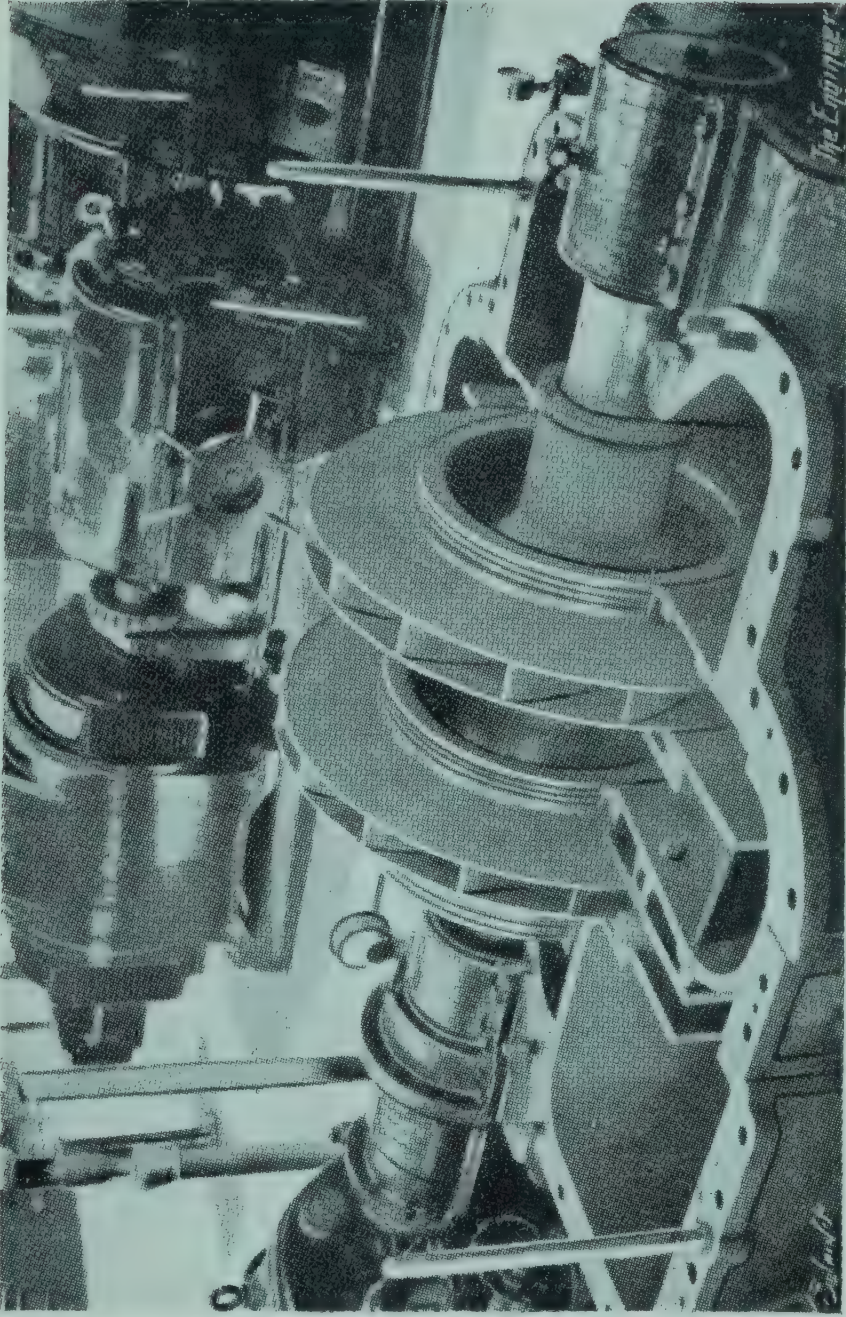


Fig. 5.31. Turboblower
(Courtesy of *Rearell & Co. Ltd.*)

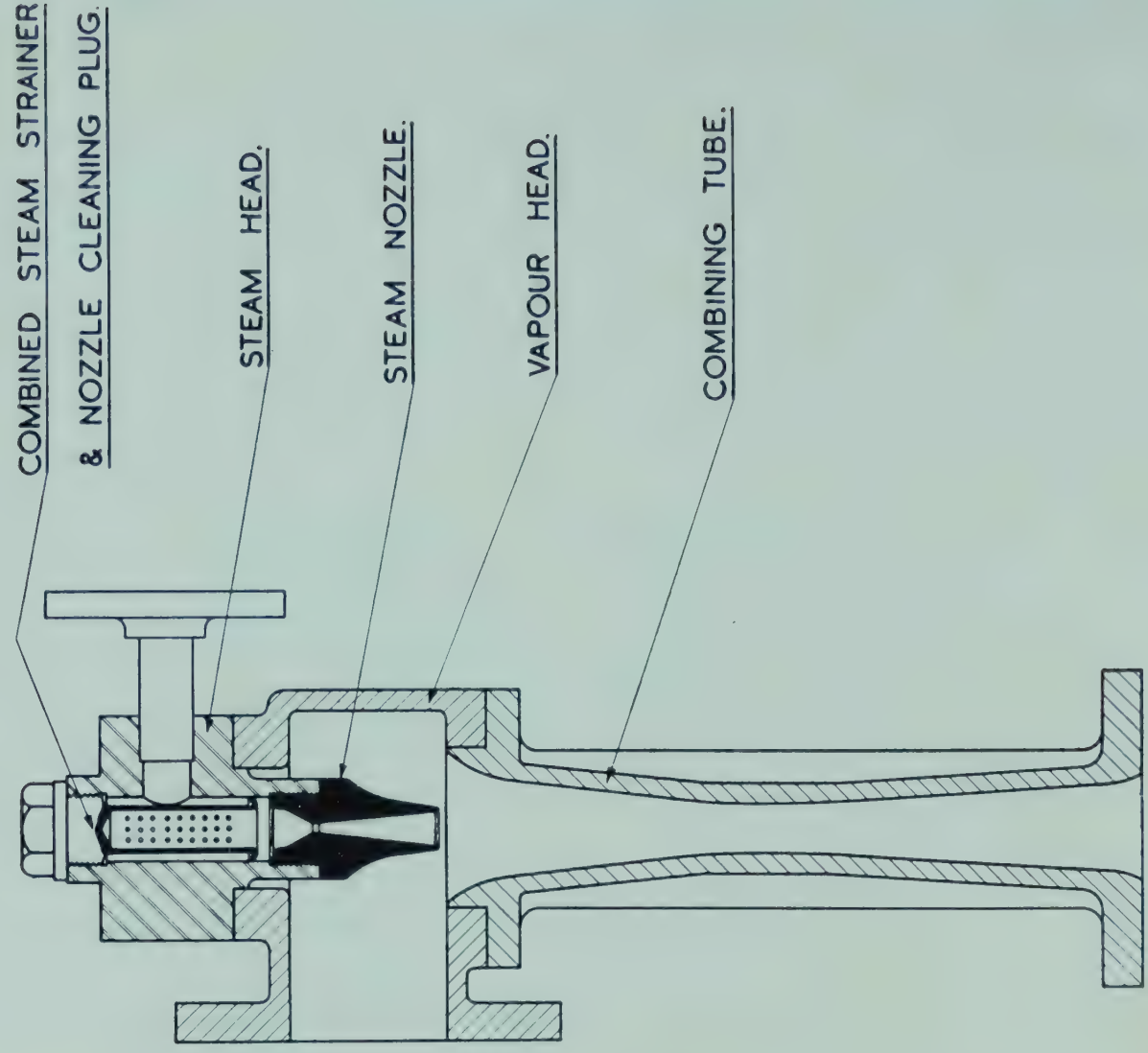
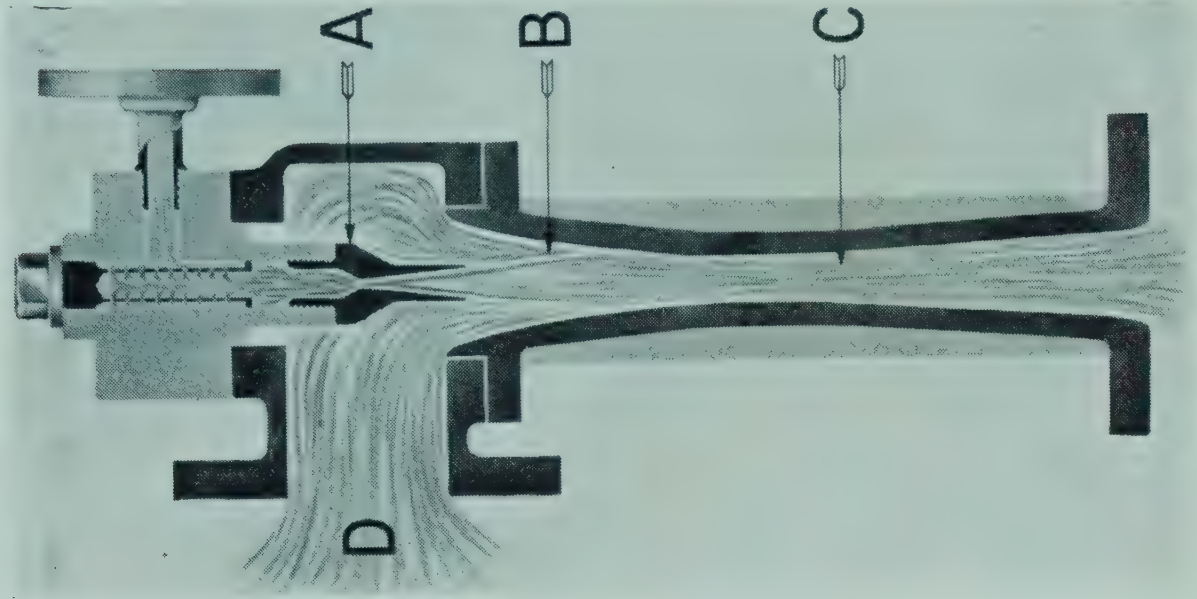
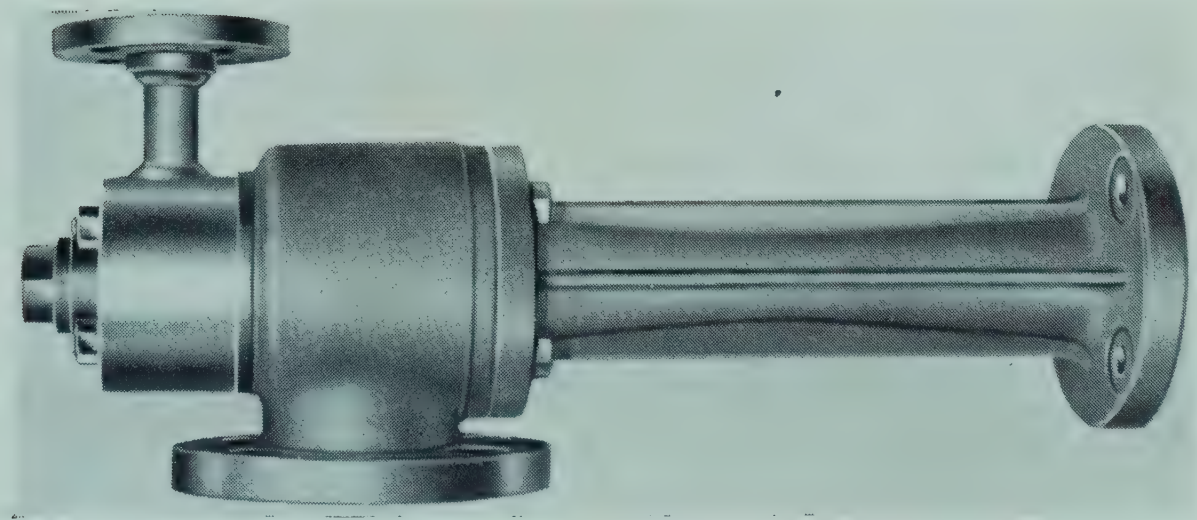


Fig. 5.34. Steam jet ejector

A—Steam nozzle. B—Mixing region. C—Mixed fluids. D—Entrained fluid.

(Courtesy of Hick, Hargreaves & Company Ltd.)

handled. Piston and rotary vane pumps will give a pressure of 10^{-2} mm mercury, whereas that given by the Nash Hytor is limited to the vapour pressure of the water used as the sealing liquid.

In steam jet ejectors (Fig. 5.34) which are extensively used for vacuum production in the chemical industry, particularly with stills and evaporators, steam is expanded through a nozzle and the pressure is reduced by the conversion of pressure energy into kinetic energy. Fluid from the system to be evacuated is sucked in at the throat of the ejector and its momentum is increased as it mixes with the steam. A large proportion of the kinetic energy of the steam is then recovered as pressure energy in the diverging outlet from the ejector. The efficiency of the ejector is largely dependent on recovery of the pressure in the diverging section, and nozzles of high efficiency require careful manufacture and are therefore rather expensive. Single stage steam ejectors and water ejectors give pressures down to about 50 mm mercury and multistage steam ejectors operate down to about 1 mm mercury.

For the production of very high vacuum, as required for freeze drying or molecular distillation, diffusion pumps are normally used with a rotary backing pump. An arrangement is illustrated in Fig. 5.35. Gas from the vessel to be evacuated diffuses through a narrow slit and is carried away in a stream of oil vapour or mercury vapour as a result of the partial pressure difference across the slit.

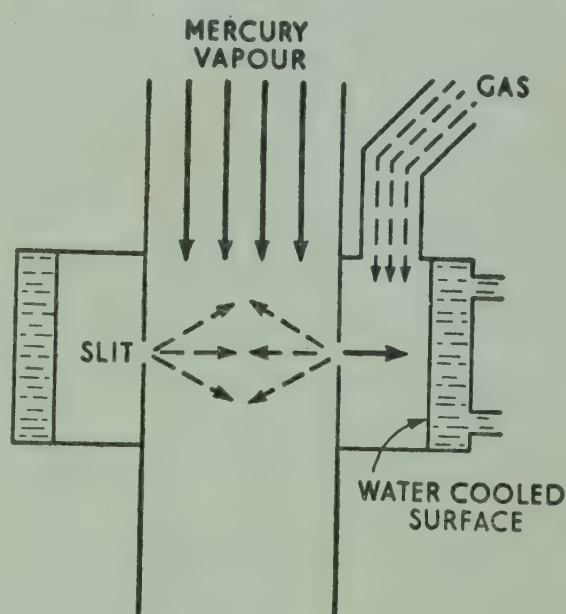


Fig. 5.35. Mercury diffusion pump
(By permission from *A Manual of Vacuum Practice* by MARTIN & HILL, Melbourne University Press)

Power Required for Compression

Consider the compression of unit mass of gas. If the volume of the gas changes by an amount dv at a pressure P , the net work done on the gas $-\delta W$ is given by

$$\begin{aligned} -\delta W &= -P \cdot dv \\ &= v \cdot dP - d(Pv) \end{aligned} \quad \dots(5.20)$$

for a reversible change, and

$$\begin{aligned} -\delta W &= -P \cdot dv + \delta F \\ &= v dP - d(Pv) + \delta F \end{aligned} \quad \dots(5.21)$$

for an irreversible change.

This equation is identical with equation 2.9 given in Chapter 2.

$$\delta W = Pdv - \delta F \quad \dots(2.9)$$

We will first consider the work done in a reversible compression, since this refers to the ideal condition for which the work of compression is a minimum. A reversible compression would have to be carried out at an infinitesimal rate

and therefore is not obtainable in practice. The actual work done will be greater than that calculated, not only because of irreversibility but also because of frictional loss and leakage in the compressor. These two factors are difficult to separate and will therefore be allowed for in the overall efficiency of the machine.

The total work of compression from a pressure P_1 to a pressure P_2 is found by integrating equation 5.20. For an ideal gas undergoing an isothermal compression, we have

$$-\int_1^2 P dv = -W = P_1 v_1 \ln \frac{P_2}{P_1} \text{ (from equation 2.36)}$$

For the isentropic compression of an ideal gas,

$$\begin{aligned} -\int_1^2 P dv = -W &= \frac{\gamma}{\gamma-1} (P_2 v_2 - P_1 v_1) - (P_2 v_2 - P_1 v_1) \text{ (from equation 2.40a)} \\ &= \frac{1}{\gamma-1} (P_2 v_2 - P_1 v_1) \end{aligned} \quad \dots (5.22)$$

$$\text{Thus } \int_1^2 v \cdot dP = -\gamma \int_1^2 P \cdot dv \text{ (comparing equations 2.40a and 5.22)}$$

for isentropic conditions and

$$\therefore -W = \frac{1}{\gamma-1} P_1 v_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \quad \dots (5.23)$$

Under these conditions the whole of the energy of compression appears as heat in the gas.

For the isentropic compression of a mass m of gas we have

$$-W \cdot m = \frac{1}{\gamma-1} P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \quad \dots (5.24)$$

where V_1 is the volume of a mass m of gas at a pressure P_1 .

If the conditions are intermediate between isothermal and isentropic, we must write k in place of γ , where $\gamma > k > 1$.

If the gas deviates appreciably from the ideal gas laws over the range of conditions considered, the work of compression is most conveniently calculated from the change in the thermodynamic properties of the gas.

$$\text{Thus } dU = T \cdot dS - P \cdot dv \text{ (from equation 2.8)}$$

$$\therefore -\delta W = -P \cdot dv = dU - T \cdot dS \quad \dots (5.25)$$

Under isothermal conditions,

$$-W = \Delta U - T \Delta S \quad \dots (5.26)$$

Under isentropic conditions,

$$-W = \Delta U \quad \dots (5.27)$$

The above equations give the work done during a simple compression of gas in a cylinder; they do not take account of the work done either during the admission of the gas prior to compression, or during the expulsion of the compressed gas.

Suppose that, after the compression of a volume V_1 of gas at a pressure P_1 to a pressure P_2 , the whole of the gas is expelled at constant pressure P_2 and a fresh charge of gas is admitted at a pressure P_1 . The cycle can be followed in Fig. 5.36, where the pressure P is plotted as ordinate against the volume V as abscissa.

Point 1 represents the initial condition of the gas (P_1 and V_1).

Line 1-2 represents the compression of gas to pressure P_2 , volume, V_2 .

Line 2-3 represents the expulsion of the gas at a constant pressure, P_2 .

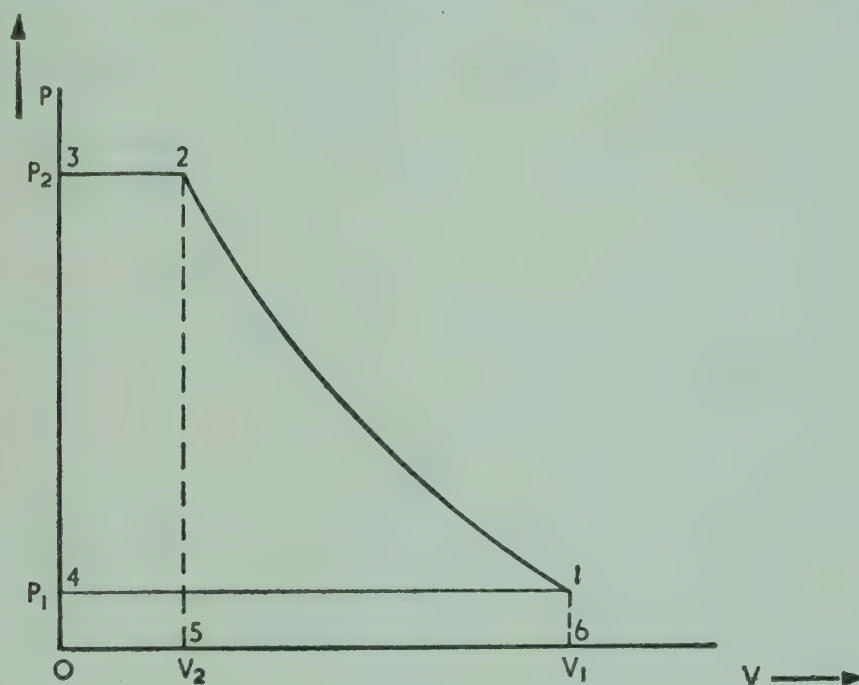


Fig. 5.36. Single stage compression cycle—no clearance

Line 3-4 represents a sudden reduction in the pressure in the cylinder from P_2 to P_1 . As the whole of the gas has been expelled, this can be regarded as taking place instantaneously.

Line 4-1 represents the suction stroke of the piston, during which a volume V_1 of gas is admitted at constant pressure, P_1 .

It will be noted that the mass of gas in the cylinder varies during the cycle. The work done by the compressor during each phase of the cycle is as follows.

$$\text{Compression,} \quad - \int_1^2 P \cdot dV \quad (\text{Area 1-2-5-6})$$

$$\text{Expulsion,} \quad P_2 V_2 \quad (\text{Area 2-3-0-5})$$

$$\text{Suction,} \quad - P_1 V_1 \quad - (\text{Area 4-0-6-1})$$

The total work done per cycle

$$\begin{aligned} &= - \int_1^2 P \cdot dV + P_2 V_2 - P_1 V_1 \quad (\text{Area 1-2-3-4}) \\ &= \int_1^2 V \cdot dP \quad \dots (5.28) \end{aligned}$$

The work of compression for an ideal gas per cycle

$$= P_1 V_1 \ln \frac{P_2}{P_1} \quad \dots(5.29)$$

under isothermal conditions.

Under isentropic conditions, work of compression

$$= P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \quad \dots(5.30)$$

Again, working in terms of the thermodynamic properties of the gas, we have,

$$\begin{aligned} v \cdot dP &= d(Pv) - P \cdot dv \\ &= d(Pv) + dU - T \cdot dS \\ &= dH - T \cdot dS \end{aligned} \quad \dots(5.31)$$

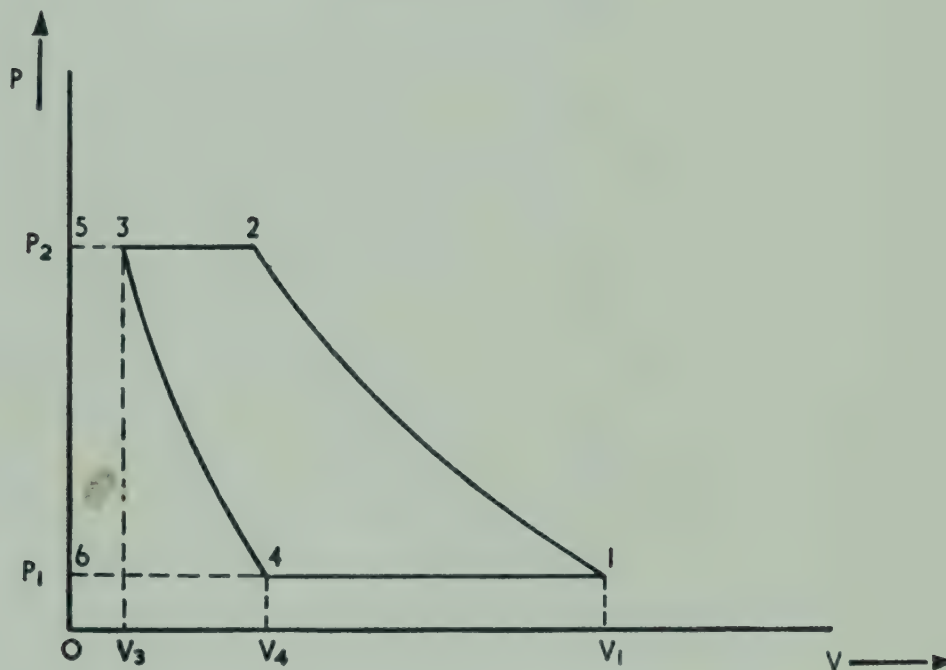


Fig. 5.37. Single stage compression cycle—with clearance

For an isothermal process,

$$- m \cdot W = m(\Delta H - T \cdot \Delta S) \quad \dots(5.32)$$

For an isentropic process,

$$- m \cdot W = m\Delta H \quad \dots(5.33)$$

where m is the mass of gas compressed per cycle.

Clearance Volume. In practice, it is not possible to expel the whole of the gas from the cylinder at the end of the compression; the volume remaining in the cylinder after the forward stroke of the piston is termed the clearance volume. The volume displaced by the piston is termed the swept volume and therefore the total volume of the cylinder is made up of the clearance volume plus the swept volume. The clearance c is defined as the ratio of the clearance volume to the swept volume.

A typical cycle for a compressor with a finite clearance volume can be

followed by reference to Fig. 5.37. A volume V_1 of gas at a pressure P_1 is admitted to the cylinder; its condition is represented by point 1.

Line 1–2 represents the compression of the gas to a pressure P_2 and volume V_2 .

Line 2–3 represents the expulsion of gas at constant pressure P_2 , so that the volume remaining in the cylinder is V_3 .

Line 3–4 represents an expansion of this residual gas to the lower pressure P_1 and volume V_4 during the return stroke.

Line 4–1 represents the introduction of fresh gas into the cylinder at constant pressure P_1 .

The work done on the gas during each stage of the cycle is as follows.

$$\text{Compression,} \quad - \int_{V_1}^{V_2} P \cdot dV$$

$$\text{Expulsion,} \quad P_2(V_2 - V_3)$$

$$\text{Expansion,} \quad \int_{V_3}^{V_4} P \cdot dV$$

$$\text{Suction,} \quad P_1(V_1 - V_4)$$

The total work done during the cycle is equal to the sum of these four components. It is represented by the area 1–2–3–4, which is equal to area 1–2–5–6 less area 4–3–5–6. If the compression and expansion are taken as isentropic, the work done per cycle is therefore

$$\begin{aligned} &= P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} - P_1 V_4 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \\ &= P_1 (V_1 - V_4) \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \quad \dots (5.34) \end{aligned}$$

Thus, theoretically, the clearance volume does not affect the work done per unit mass of gas, since $V_1 - V_4$ is the volume admitted per cycle. It does, however, influence the quantity of gas admitted and therefore the work done per cycle. In practice, however, compression and expansion are not reversible and losses arise from the compression and expansion of the clearance gases. This effect is particularly serious at high compression ratios.

Now, V_4 is not known explicitly, but can be calculated in terms of V_3 , the clearance volume.

For isentropic conditions,

$$V_4 = V_3 \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$$

and

$$V_1 - V_4 = (V_1 - V_3) + V_3 - V_3 \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$$

$$= (V_1 - V_3) \left\{ 1 + \frac{V_3}{V_1 - V_3} - \frac{V_3}{V_1 - V_3} \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \right\}$$

Now $V_1 - V_3$ is the swept volume, V_s , say; and $\frac{V_3}{V_1 - V_3}$ is the clearance c .

$$\therefore V_1 - V_4 = V_s \left\{ 1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \right\} \quad \dots (5.35)$$

The total work done on the fluid per cycle is therefore

$$P_1 V_s \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} \left\{ 1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \right\} \quad \dots (5.36)$$

The term $\left\{ 1 + c - c \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \right\}$ is called the theoretical volumetric efficiency and is a measure of the effect of the clearance on an isentropic compression. The actual volumetric efficiency will be affected, in addition, by the inertia of the valves and leakage past the piston.

If the gas is cooled during compression, the work done per cycle will be less than that given by equation 5.36; γ will be replaced by some smaller quantity k . The greater the rate of heat removal, the less is the work done. The isothermal compression is usually taken as the condition for least work of compression but clearly the energy consumption can be reduced below this value if the gas is artificially cooled below its initial temperature as it is compressed. This is not a practicable possibility because of the large amount of energy required to refrigerate the cooling fluid. It can be seen that the theoretical volumetric efficiency decreases as the rate of heat removal is increased, since γ is replaced by the smaller quantity k .

In practice, the cylinders are usually water cooled. The work of compression is thereby reduced but the effect is usually small. The reduction in temperature does, however, improve the mechanical operation of the compressor and makes lubrication easier.

Multistage Compressors. If the required pressure ratio P_2/P_1 is large it is not practicable to carry out the whole of the compression in a single cylinder because of the high temperatures which would be set up and the adverse effects of clearance volume on the efficiency. Further, lubrication would be difficult due to carbonization of the oil and there would be a risk of causing oil mist explosions in the cylinders when gases containing oxygen were being compressed. The mechanical construction also would be difficult because the single cylinder would have to be strong enough to withstand the final pressure and yet large

enough to hold the gas at the initial pressure P_1 . In the multistage compressor, the gas passes through a number of cylinders of gradually decreasing volume and can be cooled between the stages. The maximum pressure ratio normally obtained in a single cylinder is 10 but values above 6 are unusual.

The operation of the multistage compressor can conveniently be followed again on a pressure-volume diagram (Fig. 5.38). The effect of clearance volume will be neglected at first. The area 1-2-3-4 represents the work done in compressing isentropically from P_1 to P_2 in a single stage. The area 1-2-5-4 represents the necessary work for an isothermal compression. Now consider a multistage isentropic compression in which the intermediate pressures are P_{i1} , P_{i2} , etc. The gas will be assumed to be cooled to its initial temperature in an interstage cooler before it enters each cylinder.

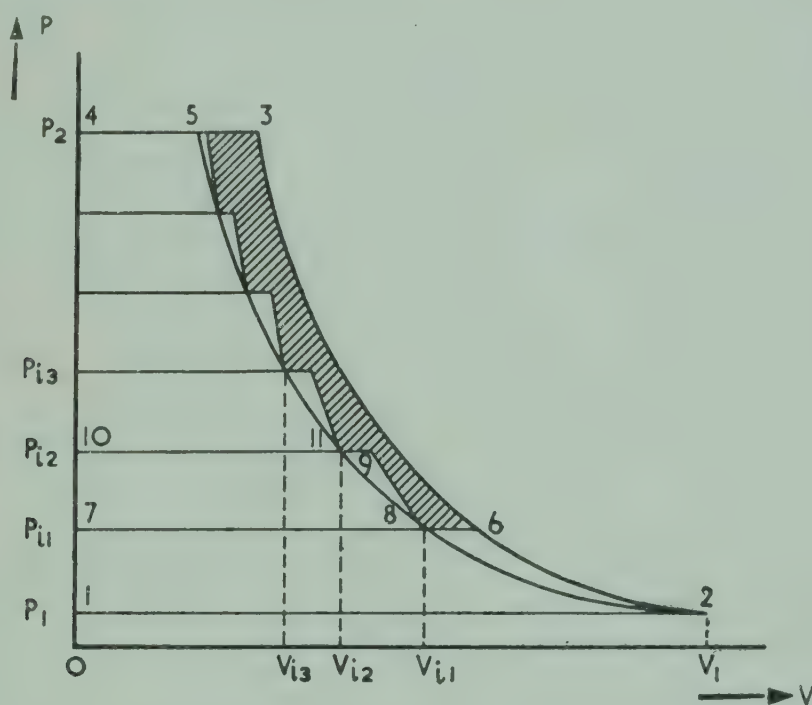


Fig. 5.38. Multistage compression cycle with interstage cooling

Line 1-2 represents the suction stroke of the first stage where a volume V_1 of gas is admitted at a pressure P_1 .

Line 2-6 represents an isentropic compression to a pressure P_{i1} .

Line 6-7 represents the delivery of the gas from the first stage at a constant pressure P_{i1} .

Line 7-8 represents the suction stroke of the second stage. The volume of the gas has been reduced in the interstage cooler to V_{i1} ; that which would have been obtained as a result of an isothermal compression to P_{i1} .

Line 8-9 represents an isentropic compression in the second stage from a pressure P_{i1} to a pressure P_{i2} .

Line 9-10 represents the delivery stroke of the second stage.

Line 10-11 represents the suction stroke of the third stage. Point 11 again lies on the line 2-5, representing an isothermal compression.

It is seen that the overall work done on the gas is intermediate between that

for a single stage isothermal compression and that for an isentropic compression. The net saving in energy is shown as the shaded area in Fig. 5.38.

The total work done per cycle W'

$$= P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_{i1}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + P_{i1} V_{i1} \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_{i2}}{P_{i1}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} + \dots$$

for an isentropic compression.

For perfect interstage cooling,

$$P_1 V_1 = P_{i1} V_{i1} = P_{i2} V_{i2} = \dots$$

$$\therefore W' = P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_{i1}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} + \left(\frac{P_{i2}}{P_{i1}} \right)^{\frac{\gamma-1}{\gamma}} + \dots - n \right\}$$

where n is the number of stages.

We now want to find how the total work per cycle W' is affected by the choice of the intermediate pressures P_{i1} , P_{i2} , etc. The work will be a minimum when $\frac{\partial W'}{\partial P_{i1}} = \frac{\partial W'}{\partial P_{i2}} = \frac{\partial W'}{\partial P_{i3}} = \dots = 0$.

$$\text{When } \frac{\partial W'}{\partial P_{i1}} = 0,$$

$$P_1 V_1 \cdot \frac{\gamma}{\gamma - 1} \left\{ \frac{\gamma - 1}{\gamma} \left(\frac{P_{i1}}{P_1} \right)^{\frac{\gamma-1}{\gamma}} P_{i1}^{-1} + \frac{1 - \gamma}{\gamma} \left(\frac{P_{i2}}{P_{i1}} \right)^{\frac{\gamma-1}{\gamma}} P_{i1}^{-1} \right\} = 0$$

$$\text{i.e.} \quad \frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} \quad \dots (5.37)$$

The same procedure is then adopted for obtaining the optimum value of P_{i2} and we obtain

$$\frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} \quad \dots (5.37a)$$

Thus the intermediate pressures should be arranged so that the compression ratio is the same in each cylinder; and equal work is then done in each cylinder.

The minimum work of compression in a compressor of n stages is therefore

$$P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ n \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{n\gamma}} - n \right\} = n P_1 V_1 \frac{\gamma}{\gamma - 1} \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{n\gamma}} - 1 \right\} \quad \dots (5.38)$$

The effect of clearance volume can now be taken into account. If the clearances in the successive cylinders are c_1, c_2, c_3, \dots , the theoretical volumetric efficiency of the first cylinder

$$= 1 + c_1 - c_1 \left(\frac{P_{i1}}{P_1} \right)^{\frac{1}{\gamma}}$$

Now assume that the same compression ratio is used in each cylinder. Then the theoretical volumetric efficiency of the first stage is

$$1 + c_1 - c_1 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}}$$

If the swept volumes of the cylinders are V_{s1} , V_{s2} , . . . , the volume of gas admitted to the first cylinder

$$= V_{s1} \left\{ 1 + c_1 - c_1 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}} \right\} \quad \dots (5.39)$$

The same mass of gas passes through each of the cylinders and, therefore, if the interstage coolers are assumed perfectly efficient, the ratio of the volumes of gas admitted to successive cylinders is $\left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}$. The volume of gas admitted to the second cylinder then

$$\begin{aligned} &= V_{s2} \left\{ 1 + c_2 - c_2 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}} \right\} = V_{s1} \left\{ 1 + c_1 - c_1 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}} \right\} \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}} \\ \therefore \frac{V_{s1}}{V_{s2}} &= \frac{1 + c_2 - c_2 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}}}{1 + c_1 - c_1 \left(\frac{P_2}{P_1} \right)^{\frac{1}{n\gamma}}} \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \quad \dots (5.40) \end{aligned}$$

In this manner the swept volume of each cylinder can be calculated in terms of V_{s1} and c_1 , c_2 , . . . , and the cylinder dimensions determined.

When the gas does not behave as an ideal gas, the change in its condition can be followed on a temperature-entropy or an enthalpy-entropy diagram. The intermediate pressures P_{i1} , P_{i2} , . . . are then selected so that the enthalpy change (ΔH) is the same in each cylinder.

Several opposing factors will influence the number of stages selected for a given compression. The larger the number of cylinders, the greater is the mechanical complexity. Against this must be balanced the higher theoretical efficiency, the smaller mechanical strains set up in the cylinders and the moving parts and the greater ease of lubrication at the lower temperatures that are experienced. Compressors with as many as nine stages are used for very high pressures.

Compressor Efficiencies. The efficiency quoted for a compressor is usually either an isothermal efficiency or an isentropic efficiency. The isothermal efficiency is the ratio of the work required for an ideal isothermal compression, to the energy actually expended in the compressor. The isentropic efficiency is defined in a corresponding manner on the assumption that the whole compression is carried out in a single cylinder. Since the energy expended in an isentropic

compression is greater than that for an isothermal compression, the isentropic efficiency is always the greater of the two. Clearly the efficiencies will depend on the heat transfer between the gas undergoing compression and the surroundings, and on how closely the process approaches a reversible compression.

The efficiency of the compression will also be affected by a number of other factors which are all connected with the mechanical construction of the compressor. Thus the efficiency will be reduced as a result of leakage past the piston and the valves and because of throttling of the gas at the valves. Further the mechanical friction of the machine will lower the efficiency and the overall efficiency will be affected by the efficiency of the driving motor and transmission.

Example. An air lift pump is used for raising 10 gal/min of a liquid of specific gravity 1.2 to a height of 60 ft. Air is available at 50 lb/sq in. gauge pressure. If the efficiency of the pump is 30%, calculate the power requirement, assuming isentropic compression of the air ($\gamma = 1.4$).

Solution. Work done per minute by pump

$$\begin{aligned} &= 10 \times 10 \times 1.2 \times 60 \times 32 \\ &= 2.30 \times 10^5 \text{ ft-poundals} \end{aligned}$$

Actual work of expansion of air

$$\begin{aligned} &= \frac{2.30 \times 10^5}{0.30} \\ &= 7.67 \times 10^5 \text{ ft-poundals/min} \end{aligned}$$

Volume of air required (at S.T.P.) per minute is given by

$$7.67 \times 10^5 = 14.7 \times 144 \times 32 \times V \ln \frac{64.7}{14.7}$$

$$\therefore V = 7.65 \text{ ft}^3/\text{min}$$

Work done in compressing 7.65 ft³ of air isentropically

$$\begin{aligned} &= 14.7 \times 144 \times 32 \times 7.65 \times \frac{1.4}{0.4} \left\{ \left(\frac{64.7}{14.7} \right)^{0.4/1.4} - 1 \right\} \\ &= 1.82 \times 10^6 (1.528 - 1) \\ &= 9.61 \times 10^5 \text{ ft-poundals} \end{aligned}$$

$$\begin{aligned} \text{Power required} &= \frac{9.61 \times 10^5}{32 \times 33,000} \\ &= 0.91 \text{ h.p.} \approx \underline{\underline{1 \text{ h.p.}}} \end{aligned}$$

Example. A single acting air compressor supplies 200 cu ft of air (at S.T.P.) per minute compressed to 55 lb/sq in. absolute from 14.7 lb/sq in. absolute pressure. If the suction temperature is 15.5°C, the stroke is 10 in. and the speed is 250 r.p.m., find the cylinder diameter. Assume the cylinder clearance is 4% and compression and re-expansion are isentropic ($\gamma = 1.4$). What is the theoretical horse-power required for the compression?

Solution. Volume per stroke

$$= \frac{200}{250} \times \frac{288.5}{273} = 0.845 \text{ cu ft}$$

$$\text{Compression ratio} = \frac{55.0}{14.7} = 3.74$$

From equation 5.35, swept volume is given by,

$$0.845 = V_s \{1 + 0.04 - 0.04(3.74)^{1/1.4}\}$$

$$\therefore V_s = \frac{0.845}{1.04 - 0.04 \times 2.7} \\ = 0.91 \text{ cu ft}$$

Therefore, cross-sectional area of cylinder

$$= 0.91 \times \frac{12}{10} = 1.09 \text{ sq ft}$$

$$\text{Hence diameter} = 1.18 \text{ ft} = \underline{\underline{14.2 \text{ in.}}}$$

From equation 5.34, work of compression per cycle

$$= 14.7 \times 144 \times 32 \times 0.845 \times \frac{1.4}{1.4 - 1} \{(3.74)^{0.4/1.4} - 1\} \\ = 201,000(1.457 - 1) \\ = 91,400 \text{ ft-poundals}$$

$$\text{Horse power} = \frac{91,400}{32} \times \frac{250}{33,000} = 21.6 \approx \underline{\underline{22}}$$

Example. Air at 20°C is compressed from 14.7 lb/sq in. to 300 lb/sq in. absolute pressure in a two-stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of the clearance gas is $PV^{1.25} = \text{constant}$. The compression ratio in each of the two cylinders is the same and the interstage cooler may be taken as perfectly efficient. If the clearances in the two cylinders are 4% and 5% respectively, calculate

- the work of compression per unit mass of gas compressed,
- the isothermal efficiency,
- the isentropic efficiency ($\gamma = 1.4$),
- the ratio of the swept volumes in the two cylinders.

Solution. Overall compression ratio

$$= \frac{300}{14.7} = 20.4$$

Volume per pound of air at 20°C

$$= \frac{359}{28.8} \times \frac{293}{273} = 13.4 \text{ ft}^3$$

From equation 5.38, work of compression per unit mass of gas

$$= 14.7 \times 144 \times 32 \times 13.4 \times 2 \times \frac{1.25}{1.25 - 1} \{(20.4)^{0.25/2.5} - 1\} \\ = 9.08 \times 10^6(1.351 - 1) \\ = 3.18 \times 10^6 \text{ ft-poundals/lb}$$

Actual energy supplied to compressor

$$= \frac{3.18 \times 10^6}{0.85} \\ = 3.75 \times 10^6 \text{ ft-poundals/lb}$$

$$1 \text{ h.p.-hour} = 33,000 \times 60 \times 32 = 6.33 \times 10^7 \text{ ft-poundals}$$

$$\text{Work of compression} = \frac{3.75 \times 10^6}{6.33 \times 10^7} = \underline{\underline{0.059 \text{ h.p.-hr/lb}}}$$

From equation 5.29, work done in isothermal compression of 1 lb of gas

$$= 14.7 \times 144 \times 32 \times 13.4 \ln 20.4$$

$$= 9.07 \times 10^5 \times 3.015$$

$$= 2.73 \times 10^6 \text{ ft-poundals/lb}$$

$$\text{Isothermal efficiency} = \frac{2.73 \times 10^6}{3.75 \times 10^6} \times 100 = \underline{\underline{73\%}}$$

From equation 5.30, work done in isentropic compression of 1 lb of gas

$$= 14.7 \times 144 \times 32 \times 13.4 \times \frac{1.4}{0.4} \{(20.4)^{0.4/1.4} - 1\}$$

$$= 3.18 \times 10^6 (2.36 - 1)$$

$$= 4.32 \times 10^6 \text{ ft-poundals/lb}$$

$$\text{Isentropic efficiency} = \frac{4.32 \times 10^6}{3.75 \times 10^6} \times 100 = \underline{\underline{115\%}}$$

Volume swept out in first cylinder in compression of unit mass of gas is given by,

$$13.4 = V_{s1} \{1 + 0.04 - 0.04(20.4)^{1/(1.25 \times 2)}\} \text{ (from equation 5.39)}$$

$$= V_{s1}(1.04 - 0.04 \times 3.34)$$

$$= 0.906 V_{s1}$$

$$\therefore V_{s1} = 14.8 \text{ cu ft}$$

Similarly, the swept volume of the second cylinder is given by,

$$13.4 \times \left(\frac{1}{20.4}\right)^{\frac{1}{2}} = V_{s2} \{1 + 0.05 - 0.05(20.4)^{1/(1.25 \times 2)}\}$$

$$\therefore 2.97 = V_{s2}(1.05 - 0.05 \times 3.34)$$

$$\therefore V_{s2} = 3.37 \text{ cu ft}$$

$$\therefore \underline{\underline{\frac{V_{s1}}{V_{s2}} = 4.4}}$$

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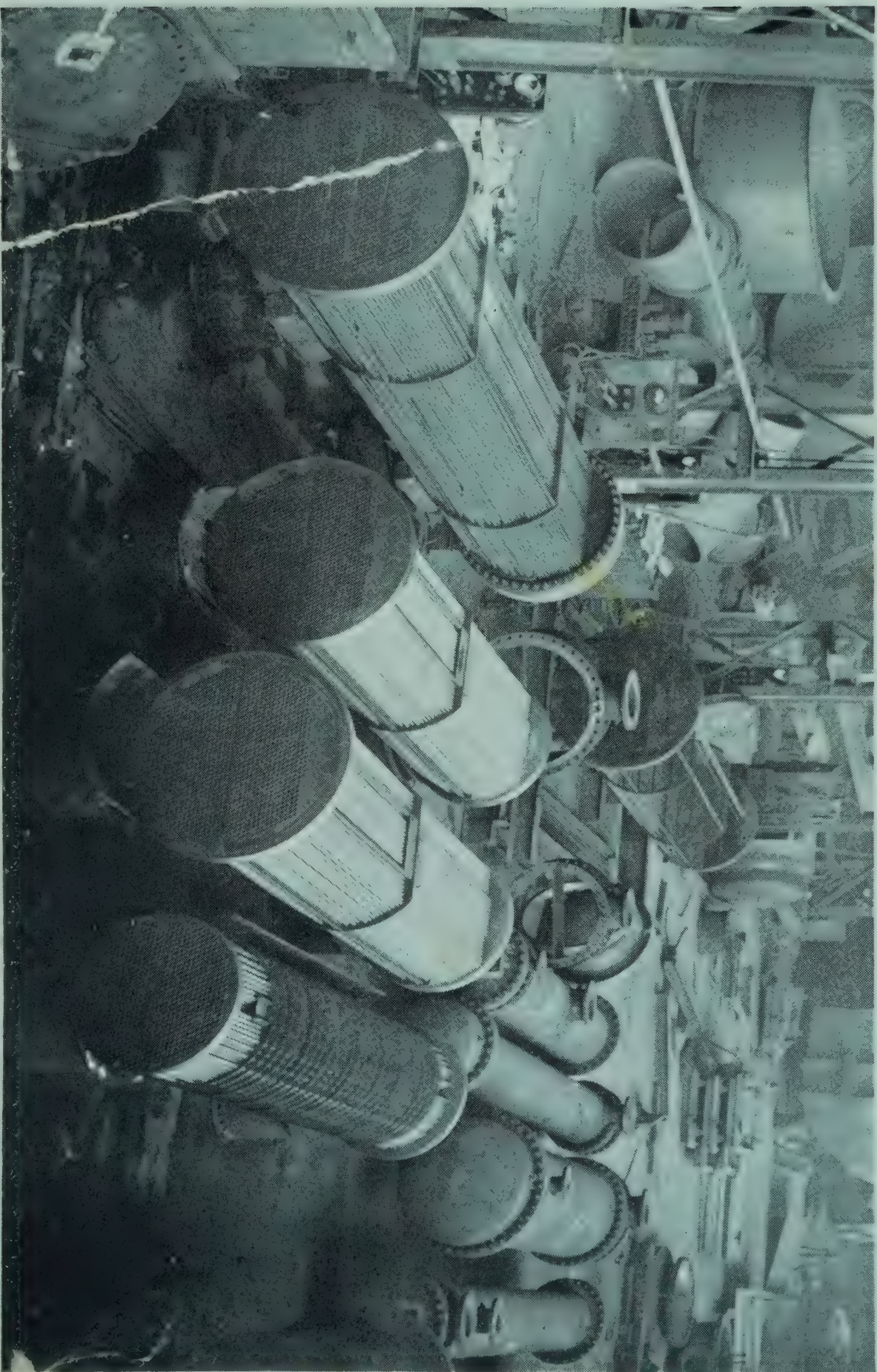
LIST OF SYMBOLS USED IN CHAPTER 5

A	Cross-sectional area of pipe	L^2
b	Width of impeller of pump	L
c	Clearance in cylinder (ratio of clearance volume to swept volume)	—
c_1, c_2, \dots	Clearances in first, second, . . . cylinders	—
E	Efficiency	—
F	Energy degraded due to irreversibility per unit mass of fluid	L^2T^{-2}
G	Mass rate of flow	MT^{-1}
g	Acceleration due to gravity	LT^{-2}
H	Enthalpy per unit mass	L^2T^{-2}
h	Virtual head developed by centrifugal pump	L
h_a	Head equivalent to atmospheric pressure	L
h_s	Submergence of air lift pump	L
h_r	Net height through which liquid is raised	L
l	Distance from top of limb of air lift pump	L
M	Mass of liquid	M
M_e	Mass of liquid contained by acid elevator	M
m	Mass of gas	M
N	Number of revolutions per unit time	T^{-1}
n	Number of stages of compression	—
P	Pressure	$ML^{-1}T^{-2}$
P_a	Atmospheric pressure	$ML^{-1}T^{-2}$
P_{i1}, P_{i2}, \dots	Pressures at delivery from first, second, . . . cylinders of multi-stage compressor	$ML^{-1}T^{-2}$
p	Power developed by pump	ML^2T^{-3}
Q	Volumetric rate of flow	L^3T^{-1}
q	Net heat added from surroundings per unit mass of fluid	L^2T^{-2}
r	Radius at which fluid is rotating	L
r_1, r_2	Radius of rotation at inlet, outlet of pump	L
S	Entropy per unit mass	$L^2T^{-2}\theta^{-1}$
s	Head of liquid in air lift pump	L
T	Absolute temperature	θ

t	Time	T
U	Internal energy per unit mass	L^2T^{-2}
u	Velocity of fluid	LT^{-1}
u_1, u_2	Velocity of fluid at inlet, outlet of pump	LT^{-1}
u_r	Velocity of fluid relative to vane of impeller	LT^{-1}
u_t	Tangential velocity of tip of vane of impeller	LT^{-1}
V	Volume of mass, m , of fluid	L^3
V_a	Volume of gas at atmospheric pressure	L^3
V_b	Volume of gas and liquid in riser of air lift pump	L^3
V_s	Swept volume of cylinder	L^3
V_{s1}, V_{s2}, \dots	Swept volumes of first, second, . . . cylinders	L^3
v	Volume per unit mass of fluid	$M^{-1}L^3$
v_a	Volume per unit mass at atmospheric pressure	$M^{-1}L^3$
W	Net work done by unit mass of fluid on surroundings	L^2T^{-2}
W'	Total work of compression per cycle in multistage compressor	ML^2T^{-2}
z	Distance in vertical direction	L
α	Correction factor for kinetic energy of fluid	—
β	Angle between tangential direction and blade of impeller at its tip	—
γ	Ratio of the specific heat at constant pressure to the specific heat at constant volume	—
ρ	Density of fluid (of liquid in air lift pump)	ML^{-3}
θ	Angle between tangential direction and direction of motion of fluid	—
τ	Torque	ML^2T^{-2}
ω	Angular velocity	T^{-1}
Δ	A finite difference in a quantity	—

Section B

Heat Transfer



Heat exchangers under construction
(Courtesy of John Thompson & Co. Ltd., Wolverhampton)

Frontispiece

CHAPTER 6

Heat Transfer

IN the majority of chemical processes heat is either given out or absorbed, and in a very wide range of chemical plant we are involved in heating or cooling fluids. Thus in furnaces, evaporators, distillation units, driers, and reaction vessels one of the major problems is that of transferring heat at the desired rate. Alternatively, it may be necessary to prevent the loss of heat from a hot vessel or steam pipe. The control of the flow of heat in the desired manner forms one of the most important sections of chemical engineering. Provided that a temperature difference exists between two parts of a system, heat transfer will take place in one or more of three different ways.

Conduction. In a solid, the flow of heat by conduction is the result of the transfer of vibrational energy from one molecule to another, and in fluids it occurs in addition as a result of the transfer of kinetic energy. Heat transfer by conduction may also arise from the transfer of free electrons. This process is particularly important with metals and accounts for their high thermal conductivities.

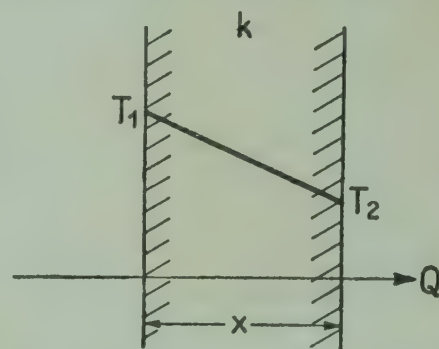


Fig. 6.1. Conduction of heat through a plane wall

Convection. Heat transfer by convection is attributable to macroscopic motion of the fluid and therefore is confined to liquids and gases. In natural convection it is caused by differences in density arising from temperature gradients in the system. In forced convection, it is due to eddy currents in a fluid in turbulent motion.

Radiation. All materials radiate thermal energy in the form of electromagnetic waves. When this radiation falls on a second body it may be partially reflected, transmitted, or absorbed. It is only the fraction that is absorbed that appears as heat in the body.

HEAT TRANSFER BY CONDUCTION

Thermal Conductivity and Thermal Resistance

For a plane wall of thickness x , as shown in Fig. 6.1, in which the two faces are kept at temperatures T_1 and T_2 , the flow of heat through area A per unit time is given by,

$$Q = \frac{k \cdot A \cdot (T_1 - T_2)}{x} \quad \dots(6.1)$$

In this equation Q is the heat transferred and k is a property of the material, known as the thermal conductivity. In general, it is a function of temperature

and the mean value over the temperature range T_1 to T_2 should be used in equation 6.1. Over a small distance dx , we have,

$$Q = -k \cdot A \cdot \frac{dT}{dx}$$

....(6.2)

the negative sign indicating that the temperature gradient is in the opposite direction to the flow of heat.

TABLE 6.1. Thermal conductivities . k . lb-cal/hr-ft²-°C/ft or B.Th.U./hr-ft²-°F/ft

	Temp. (°C)	k		Temp. (°C)	k
<i>Solids—Metals</i>			<i>Liquids</i>		
Aluminium	300	133	Acetic acid 50 %	20	0.20
Cadmium	18	54	Acetone	30	0.10
Copper	100	218	Aniline	0–20	0.1
Iron (wrought)	18	35	Benzene	30	0.09
Iron (cast)	53	27.6	Calcium chloride brine 30 %	30	0.32
Lead	100	19	Ethyl alcohol 80 %	20	0.137
Nickel	100	33	Glycerol 60 %	20	0.22
Silver	100	238	Glycerol 40 %	20	0.26
Steel 1 % C	18	26	n-Heptane	30	0.08
Tantalum	18	32	Mercury	28	4.83
Admiralty metal	30	65	Sulphuric acid 90 %	30	0.21
Bronze	—	109	Sulphuric acid 60 %	30	0.25
Stainless steel	20	9.2	Water	30	0.356
			Water	60	0.381
<i>Solids—Non-metals</i>			<i>Gases</i>		
Asbestos sheet	50	0.096	Hydrogen	0	0.10
Asbestos	0	0.09	Carbon dioxide	0	0.0085
Asbestos	100	0.11	Air	0	0.014
Asbestos	200	0.12	Air	100	0.018
Bricks (alumina)	430	1.8	Methane	0	0.017
Bricks (building)	20	0.4	Water vapour	100	0.0145
Magnesite	200	2.2	Nitrogen	0	0.0138
Cotton wool	30	0.029	Ethylene	0	0.0097
Glass	30	0.63	Oxygen	0	0.0141
Mica	50	0.25	Ethane	0	0.0106
Rubber (hard)	0	0.087			
Sawdust	20	0.03			
Cork	30	0.025			
Glass wool	—	0.024			
85 % Magnesia	—	0.04			
Graphite	0	87			

To convert above figures to g-cal/sec-cm²-°C/cm, multiply by 0.00414

It will be seen in Table 6.1 that metals have very high thermal conductivities, non-metallic solids lower values (0.03 to 2 lb-cal/hr-ft²-°C/ft), non-metallic liquids low values (0.08 to 0.4), and gases very low values. It is important to note that amongst metals stainless steel has a low value of 9, that water has a very high value for liquids (due to partial ionization), and that hydrogen has a high value for gases (due to the high mobility of the molecules). With gases k decreases with increase in molecular weight and increases with the temperature.

In addition, for gases the dimensionless group, $\frac{C_p \mu}{k}$ is approximately constant (C_p is the specific heat at constant pressure and μ is the viscosity) and can be used to evaluate k at high temperatures where it is difficult to determine experimentally because of the formation of convection currents. k does not vary significantly with pressure until this is reduced to so low a value that the mean free path of the molecules becomes comparable with the dimensions of the vessel; further reduction of pressure then causes k to decrease.

The low conductivity of heat insulating materials, such as cork, glass wool, etc., is largely accounted for by their high proportion of air space. The flow of heat through the materials is governed mainly by the resistance of the air spaces, which should be sufficiently small for convection currents to be suppressed.

It is convenient to rearrange equation 6.1 in the form

$$Q = \frac{T_1 - T_2}{\frac{x}{k}} A \quad \dots(6.3)$$

where $\frac{x}{k}$ is known as the thermal resistance.

Example. Find the heat loss per square foot of surface through a brick wall 18 in. thick when the inner surface is at 120°C and the outside at 40°C : the thermal conductivity of the brick may be taken as $0.4 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C/ft}$.

Solution. From equation 6.1

$$\begin{aligned} Q &= \frac{0.4 \times 1 \times 80}{1.5} \\ &= \underline{\underline{21 \text{ lb-cal/hr-ft}^2}} \end{aligned}$$

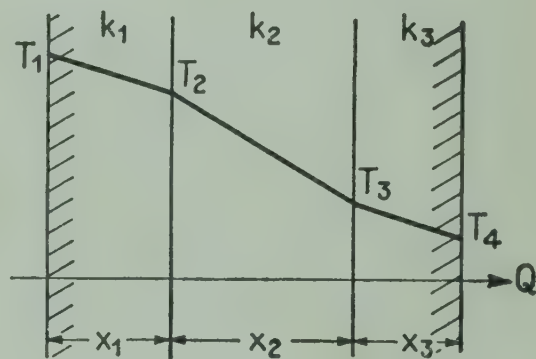


Fig. 6.2. Conduction of heat through a composite wall

Thermal Resistances in Series

Fig. 6.2 shows a composite wall made up of three materials with thermal conductivities k_1 , k_2 , and k_3 , with thicknesses as shown and with the temperatures T_1 , T_2 , T_3 , and T_4 at the faces. Applying equation 6.1 to each section in turn and noting that the same quantity of heat Q must pass through each area A , we get:

$$T_1 - T_2 = \frac{x_1}{k_1 \cdot A} \cdot Q, \quad T_2 - T_3 = \frac{x_2}{k_2 \cdot A} \cdot Q, \quad \text{and} \quad T_3 - T_4 = \frac{x_3}{k_3 \cdot A} \cdot Q$$

By addition

$$[T_1 - T_4] = \left[\frac{x_1}{k_1 \cdot A} + \frac{x_2}{k_2 \cdot A} + \frac{x_3}{k_3 \cdot A} \right] \cdot Q \quad \dots(6.4)$$

This can be arranged in the form of equation 6.3 as,

$$Q = \frac{T_1 - T_4}{\sum \frac{x_1}{k_1 \cdot A}} = \frac{\text{Total driving force}}{\text{Total } \frac{\text{thermal resistance}}{\text{area}}} \quad \dots (6.5)$$

Example. A furnace is constructed with 9 in. of firebrick, $4\frac{1}{2}$ in. of insulating brick and 9 in. of building brick. The inside temperature is 930°C and the outside temperature 55°C . If the thermal conductivities are as shown in Fig. 6.3, find the heat loss per unit area, and the temperature at the junction of the firebrick and insulating brick.

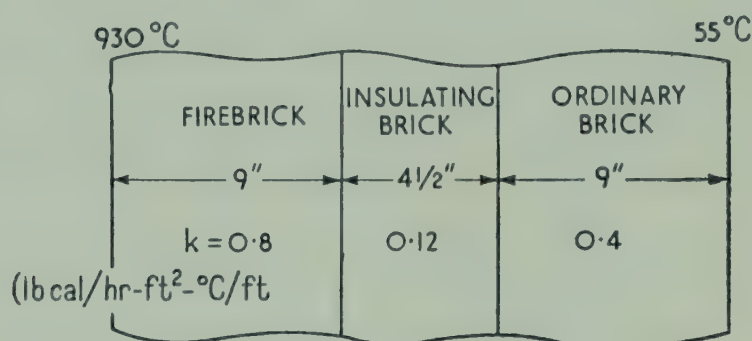


Fig. 6.3.

Solution. From equation 6.5.

$$\begin{aligned} Q &= \frac{930 - 55}{\frac{9}{12 \times 1 \times 0.8} + \frac{4.5}{12 \times 0.12} + \frac{9}{12 \times 0.4}} \\ &= \frac{875}{0.938 + 3.12 + 1.87} = \frac{875}{5.93} \\ &= \underline{\underline{148 \text{ lb-cal/hr-ft}^2}} \end{aligned}$$

$$\frac{\text{Temp. drop over firebrick}}{\text{Total temp. drop}} = \frac{0.94}{5.93}$$

$$\therefore \text{Temp. drop over firebrick } 875 \times \frac{0.94}{5.93} = 140^\circ\text{C}$$

Hence temperature at this plane is 790°C.

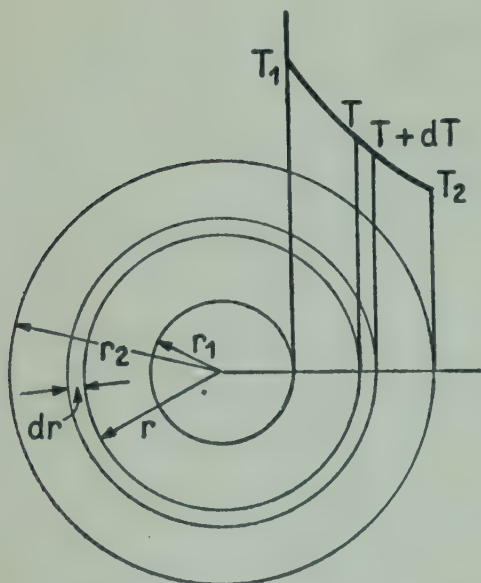


Fig. 6.4. Conduction through thick walled tube

Conduction Through a Thick Walled Tube

The conditions for heat flow through a thick walled tube when the temperature on the inside and outside are held constant are shown in Fig. 6.4. Here the area for heat flow is proportional to the radius so that the temperature gradient is inversely proportional to the radius.

The heat flow at any radius, r , is given by

$$Q = -k \cdot 2 \cdot \pi \cdot r \cdot l \cdot \frac{dT}{dr} \quad \dots (6.6)$$

where l is the length of tube.

Integrating between limits r_1 and r_2 ,

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -2 \cdot \pi \cdot l \cdot k \cdot \int_{T_1}^{T_2} dT$$

or

$$Q = \frac{2 \cdot \pi \cdot l \cdot k \cdot (T_1 - T_2)}{\ln \cdot r_2/r_1} \quad \dots(6.7)$$

This equation can be put into the form of equation 6.1, i.e.

$$Q = \frac{2 \cdot \pi \cdot r_m \cdot k \cdot l \cdot (T_1 - T_2)}{r_2 - r_1} \quad \dots(6.8)$$

where $r_m = \frac{r_2 - r_1}{\ln \cdot r_2/r_1}$, and is known as the logarithmic mean radius. For thin walled tubes it will be sufficient to use the arithmetic mean radius r_a , giving

$$Q = \frac{2 \cdot \pi \cdot k \cdot l \cdot r_a (T_1 - T_2)}{r_2 - r_1} \quad \dots(6.8a)$$

Heat Flow Through a Tube Wall from One Fluid to a Second Fluid

In heat exchangers, one fluid flowing through a tube is often heated or cooled by a second fluid flowing over the outside of the tube. This case is more complex than that of pure conduction because of the nature of the flow near a surface.

If a fluid flows in turbulent motion parallel to a solid surface, the velocity increases rapidly from zero at the wall to an almost constant value at a short distance away. This velocity gradient near the wall is accompanied by a corresponding sharp change in the temperature of the fluid. A similar condition occurs on the other side so that the temperature

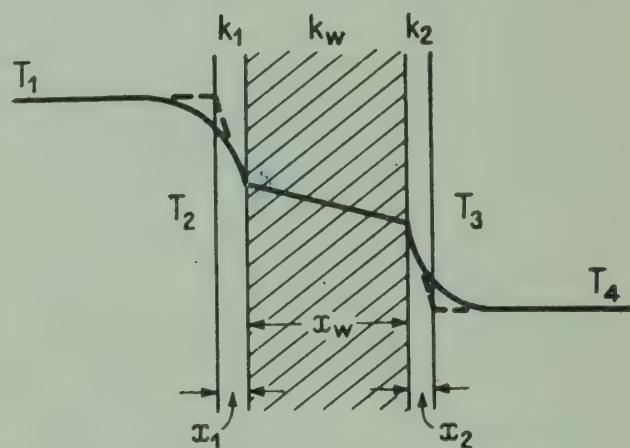


Fig. 6.5. Heat transfer between two fluids

distribution from a hot fluid on one side to a cold fluid on the other will be of the form shown in Fig. 6.5. This temperature curve may be replaced by the dotted line, by assuming that the temperature difference between the hot fluid at T_1 and the wall at T_2 all occurs across the layer of thickness x_1 and that the heat transfer across this layer is by thermal conduction. We may then write, from equation 6.1,

$$Q = \frac{k_1 \cdot A_1 \cdot (T_1 - T_2)}{x_1} \quad \dots(6.9)$$

The thickness of the layer x_1 will depend on the nature of the flow and on the surface of the wall and in general is not known. For this reason it is customary to rearrange equation 6.9 as

$$Q = h_1 \cdot A_1 \cdot (T_1 - T_2) \quad \dots(6.10)$$

or

$$T_1 - T_2 = \frac{Q}{h_1 \cdot A_1}$$

where h_1 is known as the heat transfer coefficient and $\frac{1}{h_1}$ represents the thermal resistance of the film.

We can now write equations for the heat flow through the outside film, the tube wall, and the inside film as:

$$T_1 - T_2 = \frac{Q}{h_1 \cdot A_1}; \quad T_2 - T_3 = \frac{x_w Q}{k_w \cdot A_w}; \quad T_3 - T_4 = \frac{Q}{h_2 \cdot A_2}$$

From which

$$T_1 - T_4 = \Delta T = Q \cdot \left[\frac{1}{h_1 \cdot A_1} + \frac{x_w}{k_w \cdot A_w} + \frac{1}{h_2 \cdot A_2} \right]$$

But we can define an overall coefficient of heat transfer U_1 based on the area A_1 such that

$$Q = U_1 \cdot A_1 \cdot \Delta T \quad \dots (6.11)$$

Thus

$$\frac{Q}{U_1 \cdot A_1} = \Delta T = Q \left[\frac{1}{h_1 \cdot A_1} + \frac{x_w}{k_w \cdot A_w} + \frac{1}{h_2 \cdot A_2} \right]$$

Giving

$$\frac{1}{U_1} = \frac{1}{h_1} + \frac{x_w \cdot A_1}{k_w \cdot A_w} + \frac{A_1}{h_2 \cdot A_2} \quad \dots (6.12)$$

The overall transfer coefficient may be based on the inside area, the outside area or the mean area of a tube. With thin walled tubes these will not differ appreciably and we may write,

$$\frac{1}{U} = \frac{1}{h_1} + \frac{x_w}{k_w} + \frac{1}{h_2} \quad \dots (6.12a)$$

Mean Temperature Difference

In the previous discussion it has been assumed that the temperature on each side of the surface, and hence the temperature difference was everywhere constant. In the majority of heat exchangers the temperature of at least one of the fluids changes as it passes through the unit, so that the temperature

difference varies over the length. It is therefore necessary to find the average value of the temperature difference θ_m to be used in the general equation

$$Q = U \cdot A \cdot \theta_m \quad \dots (6.13)$$

Fig. 6.6 shows the temperature conditions for both fluids flowing in the same direction, a condition known as co-current flow.

Outside stream (sp. ht. C_{p1}) at rate G_1 falls in temperature from T_{11} to T_{12} .

Inside stream (sp. ht. C_{p2}) at rate G_2 rises in temperature from T_{21} to T_{22} .

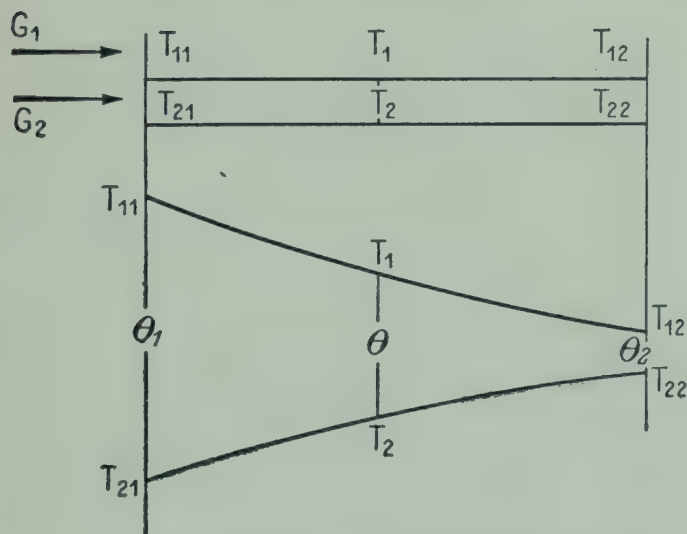


Fig. 6.6. Mean temperature difference for co-current flow

Over a small element of area dA where the temperatures of the streams are T_1 and T_2 . The temperature difference,

$$\theta = T_1 - T_2$$

$$\therefore d\theta = dT_1 - dT_2$$

Heat given out by hot stream $= dQ = -G_1 C_{p1} \cdot dT_1$.

Heat taken up by cold stream $= dQ = G_2 C_{p2} \cdot dT_2$.

$$\therefore d\theta = -\frac{dQ}{G_1 C_{p1}} - \frac{dQ}{G_2 C_{p2}} = -dQ \left[\frac{G_1 C_{p1} + G_2 C_{p2}}{G_1 C_{p1} \times G_2 C_{p2}} \right] = -\psi \cdot dQ \quad (\text{say})$$

$$\therefore \theta_1 - \theta_2 = \psi Q$$

Over this element U remains constant, so that $U \cdot dA \cdot \theta = dQ$.

$$\therefore U \cdot dA \cdot \theta = -\frac{1}{\psi} \cdot d\theta$$

$$\therefore -\psi U \cdot \int_0^A dA = \int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta}$$

$$\therefore -\psi U \cdot A = -\ln \frac{\theta_1}{\theta_2}$$

But $Q = U \cdot A \cdot \theta_m$ by definition of θ_m .

$$\therefore \theta_1 - \theta_2 = \psi Q = \psi U \cdot A \cdot \theta_m = \ln \frac{\theta_1}{\theta_2} \cdot (\theta_m)$$

$$\therefore \theta_m = \frac{\theta_1 - \theta_2}{\ln \theta_1 / \theta_2} \quad \dots (6.14)$$

If the exchanger is arranged so that the two fluids flow in opposite directions, i.e. in counter-current flow, a similar analysis will give the same value for θ_m . It is important to note that this result is dependent on assuming that U is constant over the temperature range considered. θ_m is known as the logarithmic mean temperature difference.

Unsteady Transfer of Heat

In the problems which have been considered so far, it has been assumed that the conditions at any point in the system remain constant with respect to time. The heat transfer in a medium in which the temperature is changing with time, because of thermal conduction alone, will now be treated. This problem is of importance in the calculation of the temperature distribution in a body which is being heated or cooled. In an element of dimensions dx by dy by dz , let the temperature at the point (x, y, z) be θ and at the point $(x + dx, y + dy, z + dz)$ be $\theta + d\theta$.

The rate of conduction of heat through the element

$$= -k \cdot dy \cdot dz \left(\frac{\partial \theta}{\partial x} \right)_{yz} \text{ in the } x \text{ direction}$$

$$= -k \cdot dz \cdot dx \left(\frac{\partial \theta}{\partial y} \right)_{zx} \text{ in the } y \text{ direction}$$

and
$$= -k \cdot dx \cdot dy \left(\frac{\partial \theta}{\partial z} \right)_{xy} \text{ in the } z \text{ direction}$$

Now the rate of change of heat content of the element will be equal to minus the rate of increase of heat flow from (x, y, z) to $(x + dx, y + dy, z + dz)$.

Thus rate of change of heat content of element

$$\begin{aligned} &= k \cdot dy \cdot dz \left(\frac{\partial^2 \theta}{\partial x^2} \right)_{yz} dx + k \cdot dz \cdot dx \left(\frac{\partial^2 \theta}{\partial y^2} \right)_{zx} dy + k \cdot dx \cdot dy \left(\frac{\partial^2 \theta}{\partial z^2} \right)_{xy} dz \\ &= k \cdot dx \cdot dy \cdot dz \left\{ \left(\frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right\} \end{aligned} \quad \dots (6.15)$$

But the rate of increase of heat content is also equal to the product of the heat capacity of the element and the rate of rise of temperature.

$$\text{Thus } k \cdot dx \cdot dy \cdot dz \left\{ \left(\frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right\} = C_p \cdot \rho \cdot dx \cdot dy \cdot dz \frac{\partial \theta}{\partial t}$$

i.e.
$$\begin{aligned} \frac{\partial \theta}{\partial t} &= \frac{k}{C_p \cdot \rho} \left\{ \left(\frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right\} \\ &= D_H \left\{ \left(\frac{\partial^2 \theta}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 \theta}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 \theta}{\partial z^2} \right)_{xy} \right\} \end{aligned} \quad \dots (6.16)$$

where D_H , the thermal diffusivity, $= \frac{k}{C_p \rho}$.

The above partial differential equation is most conveniently solved by the use of the Laplace transform of temperature with respect to time. As an illustration of the method of solution, the problem of the unidirectional flow of heat in a continuous medium will be considered. The basic differential equation for the process is

$$\frac{\partial \theta}{\partial t} = D_H \frac{\partial^2 \theta}{\partial x^2} \quad \dots (6.17)$$

This equation cannot be integrated directly, since the temperature θ is expressed as a function of two independent variables, distance x and time t . The method of solution involves transforming the equation so that the Laplace transform of θ with respect to time is used in place of θ . The equation then involves only the Laplace transform $\bar{\theta}$ and the distance x . The Laplace transform of θ is defined by the relation,

$$\bar{\theta} = \int_0^\infty \theta \cdot e^{-pt} \cdot dt \quad \dots (6.18)$$

where p is a parameter.

Thus $\bar{\theta}$ is a function of both θ and t . Since it is not a function of x ,

$$\frac{\partial^2 \bar{\theta}}{\partial x^2} = \frac{\partial^2 \bar{\theta}}{\partial x^2} \quad \dots (6.19)$$

Now

$$\begin{aligned}\frac{\partial \bar{\theta}}{\partial t} &= \int_0^\infty \frac{\partial \theta}{\partial t} e^{-pt} dt \\ &= \left[\theta \cdot e^{-pt} \right]_0^\infty + p \cdot \int_0^\infty e^{-pt} \cdot \theta \cdot dt \\ &= -\theta_{t=0} + p\bar{\theta}\end{aligned}\quad \dots(6.20)$$

Then taking the Laplace transforms of each side of equation 6.17,

$$\frac{\partial \bar{\theta}}{\partial t} = D_H \frac{\partial^2 \bar{\theta}}{\partial x^2}$$

i.e. $p\bar{\theta} - \theta_{t=0} = D_H \frac{\partial^2 \bar{\theta}}{\partial x^2}$ (from equations 6.19, 6.20)

i.e. $\frac{\partial^2 \bar{\theta}}{\partial x^2} - \frac{p}{D_H} \bar{\theta} = -\frac{\theta_{t=0}}{D_H}$

If the temperature everywhere is constant initially, $\theta_{t=0}$ is a constant and integration of the equation gives,

$$\bar{\theta} = B_1 e^{\sqrt{\frac{p}{D_H}} x} + B_2 e^{-\sqrt{\frac{p}{D_H}} x} + \theta_{t=0} \cdot p^{-1} \quad \dots(6.21)$$

and $\therefore \frac{\partial \bar{\theta}}{\partial x} = B_1 \sqrt{\frac{p}{D_H}} e^{\sqrt{\frac{p}{D_H}} x} - B_2 \sqrt{\frac{p}{D_H}} e^{-\sqrt{\frac{p}{D_H}} x} \quad \dots(6.22)$

The temperature θ , corresponding to the above transform $\bar{\theta}$, can now be found by reference to tables of the Laplace transform. It is first necessary, however, to evaluate the constants B_1 and B_2 using the boundary conditions for the particular problem because these constants will in general involve the parameter p which was introduced in the transformation.

Let us therefore consider the particular problem of the unidirectional flow of heat through a body with plane parallel faces a distance l apart. The heat flow is normal to these faces and the temperature of the body is initially constant throughout. The temperature scale will be so chosen that this uniform initial temperature is zero. At time, $t = 0$, one face (at $x = 0$) will be brought into contact with a source at a constant temperature θ' and the other face (at $x = l$) will be assumed to be perfectly insulated thermally.

The boundary conditions are therefore,

$$\begin{cases} t = 0, & \theta = 0 \\ t > 0, & \theta = \theta' \text{ when } x = 0, \\ t > 0, & \frac{\partial \theta}{\partial x} = 0 \text{ when } x = l \end{cases}$$

Thus

$$\bar{\theta}_{x=0} = \int_0^\infty \theta' e^{-pt} dt = \frac{\theta'}{p}$$

and

$$\left(\frac{\partial \bar{\theta}}{\partial x} \right)_{x=l} = 0$$

Substitution of these boundary conditions in equations 6.21 and 6.22 gives,

$$B_1 + B_2 = \frac{\theta'}{p}$$

and

$$B_1 e^{\sqrt{\frac{p}{D_H}} l} - B_2 e^{-\sqrt{\frac{p}{D_H}} l} = 0$$

Hence

$$B_1 = \frac{\frac{\theta'}{p} e^{-\sqrt{\frac{p}{D_H}} l}}{e^{\sqrt{\frac{p}{D_H}} l} + e^{-\sqrt{\frac{p}{D_H}} l}}$$

and

$$B_2 = \frac{\frac{\theta'}{p} e^{\sqrt{\frac{p}{D_H}} l}}{e^{\sqrt{\frac{p}{D_H}} l} + e^{-\sqrt{\frac{p}{D_H}} l}}$$

$$\begin{aligned} \text{Then } \bar{\theta} &= \frac{e^{-x\sqrt{\frac{p}{D_H}}} + e^{-(l-x)\sqrt{\frac{p}{D_H}}}}{e^{\sqrt{\frac{p}{D_H}} l} + e^{-\sqrt{\frac{p}{D_H}} l}} \frac{\theta'}{p} \\ &= \frac{\theta'}{p} \left\{ e^{(l-x)\sqrt{\frac{p}{D_H}}} + e^{-(l-x)\sqrt{\frac{p}{D_H}}} \right\} \left\{ 1 + e^{-2\sqrt{\frac{p}{D_H}} l} \right\}^{-1} \left\{ e^{-\sqrt{\frac{p}{D_H}} l} \right\} \\ &= \frac{\theta'}{p} \left\{ e^{-x\sqrt{\frac{p}{D_H}}} + e^{-(2l-x)\sqrt{\frac{p}{D_H}}} \right\} \left\{ 1 - e^{-2l\sqrt{\frac{p}{D_H}}} + \dots \right. \\ &\quad \left. + (-1)^N e^{-2Nl\sqrt{\frac{p}{D_H}}} + \dots \right\} \\ &= \sum_{N=0}^{\infty} \frac{\theta'}{p} (-1)^N \left\{ e^{-(2lN+x)\sqrt{\frac{p}{D_H}}} + e^{-\{2(N+1)l-x\}\sqrt{\frac{p}{D_H}}} \right\} \dots (6.23) \end{aligned}$$

The temperature θ is then obtained from tables of inverse Laplace transforms⁽⁵⁾ and is given by

$$\theta = \sum_{N=0}^{\infty} (-1)^N \theta' \left\{ \operatorname{erfc} \cdot \frac{2lN+x}{2\sqrt{D_H t}} + \operatorname{erfc} \cdot \frac{2(N+1)l-x}{2\sqrt{D_H t}} \right\} \dots (6.24)$$

$$\text{where } \operatorname{erfc} \cdot x = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-x^2} dx$$

Values of $\operatorname{erfc} \cdot x$ are to be found in mathematical tables⁽⁵⁾.

The numerical solution to the above problem is then obtained by inserting the appropriate values for the physical properties of the system and using as many terms in the series as are necessary for the degree of accuracy required. In most cases, the above series converge quite rapidly.

This method of solution of problems of unsteady flow is particularly useful because it is applicable when there are discontinuities in the physical properties of the material^(53a).

The boundary conditions, however, become a little more complicated, but the problem is intrinsically no more difficult.

Example. Calculate the time taken for the distant face of a brick wall, of thermal diffusivity, $D_H = 0.0042 \text{ cm}^2/\text{sec.}$ and thickness, $l = 18 \text{ in.}$, to rise from 20°C to 200°C . The whole wall is initially at a constant temperature of 20°C and the near face is suddenly raised to 600°C and maintained at that temperature. Assume that all the flow of heat is perpendicular to the faces of the wall and that the distant face is perfectly insulated.

Solution. The temperature at any distance x from the near face at time t is given by

$$\theta = \sum_{N=0}^{N=\infty} (-1)^N \cdot \theta' \left\{ \operatorname{erfc} \frac{2lN + x}{2\sqrt{D_H t}} + \operatorname{erfc} \frac{2(N+1)l - x}{2\sqrt{D_H \cdot t}} \right\} \dots (6.24)$$

The temperature at the distant face is therefore given by,

$$\theta = \sum_{N=0}^{N=\infty} (-1)^N \cdot \theta' \cdot 2 \cdot \operatorname{erfc} \frac{(2N+1) \cdot l}{2\sqrt{D_H \cdot t}}$$

Now choosing the temperature scale so that the initial temperature is everywhere zero,

$$\theta/\theta' = 180/580 = 0.31$$

$$D_H = 0.0042 \text{ cm}^2/\text{sec.}$$

Thus

$$\sqrt{D_H} = 0.065$$

$$l = 18 \text{ in.} = 46 \text{ cm}$$

Thus

$$\begin{aligned} 0.155 &= \sum_{N=0}^{N=\infty} (-1)^N \operatorname{erfc} \frac{355(2N+1)}{t^{\frac{1}{2}}} \\ &= \operatorname{erfc}(355t^{-\frac{1}{2}}) - \operatorname{erfc}(1065t^{-\frac{1}{2}}) + \operatorname{erfc}(1775t^{-\frac{1}{2}}) - \dots \end{aligned}$$

An approximate answer is obtained by taking the first term only.

Then

$$355t^{-\frac{1}{2}} = 1.0$$

giving

$$t = 127,000$$

The equation will now be solved in the following table:

t	$t^{\frac{1}{2}}$	$355t^{-\frac{1}{2}}$	$\operatorname{erfc} \cdot 355t^{-\frac{1}{2}}$	$\operatorname{erfc} \cdot 1065 \cdot t^{-\frac{1}{2}}$
1.0×10^5	316	1.12	0.1138	0.0000
1.1×10^5	332	1.07	0.1298	0.0000
1.2×10^5	347	1.02	0.1497	0.0000
1.3×10^5	361	0.981	0.1659	0.0000
1.4×10^5	374	0.946	0.1800	0.0000
1.5×10^5	388	0.912	0.1983	0.0001

By interpolation required value of

$$t = 1.27 \times 10^5 \text{ sec.}$$

$$= \underline{\underline{35 \text{ hr}}}$$

The exact mathematical solution of problems involving unsteady thermal conduction may be very difficult, and sometimes impossible, especially where bodies of irregular shapes are concerned, and other methods are therefore required.

For a body of characteristic linear dimension L initially at a uniform temperature θ_0 exposed suddenly to surroundings at a temperature θ' , the temperature θ

at any time t at a distance x from the surface can be combined with the other variables to give the dimensionless groups,

$$\frac{\theta' - \theta}{\theta' - \theta_0}, \quad \frac{hL}{k}, \quad D_H \frac{t}{L^2}, \quad \text{and} \quad \frac{x}{L}$$

where h is the heat transfer coefficient at the surface. Where the resistance to heat transfer at the surface can be neglected, i.e. where the temperature of the surface of the body is maintained at the temperature θ' the temperature will no longer be a function of h or L and we have only two dimensionless groups,

$$\frac{\theta' - \theta}{\theta' - \theta_0} \quad \text{and} \quad D_H \frac{t}{x^2}$$

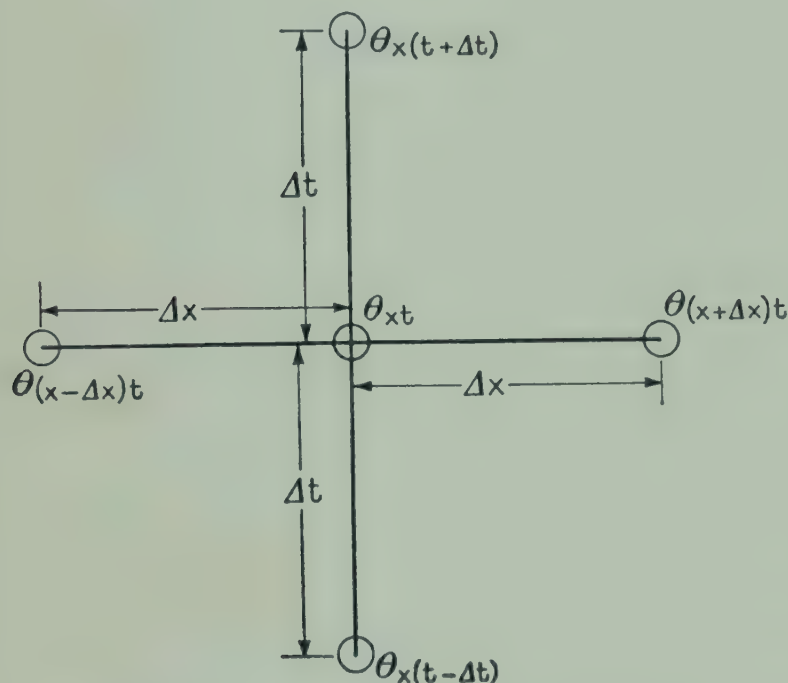


Fig. 6.7. Variation of temperature with time and distance

Curves connecting these groups have been plotted by a number of workers for bodies of various shapes. The method is limited, however, to those shapes which have been studied experimentally.

A general method of estimating the temperature distribution in a body of any shape consists of replacing the heat flow problem by the analogous electrical problem and measuring the electrical potentials at various points. The heat

capacity per unit volume $C_p \rho$ is represented by an electrical capacitance, and the thermal conductivity k by an electrical conductivity. The method can be used to take account of variations in the thermal properties over the body.

A series of numerical methods has been developed by replacing the differential equation by a finite difference equation. Thus in a problem of unidirectional flow of heat, we can write

$$\begin{aligned} \frac{\partial \theta}{\partial t} &\approx \frac{\theta_{x(t+\Delta t)} - \theta_{x(t-\Delta t)}}{2\Delta t} \approx \frac{\theta_{x(t+\Delta t)} - \theta_{xt}}{\Delta t} \\ \frac{\partial^2 \theta}{\partial x^2} &\approx \frac{\frac{\theta_{(x+\Delta x)t} - \theta_{xt}}{\Delta x} - \frac{\theta_{xt} - \theta_{(x-\Delta x)t}}{\Delta x}}{\Delta x} \\ &= \frac{\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}}{(\Delta x)^2} \end{aligned}$$

where θ_{xt} is the value of θ at time t and distance x from the surface, and the other values of θ are at intervals Δx and Δt as shown in Fig. 6.7.

Substituting these values in equation 6.17,

$$\theta_{x(t+\Delta t)} - \theta_{x(t-\Delta t)} = D_H \frac{2\Delta t}{(\Delta x)^2} (\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}) \dots (6.25)$$

and

$$\theta_{x(t+\Delta t)} - \theta_{xt} = D_H \frac{\Delta t}{(\Delta x)^2} (\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t} - 2\theta_{xt}) \dots (6.26)$$

Thus, if the temperature distribution at time, t , is known, the corresponding distribution at time $t + \Delta t$ can be calculated by the application of equation 6.26 over the whole extent of the body in question. The intervals Δx and Δt are so chosen that the required degree of accuracy is obtained.

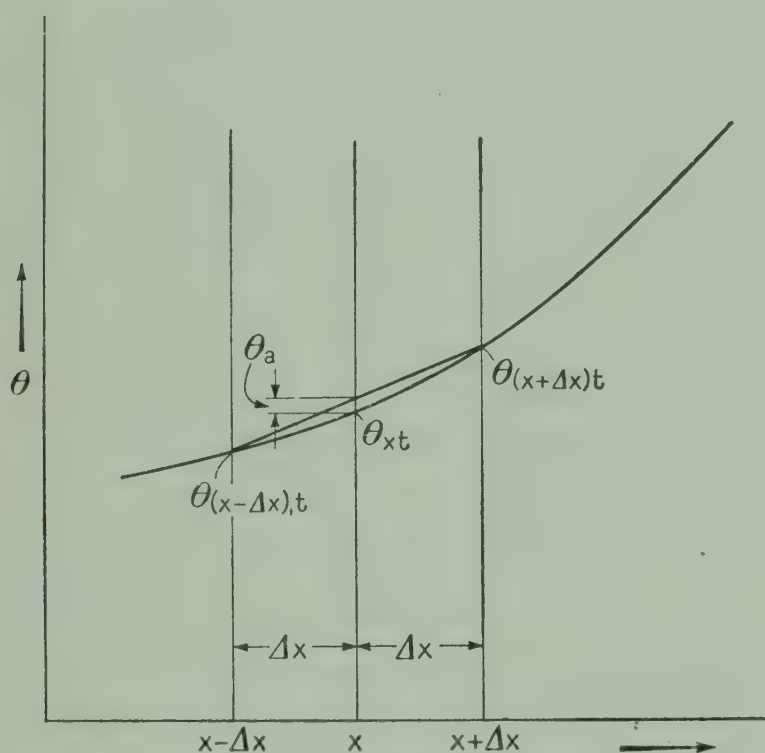


Fig. 6.8. Schmidt's method

A graphical method of procedure is due to SCHMIDT⁽¹⁾. Suppose that the temperature distribution at time t is represented by the curve AB as shown in Fig. 6.8. If the points representing the temperatures at $x - \Delta x$ and $x + \Delta x$ are joined by a straight line, the distance θ_a is given by

$$\begin{aligned} \theta_a &= \frac{\theta_{(x+\Delta x)t} + \theta_{(x-\Delta x)t}}{2} - \theta_{xt} \\ &= \frac{(\Delta x)^2}{2D_H \Delta t} (\theta_{x(t+\Delta t)} - \theta_{xt}) \quad (\text{from equation 6.26}) \end{aligned}$$

Thus θ_a represents the change in θ_{xt} after a time interval, Δt , such that

$$\Delta t = \frac{(\Delta x)^2}{2D_H}$$

If this simple construction is carried out over the whole of the body, the temperature distribution after time Δt is obtained. The temperature distribution after an interval $2\Delta t$ is then obtained by a repetition of this procedure.

SOUTHWELL's relaxation method⁽⁴⁾ can also be applied to determine the temperature distribution at any time by using the finite difference equation in the form of equation 6.25.

HEAT TRANSFER BY CONVECTION

Determination of Film Coefficients for Convection

To calculate the area of surface required to give a heat transfer rate Q , it is necessary to know the overall coefficient U or the values of the individual film coefficients, h_1, h_2 In this section we shall consider methods for calculating individual coefficients when the heat is transferred by convection.

So many factors influence the value of h that it is almost impossible to determine their individual effects by direct experimental methods. By arranging these terms in a series of dimensionless groups, equations have been developed for calculating the values of h for many practical conditions. The form of these equations may be deduced from considerations of the boundary layer (Chapter 9), but it is easier to get a general picture in the first place by the method of dimensional analysis⁽²⁾.

Application of Dimensional Analysis to Heat Transfer by Convection

The heat transferred per unit area per unit time q will be considered to depend on the velocity u , the density ρ , the viscosity μ , the thermal conductivity k , the coefficient of thermal expansion β and the specific heat C_p of the fluid, and the temperature difference ΔT between the fluid and the surface. It will also depend on the characteristic dimension L of the surface across which the heat is transferred. These quantities, with the dimensions in which they are measured are given below. Then over the range considered it will be assumed that the relation between the heat flow q and the variables can be expressed as a simple exponential series, i.e.

$$q \propto [u^{w_1} . L^{w_2} . \Delta T^{w_3} . \mu^{w_4} . k^{w_5} . \rho^{w_6} . C_p^{w_7} . (\beta g)^{w_8}] \dots (6.27)$$

Dimensions of the various terms in length **L**, mass **M**, time **T**, temperature θ , heat **Q**:

q	Heat transferred/unit area . unit time	$QL^{-2}T^{-1}$
u	Velocity	LT^{-1}
L	Linear dimension	L
μ	Viscosity	$ML^{-1}T^{-1}$
ρ	Density	ML^{-3}
k	Thermal conductivity	$QT^{-1}L^{-1}\theta^{-1}$
C_p	Specific heat at constant pressure	$QM^{-1}\theta^{-1}$
ΔT	Temperature difference	θ
$(\beta . g)$	Coefficient of thermal expansion times g	$LT^{-2}\theta^{-1}$

It will be noted that both temperature and heat are here taken as fundamental units; this provides an additional equation below.

Then equating the indices on each side of equation 6.27,

$$\begin{array}{ll} \text{in length} & w_1 + w_2 - w_4 - w_5 - 3 \cdot w_6 + w_8 = -2 \\ \text{in mass} & w_4 + w_6 - w_7 = 0 \\ \text{in time} & -w_1 - w_4 - w_5 - 2 \cdot w_8 = -1 \\ \text{in temperature} & w_3 - w_5 - w_7 - w_8 = 0 \\ \text{in heat} & w_5 + w_7 = 1 \end{array}$$

These equations may be solved in terms of w_1 , w_7 , w_8 to obtain groups each containing only one of the variables u , C_p , and βg .

Thus

$$\begin{aligned} w_5 &= 1 - w_7; \quad w_4 = -w_1 + w_7 - 2w_8; \quad w_3 = 1 + w_8; \\ w_2 &= -1 + w_1 + 3w_8; \quad \text{and } w_6 = w_1 + 2w_8 \end{aligned}$$

Hence

$$q \propto u^{w_1} \cdot L^{-1+w_1+3w_8} \cdot \Delta T^{1+w_8} \cdot \mu^{-w_1+w_7-2w_8} \cdot k^{1-w_7} \cdot \rho^{w_1+2w_8} \cdot C_p^{w_7} \cdot (\beta \cdot g)^{w_8}$$

or

$$q \propto \frac{k \Delta T}{L} \cdot \left(\frac{L \cdot u \cdot \rho}{\mu} \right)^{w_1} \cdot \left(\frac{C_p \cdot \mu}{k} \right)^{w_7} \cdot \left(\frac{\beta g \cdot \Delta T \cdot L^3 \cdot \rho^2}{\mu^2} \right)^{w_8}$$

i.e.

$$\frac{qL}{\Delta T \cdot k} = \frac{hL}{k} = C \left(\frac{Lu\rho}{\mu} \right)^{w_1} \left(\frac{C_p \mu}{k} \right)^{w_7} \left(\frac{\beta g \Delta T L^3 \rho^2}{\mu^2} \right)^{w_8} \quad \dots (6.28)$$

This general equation involves the use of four dimensionless groups but may frequently be simplified for design purposes. In equation 6.28,

$\frac{hL}{k}$ is known as the Nusselt Group, $Nu.$,

$\frac{Lu\rho}{\mu}$ the Reynolds Group, $Re.$,

$\frac{C_p \mu}{k}$ the Prandtl Group, $Pr.$, and

$\frac{\beta g \Delta T L^3 \rho^2}{\mu^2}$ the Grashof Group, $Gr.$, and

C is a constant.

For conditions in which only natural convection occurs the velocity is dependent solely on the buoyancy effects, represented by the Grashof Number and the Reynolds Group can be omitted. Again, when forced convection occurs the effects of natural convection are usually negligible and the Grashof Number may be omitted. Thus,

$$\text{for natural convection, } Nu. = f(Pr. Gr) \quad \dots (6.29)$$

$$\text{and for forced convection, } Nu. = f(Re. Pr) \quad \dots (6.30)$$

Convection in Gases. For most gases over a wide range of temperature and pressure, the product $C_p \mu / k$ is constant and the Prandtl Group may often be omitted, making the design equations for calculation of film coefficients with gases simpler.

Forced Convection in Tubes

The results of a number of workers who have used a variety of gases such as air, carbon dioxide, and steam, and of others who have used liquids such as water, acetone, kerosene, and benzene, can be expressed in the general form of

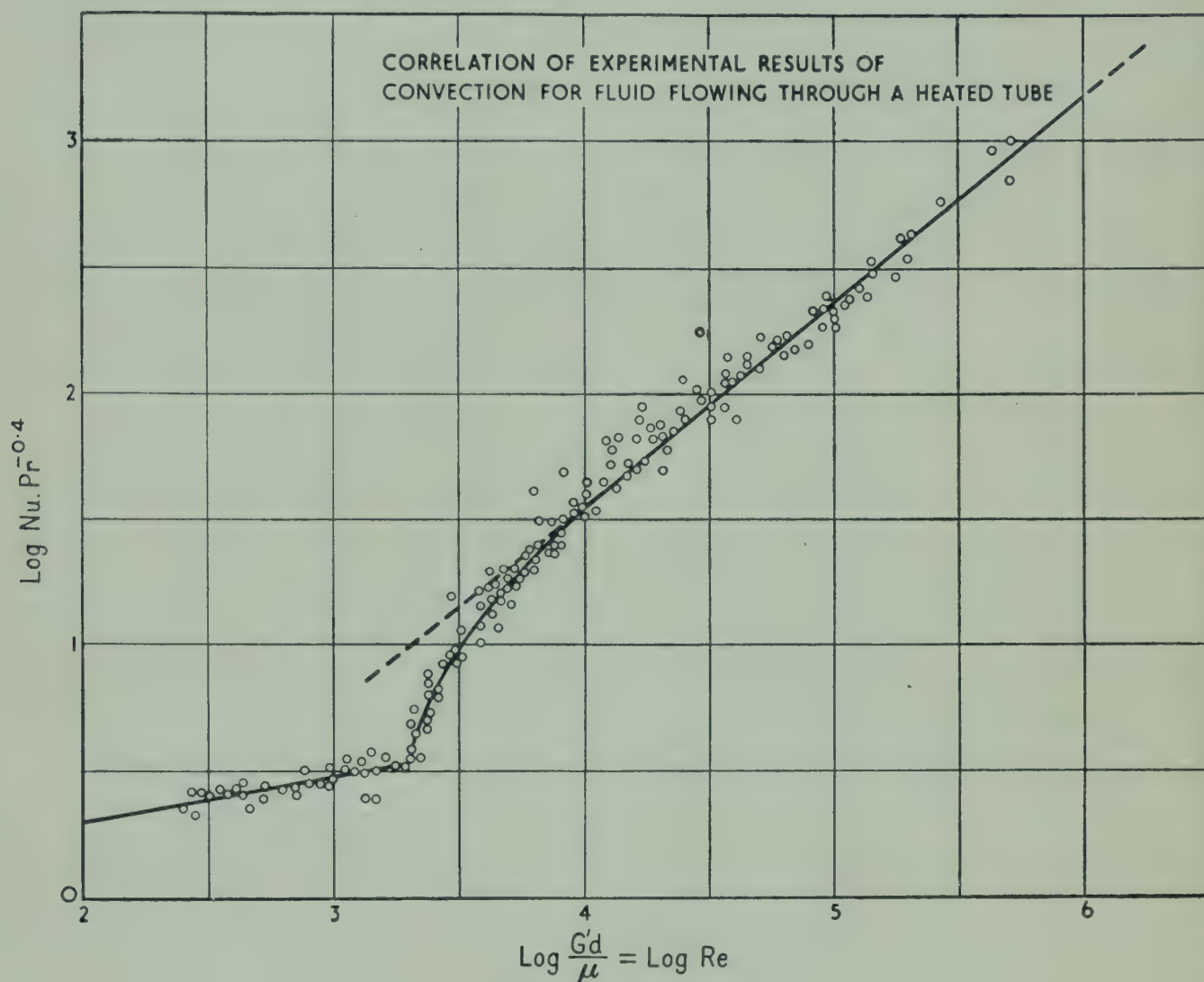


Fig. 6.9. Forced convection in tubes

equation 6.30 by plotting $\log Nu./Pr^{0.4}$ against $\log Re.$, as shown in Fig. 6.9, where the characteristic linear dimension is taken as the diameter, d . In this plot $Re.$ is shown as $G'd/\mu$, where G' is the mass rate of flow per unit area. This form of $Re.$ is particularly convenient for gases, since the mass flow is independent of pressure and temperature, whereas the velocity varies. For values of $Re.$ greater than 4000 the points are well represented by

$$Nu. = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \quad \dots (6.31)$$

For $Re. < 1,500$ the flow is streamline, and for $1,500 < Re. < 2,100$ there is a transitional region leading to turbulent conditions. It is therefore necessary to divide the problem into three sections, turbulent flow, transitional flow, and streamline flow. Further examination shows that it is again necessary to

distinguish between liquids of high viscosity and those of low viscosity such as water. These conditions will be examined separately.

Turbulent Flow of Liquids. In using equation 6.31 the physical properties of the fluid are taken at the arithmetic mean of the temperatures at the two ends of the tube. For liquids with viscosities little above that of water this equation may be used for heating or cooling. With more viscous liquids there will at any cross-section be a marked difference in the viscosity of the fluid along the axis and of that near the wall. The conditions for heating will not in this case be the same as for cooling and it is considered better to use a modified equation due to SIEDER and TATE⁽³³⁾.

$$Nu. = 0.027 \left(\frac{\mu}{\mu_s} \right)^{0.14} . Re^{0.8} . Pr^{0.33} \quad \dots(6.32)$$

where μ is the viscosity at the mean bulk temperature and μ_s at the surface of the wall. Whilst these equations have been tested for a number of liquids, only small diameter pipes have been used and there is little evidence to show the effect of the nature of the pipe surface on the results. From dimensional considerations we might expect the ratio of the length to the diameter of the tube l/d to be important but results of LAWRENCE and SHERWOOD⁽²²⁾, for values of l/d from 50 to 224, showed no important influence of this term for turbulent flow. This is because fully developed conditions of flow are attained at only a short distance from the inlet (see Chapter 9). If the viscosity is measured at the film temperature T_f , taken as $\frac{T_w + T}{2}$, COLBURN has shown that in equation 6.31, the index of the Prandtl Group becomes 0.33. (T_w is the temperature of the surface.)

For water, the equation may be simplified by including the variation of physical properties with temperature to give

$$h = 200(1 + 0.015T) \frac{u^{0.8}}{d^{0.2}} \quad \dots(6.33)$$

where d is the inside diameter in inches, u is in ft/sec., T is in degrees C, and h is in lb-cal/ft²-hr-°C. With degree Fahrenheit units,

$$h = 150(1 + 0.011T) \frac{u^{0.8}}{d^{0.2}} \quad \dots(6.33a)$$

For a detailed discussion on experiments with water see EAGLE and FERGUSON⁽²¹⁾.

Turbulent Flow of Gases. Data from a number of workers fit equation 6.31 but since the Prandtl Group is about 0.74 for most gases under wide conditions this equation may be simplified to

$$Nu. = 0.02 . Re^{0.8} \quad \dots(6.34)$$

From Table 6.2 it will be seen that h for water is many times that for air flowing at the same linear velocity.

TABLE 6.2. *Film coefficients for air and water (16°C and atmospheric pressure)⁽⁶⁾*

Inside diameter of tube (in.)	Values of h in lb-cal/hr-ft ² -°C						
	Air				Water		
	Velocity (ft/sec.)						
	20	80	140	200	2	4	8
	lb/hr-ft ²						
	5,500	22,000	40,000	55,000	450,000	900,000	1,800,000
1	6	17·8	28	37	440	770	1340
2	5·2	15·8	24·2	32·4	380	670	1160
3	4·6	14·2	22·2	29·8	360	620	1080

Streamline Flow. Although heat transfer to a fluid in streamline flow takes place solely by conduction, it is convenient to consider it here so that the results

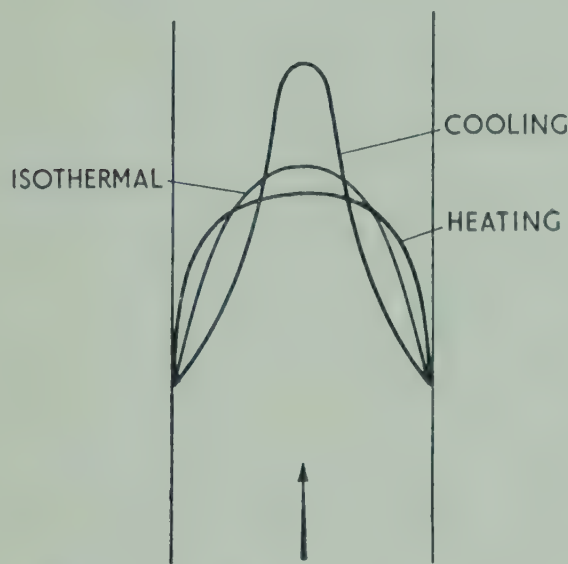


Fig. 6.10. Effect of heat transfer on velocity distribution

can be compared with those for turbulent flow. We have seen in Chapter 3 that for streamline flow through a tube the velocity distribution across a diameter is parabolic (see Fig. 6.10). If a liquid enters a section heated on the outside, the fluid near the wall will be at a higher temperature than that in the centre and its viscosity will be lower. The velocity of the fluid near the wall will therefore be greater in the heated section, and correspondingly less at the centre. The velocity distribution will therefore be altered, as shown. If the fluid enters a section where it is cooled, the same reasoning will show that the distribution in velocity will

be altered to that shown. With a gas the conditions are reversed, because of the increase of viscosity with temperature. The heat transfer problem is therefore complex.

In Chapter 9 equations will be derived for h based on the assumptions of a parabolic distribution of velocity and of heat transfer by radial conduction. h will be shown to be infinite at the entrance and to fall slowly along the pipe. For values of $[Re. Pr. d/l]$ greater than 12, the following empirical equation is applicable,

$$Nu. = 1.62 \left[Re. Pr. \frac{d}{l} \right]^{\frac{1}{3}} = 1.75 \left[\frac{G \cdot C_p}{k \cdot l} \right]^{\frac{1}{3}} \dots (6.35)$$

where $G = \frac{\pi \cdot d^2}{4} \cdot \rho \cdot u$, i.e. the mass rate of flow.

In this equation the temperature difference is taken as the arithmetic mean of the terminal values, i.e.

$$\left[\frac{(T_w - T_1) + (T_w - T_2)}{2} \right]$$

where T_w is the temperature of the tube wall which is taken as constant.

If the liquid is heated almost to the wall temperature T_w (i.e. when $\frac{G \cdot C_p}{k \cdot l}$ is very small) we have, on equating the heat gained by the liquid to that transferred from the pipe,

$$G \cdot C_p \cdot (T_2 - T_1) = \pi \cdot d \cdot l \cdot h \left(\frac{T_2 - T_1}{2} \right)$$

or

$$h = \frac{2 \cdot G \cdot C_p}{\pi \cdot d \cdot l} \quad \dots (6.36)$$

Viscous Liquids. Experimental values of h for viscous oils are greater than those given by equation 6.35 for heating and less for cooling. This is due to the large variation of viscosity with temperature and the correction introduced for turbulent flow may also be used here, giving the equation as

$$\frac{h \cdot d}{k} \left(\frac{\mu_s}{\mu} \right)^{0.14} = 1.86 \left[Re \cdot Pr \cdot \frac{d}{l} \right]^{\frac{1}{3}} = 2.01 \left[\frac{G \cdot C_p}{k \cdot l} \right]^{\frac{1}{3}} \quad \dots (6.37)$$

When $\left(\frac{G \cdot C_p}{k \cdot l} \right) < 10$, the outlet temperature closely approaches that of the wall and equation 6.36 applies. These equations have been obtained with tubes from about 0.4 to 1.6 in. diameter and the length of unheated tube preceding the heated section is important. They are not entirely consistent since for very small values of ΔT we should expect the constants in equations 6.35 and 6.37 to be the same. It is important to note, when using these equations for design purposes, that the error may be as much as $\pm 25\%$ for turbulent flow and greater for streamline conditions.

Whenever possible, streamline conditions of flow are avoided in heat exchange equipment because of the very low heat transfer coefficients which are obtained. With very viscous liquids, however, turbulent conditions can be produced only if a very high pressure drop across the plant is permissible. In the chemical industry, streamline flow in heat exchangers is most commonly experienced with heavy oils and brines at low temperatures. Since the viscosity of these materials is critically dependent on temperature, the equations would not be expected to apply with a high degree of accuracy.

Forced Convection Outside Tubes

If a fluid passes at right angles across a single tube, the distribution of velocity around the tube will not be uniform. In the same way the rate of heat flow around a hot pipe across which air is passed is not uniform but is a maximum at the front and back and a minimum at the sides, where the rate is only some

40% of the maximum. The general picture is shown in Fig. 6.11. For design purposes we refer to the average value.

Flow Across Single Cylinders. A number of workers (REIHER⁽¹⁴⁾, HILPERT⁽²⁶⁾, GRIFFITHS and AWBERY⁽²⁵⁾) have studied the flow of a hot gas past a single cylinder varying from a thin wire to a tube of 6 in. diameter. They have used temperatures up to 800°C and air velocities up to 100 ft/sec. with Reynolds Numbers ($d_o u \rho / \mu$) from 1000 to 100,000. Their results may be expressed by the relation,

$$Nu. = 0.26 \cdot Re^{0.6} \cdot Pr^{0.3} \quad \dots (6.38)$$

Taking $Pr.$ as 0.74 for gases, this reduces to

$$Nu. = 0.24 \cdot Re^{0.6} \quad \dots (6.39)$$

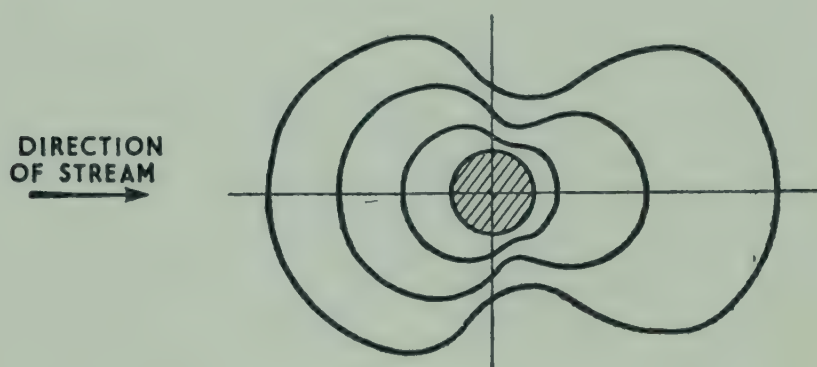


Fig. 6.11. Distribution of film heat transfer coefficient round a cylinder with flow normal to axis for three different values of Re .

DAVIS⁽¹³⁾ has also worked with water, paraffin, and light oils and obtained similar results. For very low values of Re . (from 0.2 to 200) with liquids the data are better represented by the equation,

$$Nu. = 0.86 \cdot Re^{0.43} \cdot Pr^{0.3} \quad \dots (6.40)$$

In each case the physical properties of the fluid are measured at the mean film temperature T_f , taken as the average of the surface temperature T_w and the mean fluid temperature T_m ($T_m = \frac{T_1 + T_2}{2}$).

Flow at Right Angles to Tube Bundles. One of the great difficulties with this system is that the area for flow is continually changing. Moreover the degree of turbulence is considerably less for banks of tubes in line, as at a , than for staggered tubes, as at b in Fig. 6.12. With the small bundles which are common in the chemical industry the selection of the true mean area for flow is further complicated by the change in number of tubes in the rows.

The results of a number of workers for the flow of gases across tube banks can be put in the form,

$$Nu. = 0.33 \cdot C_h \cdot Re_{\max}^{0.6} \cdot Pr^{0.3} \quad \dots (6.41)$$

where C_h depends on the geometrical arrangement of the tubes (Table 6.3).

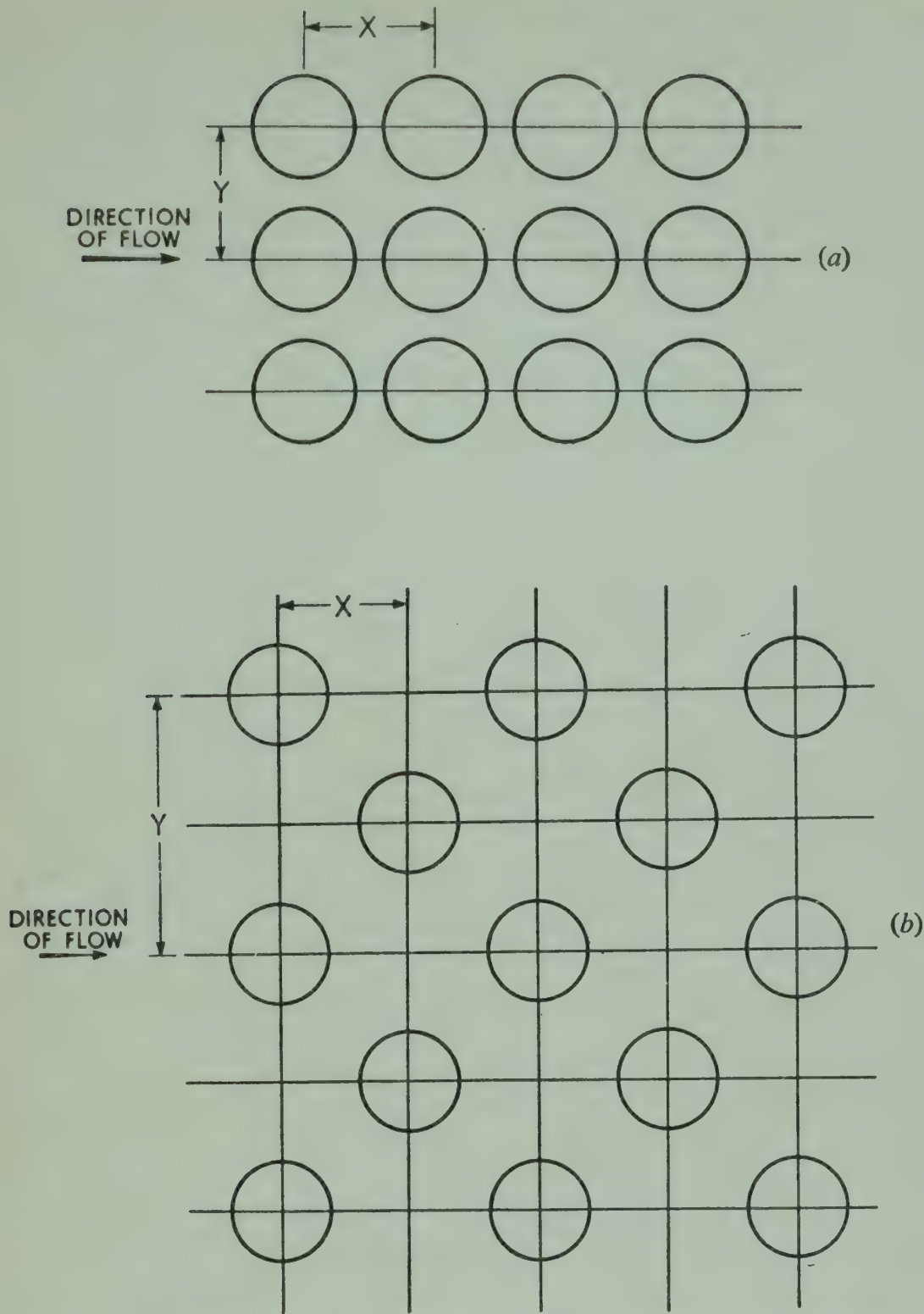


Fig. 6.12. Arrangements of tubes in heat exchangers ((a) in-line, (b) staggered)

TABLE 6.3.⁽⁶⁾ Values of C_h and C_f

Re_{max}	$X = 1.25d_o$				$X = 1.5d_o$			
	In-line		Staggered		In-line		Staggered	
	C_h	C_f	C_h	C_f	C_h	C_f	C_h	C_f
$Y = 1.25d_o$								
2,000 .	1.06	1.68	1.21	2.52	1.06	1.74	1.16	2.58
20,000 .	1.00	1.44	1.06	1.56	1.00	1.56	1.05	1.74
40,000 .	1.00	1.20	1.03	1.26	1.00	1.32	1.02	1.50
$Y = 1.5d_o$								
2,000 .	0.95	0.79	1.17	1.80	0.95	0.97	1.15	1.80
20,000 .	0.96	0.84	1.04	1.10	0.96	0.96	1.02	1.16
40,000 .	0.96	0.74	0.99	0.88	0.96	0.85	0.98	0.96

GRIMISON⁽⁴⁰⁾ proposed this form of expression to correlate the data of HUGE⁽³⁷⁾ and PIERSON⁽³⁶⁾, who worked with small electrically heated tubes in rows of ten deep. Other workers have used similar equations. Some correction factors have been given by PIERSON for bundles with less than ten rows but there are insufficient reported data from commercial exchangers to fix these values with accuracy. Thus for five rows a factor of 0.92 and for eight rows 0.97 is suggested.

These equations are based on the maximum velocity through the bundle.

Thus for an in-line arrangement as in Fig. 6.12, $G'_{\max} = \frac{Y}{Y - d_o} G'$, where

Y is the pitch of the pipes at right angles to direction of flow; it is more convenient here to use the mass flow rate per unit area in place of the velocity. For staggered arrangements the maximum velocity may be based on the distance

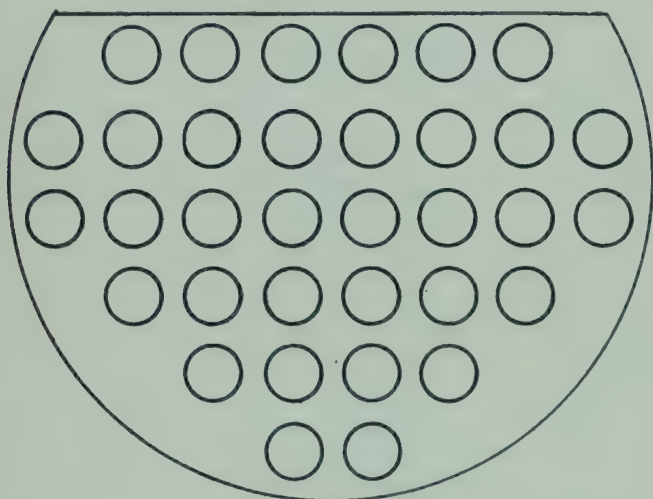


Fig. 6.13. Baffle for heat exchanger

between the tubes in a horizontal line or on the diagonal of the tube bundle, whichever is the less.

It has been suggested that for in-line arrangements the constant in equation 6.41 should be reduced to 0.26 but there is insufficient evidence from commercial exchangers to settle this question.

With liquids the same equation may be used, but for $Re.$ less than 2000, there is insufficient published work to

justify an equation. MCADAMS⁽³⁾, however, has given a curve for h for a bundle with staggered tubes ten rows deep.

Effect of Baffles on Transfer Coefficients. To increase the velocity over the tubes it is common practice to fit baffles across the bundle. The commonest form is shown in Fig. 6.13, where it is seen that the cut-away section is about 25% of the total area. With such an arrangement, the flow pattern becomes more complex and the extent of leakage between the tube holes and the baffle, and between the baffle and the inside of the shell of the exchanger, complicates the problem. For more detail the reader is referred to papers by SHORT⁽⁴⁶⁾, DONOHUE⁽⁵²⁾ and TINKER⁽⁵⁶⁾. The various methods are all concerned with developing a method of calculating the true area of flow and of assessing the probable influence of leaks. When using baffles, the value of h , as found from equation 6.41, is commonly multiplied by 0.6 to allow for leakage.

Heat Transfer and Pressure Drop over Tube Bundles. The drop in pressure across a tube bundle may be important from the point of view of the power required and it is useful to examine the change in the ratio of heat transfer to pressure drop as the geometrical spacing of the tubes is altered. The pressure drop depends on ρu_t^2 , where u_t is the velocity through the minimum area. In Chapter 3 an equation from GRIMISON⁽⁴⁰⁾ was expressed as

$$-\Delta P_f = \frac{1}{6} C_f \cdot j \cdot \rho \cdot u_t^2 \quad \dots (3.35)$$

where C_f depends on the geometry of the tube spacing and j is the number of rows of tubes. Some indication of the values of this ratio C_h/C_f are given in Table 6.4, where Re_{\max} is kept at 20,000.

TABLE 6.4.⁽⁶⁾ *Ratio of heat transfer to friction for tube bundles*

	$X = 1.25d_o$			$X = 1.5d_o$		
	C_h	C_f	C_h/C_f	C_h	C_f	C_h/C_f
In-line						
$Y = 1.25d_o$	1	1.44	0.69	1	1.56	0.64
$Y = 1.5d_o$	0.96	0.84	1.14	0.96	0.96	1.0
Staggered						
$Y = 1.25d_o$	1.06	1.56	0.68	1.05	1.74	0.60
$Y = 1.5d_o$	1.04	1.10	0.95	1.02	1.16	0.88

The ratio C_h/C_f is higher for the in-line arrangement than for staggered but the actual heat transfer for the staggered arrangement is higher. For the closely spaced units usual in the chemical industry this factor is not very important but if Y is increased to about $3d_o$ although the ratio C_h/C_f improves considerably, the size of the exchanger for a given heat load is much increased.

Flow in Non-circular Sections

For the heat transfer for fluids flowing in non-circular ducts, such as rectangular ventilating ducts, the equations developed for turbulent flow inside a circular pipe may be used if some equivalent diameter is used in the place of d . The use of the hydraulic mean diameter d_e has already been discussed in Chapter 3.

The data for heating and cooling water in turbulent flow in rectangular ducts are reasonably well expressed by the use of equation 6.31 in the form

$$\frac{h \cdot d_e}{k} = 0.023 \left(\frac{d_e \cdot G'}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.4} \dots (6.42)$$

Whilst the experimental data of COPE and BAILEY⁽²⁷⁾ are somewhat low, data by WASHINGTON and MARKS⁽³⁸⁾ for heating air in ducts are well represented by this equation.

Annular Sections between Concentric Tubes. Concentric tube heat exchangers are widely used because of their simplicity of construction and the ease with which additions may be made to increase the area. They are also used to give turbulent conditions where the quantity flowing is small.

In presenting equations for the film coefficient in the annulus, one of the difficulties has been to select the best equivalent diameter to use. When

considering the film on the outside of the inner tube, DAVIS^(46a) has proposed the equation,

$$\frac{hd_1}{k} = 0.031 \left(\frac{d_1 G'}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.33} \left(\frac{\mu}{\mu_s} \right)^{0.14} \left(\frac{d_2}{d_1} \right)^{0.15} \dots (6.43)$$

CARPENTER *et al* ⁽⁴⁹⁾ suggested using the hydraulic mean diameter $d_e = d_2 - d_1$ in the Sieder and Tate equation (6.32) and recommend the expression

$$\frac{h \cdot d_e}{k} \left(\frac{\mu_s}{\mu} \right)^{0.14} = 0.027 \left(\frac{d_e \cdot G'}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.33} \dots (6.44)$$

Their results, which were obtained using a small annulus, were somewhat below those given by equation 6.44 for values of $\frac{d_e G'}{\mu}$ less than 10,000, but this may have been because the flow was not fully turbulent: with an index on the Reynolds group of 0.9, the equation fitted the points much better. There is little to choose between these two equations but they both give rather high values for h .

For the viscous region CARPENTER's results are reasonably well covered by the equation,

$$\begin{aligned} \frac{h \cdot d_e}{k} \left(\frac{\mu_s}{\mu} \right)^{0.14} &= 2.01 \left(\frac{G \cdot C_p}{k \cdot l} \right)^{0.33} \dots (6.45) \\ &= 1.86 \left[\left(\frac{d_e \cdot G'}{\mu} \right) \left(\frac{C_p \mu}{k} \right) \left(\frac{d_1 + d_2}{l} \right) \right]^{1/3} \end{aligned}$$

Equation 6.45 is the same as equation 6.37 with d_e in the place of d .

These results have all been obtained with small units and mainly with water as the fluid in the annulus, and no entirely satisfactory solution has yet been obtained.

Natural Convection

If a beaker containing water rests on a hot plate, the water at the bottom of the beaker becomes hotter than that at the top. Since the density of the hot water is lower than that of the cold, the water in the bottom rises and heat is transferred by natural convection. In the same way air in contact with a hot plate will be heated by natural convection currents, the air near the surface being hotter and of lower density than that some distance away. In these cases there is no external agency providing forced convection currents and the transfer of heat occurs at a correspondingly lower rate, since the natural convection currents move rather slowly.

For these processes which depend on buoyancy effects, we have seen from equation 6.29 that the rate of heat transfer would be expected to follow a relation,

$$Nu. = \psi(Pr.Gr.) \dots (6.29)$$

Measurements by SCHMIDT⁽¹⁶⁾, of the upward air velocity near a 1-ft vertical plate, show that the velocity rises rapidly to a maximum at a distance of about 2 mm from the plate and then falls rapidly. However, the temperature evens

out at about 1 cm from the plate. Temperature measurements around horizontal cylinders have been made by RAY⁽¹¹⁾.

Horizontal Surfaces. Natural convection from horizontal surfaces to air, nitrogen, hydrogen, and carbon dioxide, and to liquids (water, aniline, carbon tetrachloride, glycerol) have been made by a number of workers (DAVIS⁽¹²⁾, ACKERMAN⁽²⁴⁾, SAUNDERS⁽³⁴⁾, and others). Most of the results are for thin wires and tubes up to about 2 in. diameter; the temperature differences used are up to about 1100°C with gases and about 85°C with liquids. The

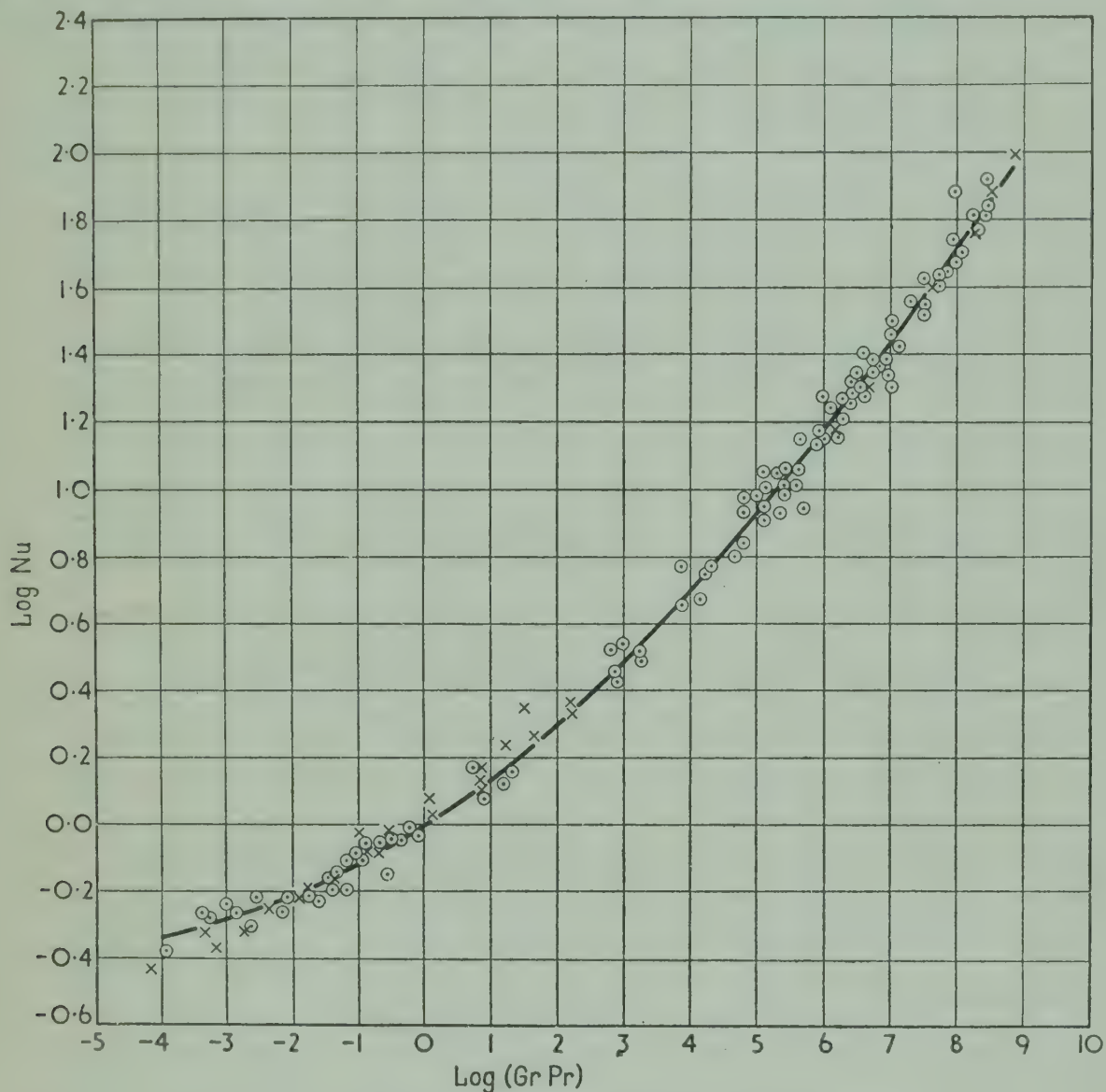


Fig. 6.14. Natural convection from horizontal tubes

general form of the results is shown in Fig. 6.14, where $\log [Pr.Gr.]$ is plotted against $\log Nu$. for streamline conditions. The curve can be represented by the relation

$$Nu. = C' . (Pr.Gr.)^n \quad \dots (6.46)$$

where n is 0.25 for values of $Gr.Pr.$ between 10^3 and 10^8 . For turbulent conditions n is found to be about 0.33.

Vertical Surfaces. Work by SAUNDERS^(34, 42) and others can also be expressed by equation 6.46, with values of C' as given in Table 6.5. The results are for air with surfaces up to several feet in height and for liquids with surfaces up to only about 1 ft high. Under these conditions turbulence occurs at a value of $Gr.Pr.$ of about 2×10^9 . Again n is 0.25 for streamline and 0.33 for turbulent conditions.

TABLE 6.5. Values of C' for natural convection for use in equation 6.46

Nature of surface	Characteristic dimension	Stream-line	Turbulent
Horizontal or vertical cylinders	Diameter	0.47	0.10
Vertical planes or vertical cylinders of large diameter	Height	0.56	0.12
Horizontal planes facing upwards	Mean length of side	0.54	0.14
Horizontal planes facing downwards	Mean length of side	0.25	Not reached

In using these equations the values of the physical properties ρ , C_p , k , μ are taken at the mean of the surface and bulk temperatures. The coefficient of cubical expansion β is taken as $1/T$, where T is the absolute temperature.

Simplified Equations for Natural Convection to Air. For the special case of convection from a hot body to air these equations may be simplified. Thus for streamline flow equation 6.46 may be written as,

$$\frac{hl}{k} = C' \left[\frac{\beta g \Delta T l^3 \rho^2}{\mu^2} \cdot \frac{C_p \mu}{k} \right]^{\frac{1}{4}} \text{ or } h = C' \left(\frac{\Delta T}{l} \right)^{\frac{1}{4}} \cdot k \cdot \left[\frac{\beta g \rho^2 C_p}{\mu k} \right]^{\frac{1}{4}} \dots (6.47)$$

Over a wide range in temperature the group $k \left[\frac{\beta g \rho^2 C_p}{\mu k} \right]^{\frac{1}{4}}$ remains reasonably constant. The heat transfer coefficient h may thus be obtained from the relation

$$h = C' \cdot (0.58) \cdot \left(\frac{\Delta T}{l} \right)^{\frac{1}{4}}$$

where ΔT is in $^{\circ}\text{C}$, l is in feet, and h is in $\text{lb-cal/ft}^2\text{-hr-}^{\circ}\text{C}$.

For turbulent conditions

$$\frac{hl}{k} = C' \left[\frac{\beta g \Delta T l^3 \rho^2}{\mu^2} \cdot \frac{C_p \mu}{k} \right]^{\frac{1}{4}} \text{ or } h = C' (\Delta T)^{\frac{1}{4}} \left[\frac{k^2 \Delta T^{\frac{1}{4}} \beta g \rho^2 C_p}{\mu} \right]^{\frac{1}{4}} \dots (6.48)$$

h is then independent of l and since $\frac{k^2 \Delta T^{\frac{1}{4}} \beta g \rho^2 C_p}{\mu}$ is approximately constant, h is proportional to $\Delta T^{\frac{1}{4}}$.

For the following geometrical conditions, equations can be obtained as shown below.

	Streamline	Turbulent
Horizontal or vertical pipes	$h = 0.28 \left(\frac{\Delta T}{d_0} \right)^{\frac{1}{4}}$	$0.29 \Delta T^{\frac{1}{4}}$
Vertical planes	$0.32 \left(\frac{\Delta T}{l} \right)^{\frac{1}{4}}$	$0.35 \Delta T^{\frac{1}{4}}$
Horizontal planes facing upwards	$0.31 \left(\frac{\Delta T}{l} \right)^{\frac{1}{4}}$	$0.41 \Delta T^{\frac{1}{4}}$
Horizontal planes facing downwards	$0.14 \left(\frac{\Delta T}{l} \right)^{\frac{1}{4}}$	

Fluid Layers between Two Surfaces. For the transfer of heat from a hot surface across a thin layer of fluid to a parallel cold surface,

$$\frac{Q}{Q_k} = \frac{h \cdot \Delta T}{\frac{k}{x} \Delta T} = \frac{hx}{k} = Nu.$$

where Q_k is the rate at which heat would be transferred by pure thermal conduction between the layers, a distance x apart, and Q is the actual rate.

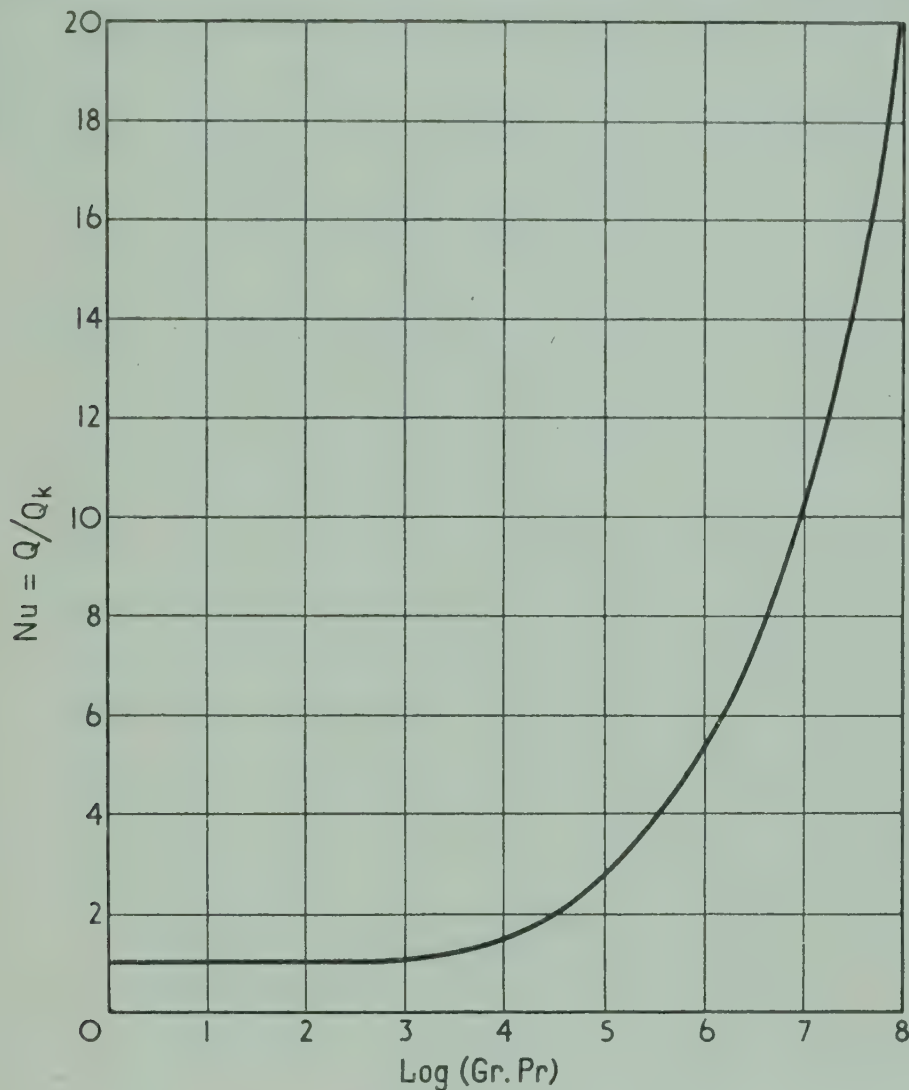


Fig. 6.15. Natural convection between surfaces

For $(Gr.Pr.)$ equal to 10^3 , the heat transferred is approximately equal to that due to conduction alone, but for $(Gr.Pr.)$ from 10^4 to 10^6 , the heat transferred is given by

$$\frac{Q}{Q_k} = 0.15(Gr.Pr.)^{0.25} \quad \dots (6.49)$$

In this equation the characteristic dimension to be used for the Grashof group is x , the distance between the planes, and the heat transfer is independent of the extent of the surfaces provided they are large compared with x . For higher values of $(Gr.Pr.)$, Q/Q_k is proportional to $(Gr.Pr.)^{\frac{1}{3}}$, showing that the heat transferred is now entirely by convection and is not influenced by the distance x between the surfaces.

A similar form of analysis has been given by KRAUSSOLD⁽²⁹⁾ for air between two concentric cylinders. It is important to note from this general analysis that a single layer of air will not be a good insulator because convection currents

set in before it becomes 1 in. thick. The good insulating properties of porous materials are attributable to the fact that they offer a series of very thin layers of air in which convection currents do not arise.

HEAT TRANSFER BY RADIATION

A heated body emits energy in the form of electromagnetic waves. This energy is radiated in all directions and on falling on a second body is partially absorbed, partially reflected and partially transmitted as indicated in Fig. 6.16. The fraction of the incident radiation absorbed is known as the absorptivity a , and the fraction reflected the reflectivity of the body. The amount transmitted

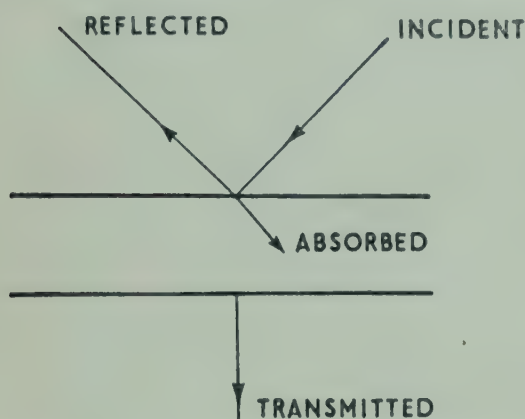


Fig. 6.16. Reflection, absorption, and transmission of radiation

will therefore depend on these two properties. If the amount transmitted is negligible the material is termed opaque.

Kirchhoff's Law

Suppose two bodies A and B of areas A_1 and A_2 to be in a large enclosure from which no energy is lost to the outside. Then the energy absorbed by A from the enclosure will be $A_1 a_1 I'$, where I' is the rate at which radiation is falling on A per unit area, and a_1 is termed the absorptivity. The energy given out by A will be $E_1 A_1$, where E_1 , the emissive power, is the energy emitted per unit area per unit time. At equilibrium these quantities will be equal. In the same way the energy emitted by B will equal the energy received. Thus we have:

$$I' A_1 a_1 = A_1 E_1 \text{ and } I' A_2 a_2 = A_2 E_2$$

or
$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = \frac{E}{a} \text{ for any other body.}$$

Thus for all bodies the ratio of the emissive power to the absorptivity will be the same. The maximum possible value for E will occur when a has its maximum value of unity. This condition applies to a "black body" which is defined as one which absorbs all the radiation falling on it, i.e. its absorptivity, $a_b = 1$.

Now the emissivity e of a body is defined as the ratio of its emitting power to that of a black body; i.e. $e = \frac{E}{E_b}$. Since $\frac{E}{a}$ is constant for all bodies, $\frac{E}{a} = \frac{E_b}{a_b}$.

Thus
$$e = \frac{E}{E_b} = \frac{a}{a_b}$$

Then, since $a_b = 1$, the emissivity, e , of any body is equal to its absorptivity, a .

Energy Emitted by a Black Body

The energy emitted per unit time by a black body depends only on its temperature. This energy is given out over a range of wavelengths and the general

distribution of energy from a black body at various temperatures is shown in Fig. 6.17. At any temperature there is a wavelength Z_m at which the maximum energy is emitted. The energy at wavelengths less than this falls off rapidly but for wavelengths greater than Z_m the drop in energy is much less. It will be seen that the total emission increases rapidly with temperature and that the higher the temperature the smaller is the wavelength at which the maximum occurs and the more pronounced is the peak.

The wavelength of the energy radiated at temperatures below 550°C is too great for it to be visible but at higher temperatures the wavelengths correspond to those of the visible spectrum and the colour of a radiating body under these conditions is a good indication of its temperature.

The total energy emitted per unit area per unit time is given by the Stefan-Boltzmann law,

$$E = \sigma T^4 \quad \dots(6.50)$$

where T is the absolute temperature, and σ is the Stefan-Boltzmann constant which has the value 1.01×10^{-8} lb-cal/hr-ft²-°K⁴, 1.73×10^{-9} B.Th.U./hr-ft²-°R⁴, 5.71×10^{-5} ergs/sec.-cm²-°K⁴, or 4.92×10^{-8} Kg-cal/hr-m²-°K⁴. The wavelength Z_m at which the maximum energy is radiated was found by Wien to vary inversely as the absolute temperature. Taking the temperature in degrees Kelvin, $TZ_m = 0.288$, where the wavelength is in centimetres.

If E_Z' is the radiation between wavelengths Z and $Z + dZ$, the total radiation is given by,

$$E = \int_0^\infty E_Z' dZ \quad \dots(6.51)$$

It can be shown that this integral is equal to σT^4 , so that —

$$E = \sigma T^4 \quad \dots(6.50)$$

Emissivity e . The ratio of the energy emitted by a body to that emitted by a black body at the same temperature has been defined above as the emissivity. Strictly this should be taken for each wavelength, since the ratio e will not remain constant over a wide range of wavelengths. A grey body is defined as one which has a constant value of e , so that for any temperature range it radiates the same proportion of the energy radiated by a black body. Similarly it will have a constant absorptivity.

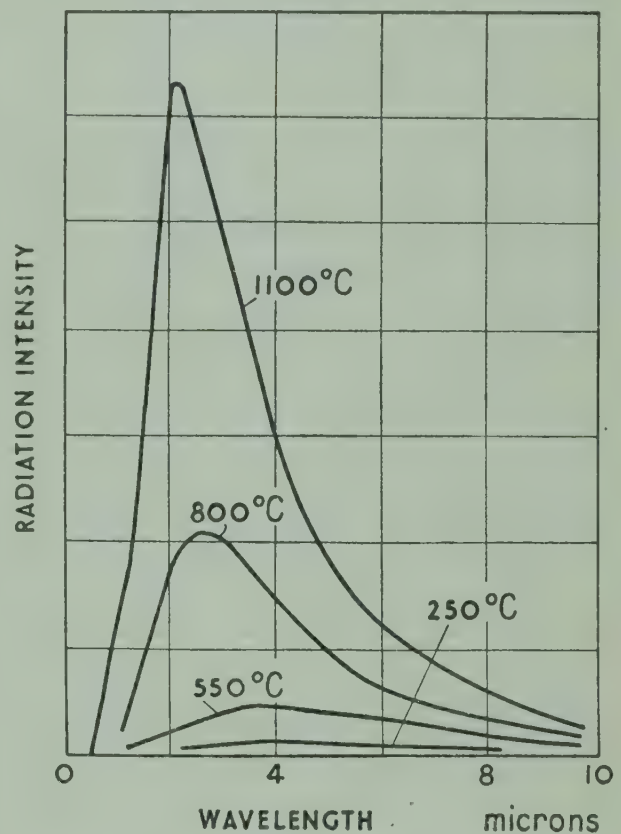


Fig. 6.17. Emission from a black body

Values of e have been measured for many materials and it is found that for most industrial non-metallic surfaces and for non-polished metals e may be taken as about 0.9; for highly-polished surfaces such as copper or aluminium values of e as low as 0.03 are obtained.

Heat Transferred by Radiation ✓

A grey body of emissivity e at an absolute temperature T_1 in black surroundings emits energy $e\sigma \cdot T_1^4$. The surroundings reflect back none of this radiation, but if they are at an absolute temperature T_2 they will emit radiation σT_2^4 . The grey body will absorb a fraction e , so that the net radiation per unit area from the grey body will be,

$$q = e\sigma(T_1^4 - T_2^4) \quad \dots(6.52)$$

This relation will still be true where the grey body is so small that a negligible proportion of its radiation is reflected back to it from the surroundings; for example, a body radiating to the atmosphere.

For a material that does not behave as a grey body, but as a selective emitter, the absorptivity of the surface at T_1 for radiation from surroundings at T_2 will be a_{T_2} . This will not be equal to its emissivity (e_{T_1}) at T_1 but to its emissivity at T_2 , i.e. e_{T_2} . Under these conditions the general equation for the net exchange of heat becomes,

$$q = \sigma[e_{T_1}T_1^4 - e_{T_2}T_2^4] \quad \dots(6.53)$$

Large Parallel Plates and Concentric Cylinders. Under these conditions all of the radiation reflected from the second plane or the inside of the outer cylinder will be received by the radiating surface at T_1 . For grey surfaces it may be shown that for parallel planes

$$q = \frac{e_1 e_2 \sigma}{e_1 + e_2 - e_1 e_2} [T_1^4 - T_2^4] \quad \dots(6.54)$$

and for concentric cylinders⁽¹⁷⁾

$$q = \frac{e_1 e_2 \sigma}{e_2 + e_1(1 - e_2)r_1/r_2} [T_1^4 - T_2^4] \quad \dots(6.55)$$

where r_1 and r_2 are the radii of the inner and outer cylinders.

Intensity of Radiation in a Given Direction. If experiments are made on the intensity of radiation emitted by a small plane surface in a given direction it is found that the strongest radiation is in a line normal to the surface. This intensity normal to the plane I is defined as the radiation given out per unit time per unit area of a small surface dA over unit solid angle. Then the intensity of radiation at an angle α to the normal is seen from Fig. 6.18 to be $I \cos \alpha$. This form of relation shows the similarity of the transfer process to the radiation of light. It is followed accurately by black bodies but metallic surfaces may show wide divergences.

Radiation intensity I = energy per unit area per unit time per unit solid angle $= \frac{dq}{d\omega}$.

Radiation from small surface dA , through small solid angle $d\omega$, at angle α to normal is $I dA \cos \alpha \cdot d\omega$.

$$\therefore \text{Total energy emitted in forward direction} = \int_{\alpha=0}^{\alpha=\pi/2} I dA \cos \alpha \cdot d\omega.$$

But $d\omega = \frac{2\pi r \sin \alpha r d\alpha}{r^2}$, where r is radius of described circle.

$$\therefore \text{Emissive power } (\sigma T^4 \text{ for black body}) = \int_0^{\pi/2} I \cos \alpha \cdot 2\pi \sin \alpha d\alpha = \pi I.$$

Geometric Factor F . This is defined as the fraction of the total radiation from a surface of area A_1 which is intercepted by another surface of area A_2 ,

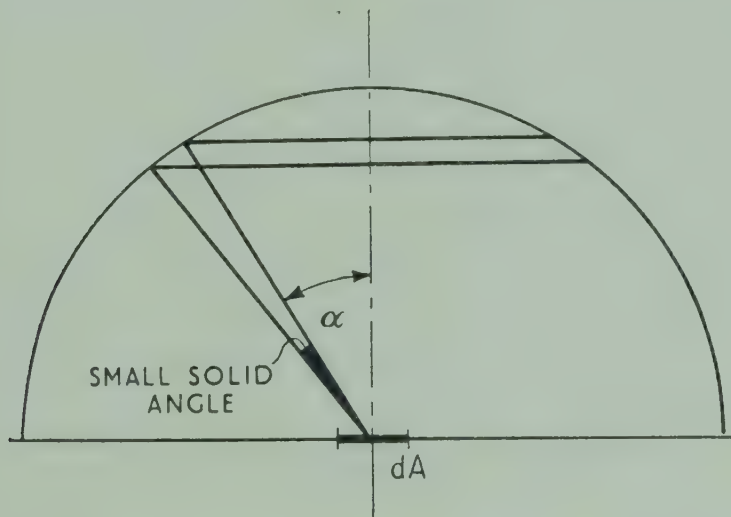


Fig. 6.18. Intensity of radiation in a given direction

and depends on the geometrical arrangement of the two surfaces. Thus for two black bodies, the radiation from A_1 received by A_2 will be $F_{12}A_1\sigma \cdot T_1^4$. Similarly the radiation from A_2 received by A_1 will be $F_{21}A_2\sigma \cdot T_2^4$. The net radiation between the two surfaces will then be

$$F_{12}A_1\sigma \cdot T_1^4 - F_{21}A_2\sigma \cdot T_2^4 = FA_1\sigma \cdot [T_1^4 - T_2^4] \quad \dots(6.56)$$

where F is the geometric factor based on A_1 . Values of F have been calculated or determined by experiments with models.

Transfer Coefficient for Radiation. The net heat transfer from unit surface of a grey body at a temperature T_1 to an enclosure at T_2 can be written as

$$q = h_r[T_1 - T_2] \quad \dots(6.57)$$

Thus

$$h_r = \frac{q}{T_1 - T_2} = \frac{\sigma \cdot e}{T_1 - T_2} [T_1^4 - T_2^4]$$

and h_r may be looked upon as a radiation transfer coefficient.

Example. Calculate the total heat loss by radiation and convection from an unlagged horizontal steam pipe of 2 in. outside diameter at 140°C to air at 10°C .

Solution. Loss by radiation

$$q_r = 1.01 \times 10^{-8} \times 0.9(377^4 - 283^4)$$
$$= 125 \text{ lb-cal/hr-ft}^2$$

(from equation 6.52)

taking an emissivity of 0.9.

Loss by natural convection
(assuming streamline flow)

$$= 0.28 \left(\frac{\Delta T}{d_o} \right)^{\frac{1}{4}} \times \Delta T$$
$$= 0.28 \left(\frac{94}{1/6} \right)^{\frac{1}{4}} \times 94$$
$$= 128 \text{ lb-cal/hr-ft}^2$$

Thus the total loss of heat is given by the sum of these quantities or 253 lb-cal/hr-ft². In practical conditions the heat loss for an unlagged pipe will be rather higher, particularly if vertical, when the convection currents become stronger.

Gas Radiation

Most of the simple monatomic and diatomic gases such as helium, hydrogen, oxygen, and nitrogen are transparent to thermal radiation, but some polyatomic gases—notably carbon dioxide, water vapour, carbon monoxide, ammonia and hydrocarbons—absorb a considerable amount of radiation of certain frequencies. These gases, which are industrially very important, radiate appreciably in the same wave bands. In contrast with the behaviour of solids a considerable thickness of gas is required to absorb a large fraction of the radiation falling on it. Thus if *I'* is the intensity of the incident radiation, the intensity *I* after the radiation has passed through a layer of thickness *x* is given by

$$I = I' e^{-m'x}$$

....(6.58)

where *m'* will, in general, vary with the wavelength and is approximately proportional to the partial pressure of the gas for any one wavelength, i.e. to the number of molecules per unit volume. The absorption will then be a function of the product *P_g · l*, where *P_g* is the partial pressure of the gas and *l* is the equivalent thickness of the gas stream. The absorptivity of a gas is conveniently plotted against temperature for a constant value of *P_g · l*. The thickness *l* is assumed the same in all directions but this will only be true for the case of a hemisphere where the gas is radiating to the mid-point of its base. It has been found that for a wide range of conditions *l* may be taken as 3.4 times the gas volume divided by the area of the retaining walls. A few values on this basis are given in Table 6.6.

TABLE 6.6

Shape	Characteristic dimension <i>L</i>	Equivalent dimension for hemispherical radiation
Sphere	Diameter	0.57 <i>L</i>
Infinite cylinder radiating to walls	Diameter	0.85 <i>L</i>
Space between infinite parallel planes	Distance apart	1.70 <i>L</i>
Space outside bank of tubes with centres on equilateral triangle; clearance = diameter of tube	Clearance	2.89 <i>L</i>
Ditto, with tube centres on square pitch; tube diameter = clearance	Clearance	3.49 <i>L</i>

Curves showing the emissivity of water vapour at a total pressure of one atmosphere, (Fig. 6.19) are taken from the work of HOTTEL⁽³¹⁾. The net radiant heat exchange between a gas at T_1 and unit area of enclosure at T_2 acting as a black body will be,

$$q = \sigma[e_g T_1^4 - a_g T_2^4] \quad \dots(6.59)$$

where e_g and a_g are the emissivity and absorptivity of the gas.

If the enclosure acts as a grey surface some of the radiation falling on it is reflected back; of this, part is absorbed by the gas and part by the surface. For

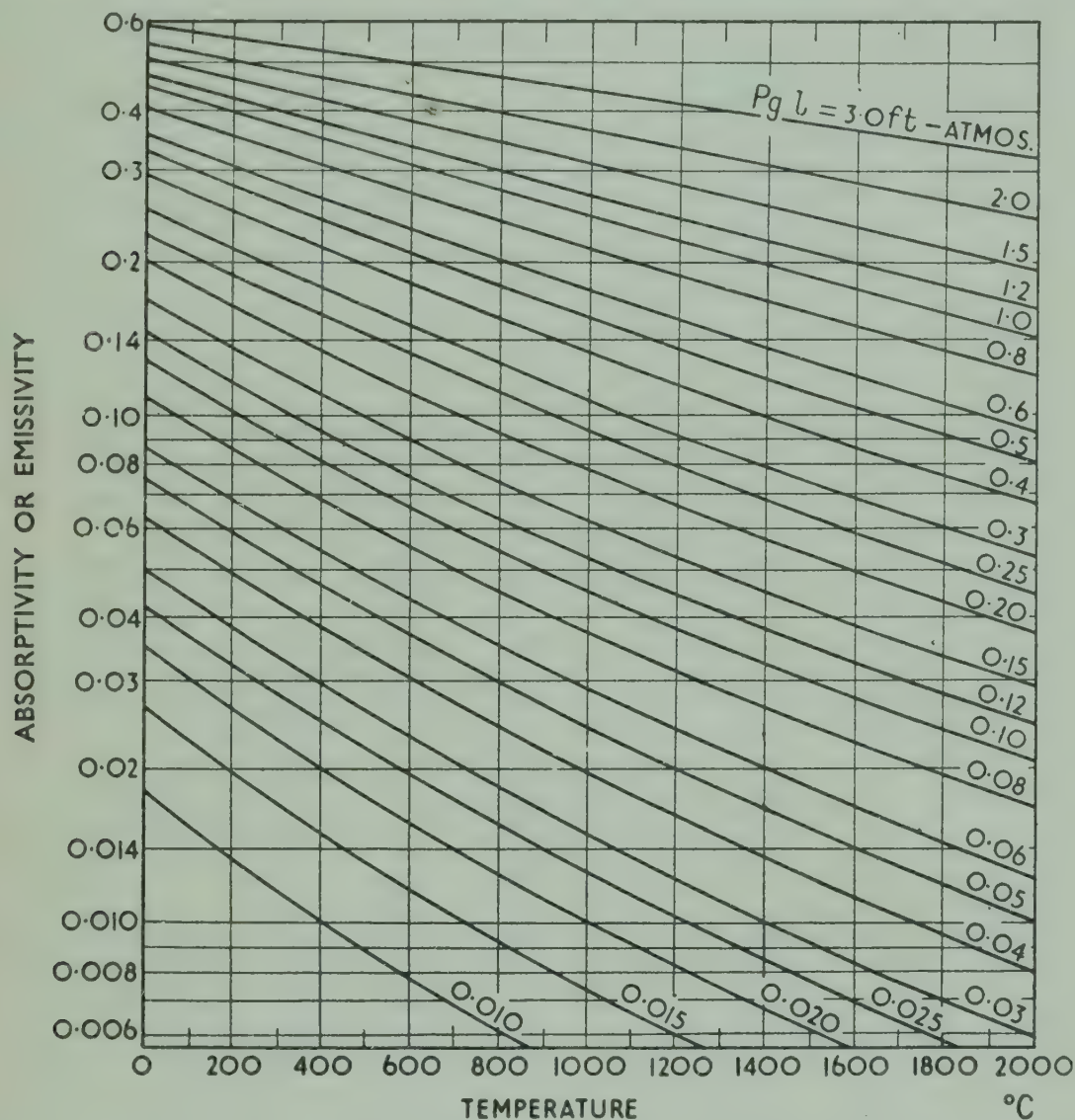


Fig. 6.19. Absorptivity and emissivity of water vapour

these general conditions it is usually sufficiently accurate to take the emissivity of the enclosure as e' , where $e' = \left(\frac{e_s + 1}{2}\right)$. This is permissible because e_s , the emissivity of the surface, will probably lie between 0.7 and 1. The radiation exchange will then be,

$$q = e' \cdot \sigma[e_g T_1^4 - a_g T_2^4] \quad \dots(6.60)$$

HEAT TRANSFER IN THE CONDENSATION OF VAPOURS

When a saturated vapour is brought into contact with a cool surface, heat is transferred from the vapour to the surface and a film of condensate is produced.

In considering the heat that is transferred in such a case we shall follow the method first put forward by NUSSELT⁽¹⁰⁾ and later modified by JAKOB and others. If the vapour is condensing on a vertical surface, the condensate film flows downwards under the influence of gravity, but is retarded by the viscosity of the liquid. This flow will normally be streamline and the heat will flow through the film by conduction. Nusselt supposed that the temperature of the film at the cool surface was equal to that of the surface, and at the other side was at the temperature of the vapour. Actually there must be some small difference in temperature between the vapour and the film but this may generally be neglected, except where non-condensable gas is present in the vapour.

Calculation of Film Coefficient

It is first necessary to calculate the velocity distribution in a liquid which is flowing down a surface inclined at an angle to the horizontal.

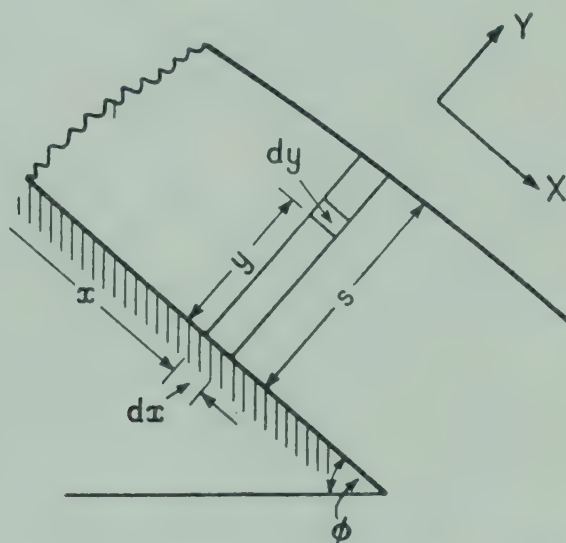


Fig. 6.20. Flow of liquid over a surface

Consider a plane surface of unit width, inclined at an angle ϕ to the horizontal. At a distance x from the top of the surface, the thickness of the liquid layer is s , say (Fig. 6.20).

In an element of condensate of thickness dx situated at a distance x from the top of the surface, the accelerating force in the X -direction, acting on the liquid at a distance greater than y from the surface, $= (s - y) \cdot dx \cdot \rho \cdot g \cdot \sin \phi$. The retarding force is made up of the drag of the gas or vapour on the free surface and the drag at the inner boundary of the element. The shear stress R' at the free surface will be small, except at high vapour velocities and will be neglected at present. Thus the retarding force in the X -direction $= \mu \frac{\partial u_y}{\partial y} dx$, where u_y is the velocity at a distance y from the surface. Under equilibrium conditions therefore,

$$(s - y) \cdot dx \cdot \rho \cdot g \cdot \sin \phi = \mu \frac{\partial u_y}{\partial y} dx \quad \dots (6.61)$$

Thus

$$du_y = \frac{\rho \cdot g \cdot \sin \phi}{\mu} \cdot (s - y) dy$$

and

$$u_y = \frac{\rho \cdot g \cdot \sin \phi}{\mu} \cdot (sy - \frac{1}{2}y^2) + \text{constant}$$

Since the liquid in contact with the surface must be at rest, $u_y = 0$ when $y = 0$, and the constant is, therefore, zero.

The mass rate of flow G of liquid over the surface can now be calculated.

$$\begin{aligned}
 G &= \int_0^s \left\{ \frac{\rho \cdot g \cdot \sin \phi}{\mu} (sy - \frac{1}{2}y^2) \right\} \rho \cdot dy \\
 &= \frac{\rho^2 \cdot g \cdot \sin \phi}{\mu} \left(\frac{s^3}{2} - \frac{s^3}{6} \right) \\
 &= \frac{\rho^2 \cdot g \cdot \sin \phi \cdot s^3}{3\mu}
 \end{aligned}$$

so that the mean velocity of the fluid,

$$u_m = \frac{\rho \cdot g \cdot \sin \phi \cdot s^2}{3\mu} \quad \dots (6.62)$$

For a vertical surface, $\sin \phi = 1$ and $u_m = \frac{\rho \cdot g \cdot s^2}{3\mu} \quad \dots (6.62a)$

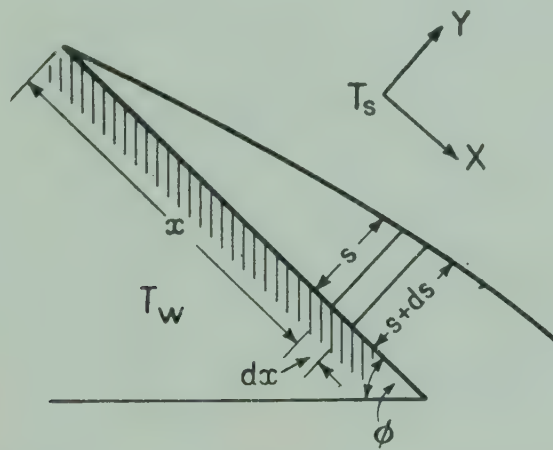


Fig. 6.21. Condensation on an inclined surface

The maximum velocity, which occurs at the free surface,

$$= \frac{\rho \cdot g \cdot \sin \phi \cdot s^2}{2\mu}$$

and is 1.5 times the mean velocity of the liquid.

Since the liquid is produced by condensation, the thickness of the film will be zero at the top and will gradually increase towards the bottom. Under stable conditions, the difference in the mass rates of flow at distances x and $x + dx$ from the top of the surface will result from condensation over the small element of the surface of length dx (Fig. 6.21).

If the thickness of the liquid film increases from s to $s + ds$ in that distance, the increase in the mass rate of flow of liquid dG

$$\begin{aligned}
 &= \frac{d}{ds} \left\{ \frac{\rho^2 \cdot g \cdot \sin \phi \cdot s^3}{3\mu} \right\} ds \\
 &= \frac{\rho^2 \cdot g \cdot \sin \phi \cdot s^2 \cdot ds}{\mu}
 \end{aligned}$$

If the vapour temperature is T_s and the wall temperature is T_w , the heat transferred by thermal conduction to an element of surface of length dx

$$= k \cdot (T_s - T_w) \frac{dx}{s}$$

where k is the thermal conductivity of the condensate.

Thus the mass rate of condensation on this small area of surface

$$= k \cdot (T_s - T_w) \cdot \frac{dx}{s \cdot \lambda}$$

where λ is the latent heat of vaporisation of the liquid.

$$\text{Thus} \quad k \cdot (T_s - T_w) \cdot \frac{dx}{s \cdot \lambda} = \frac{\rho^2 \cdot g \cdot \sin \phi \cdot s^2 \cdot ds}{\mu}$$

On integration, this equation gives

$$\mu \cdot k \cdot (T_s - T_w) \cdot x = \rho^2 \cdot g \cdot \sin \phi \cdot s^4 \cdot \frac{1}{4} \cdot \lambda$$

(since $s = 0$ when $x = 0$).

$$\text{Thus} \quad s = \sqrt[4]{\frac{4\mu \cdot k \cdot x \cdot (T_s - T_w)}{g \cdot \sin \phi \cdot \lambda \rho^2}} \quad \dots (6.63)$$

Now the heat transfer coefficient h at $x = x$, $= k/s$.

$$\text{Thus} \quad h = \sqrt[4]{\frac{\rho^2 \cdot g \cdot \sin \phi \cdot \lambda \cdot k^3}{4\mu x (T_s - T_w)}} \quad \dots (6.64)$$

$$\text{and} \quad Nu. = \frac{hx}{k} = \sqrt[4]{\frac{\rho^2 \cdot g \cdot \sin \phi \cdot \lambda \cdot x^3}{4\mu k (T_s - T_w)}} \quad \dots (6.65)$$

These expressions give point values of h and Nu_x at $x = x$. It is seen that the coefficient decreases from a theoretical value of infinity at the top as the condensate film thickens. The mean value of the heat transfer coefficient over the whole surface, between $x = 0$ and $x = x$ is given by

$$\begin{aligned} h_m &= \frac{1}{x} \int_0^x h \cdot dx \\ &= \frac{1}{x} \int_0^x K \cdot x^{-\frac{1}{4}} \cdot dx \quad (\text{where } K \text{ is independent of } x) \\ &= \frac{1}{x} \cdot K \cdot \frac{x^{\frac{3}{4}}}{\frac{3}{4}} \\ &= \frac{4}{3} \cdot K \cdot x^{-\frac{1}{4}} \\ &= \frac{4}{3} h \\ &= 0.943 \sqrt[4]{\frac{\rho^2 \cdot g \cdot \sin \phi \cdot \lambda \cdot k^3}{\mu \cdot x \cdot \Delta T_f}} \quad \dots (6.66) \end{aligned}$$

where ΔT_f is the temperature difference across the condensate film.

For a vertical surface, $\sin \phi = 1$ and

$$h_m = 0.943 \sqrt[4]{\frac{\rho^2 g \lambda k^3}{\mu x \Delta T_f}} \quad \dots (6.67)$$

Dimensionless Form of Nusselt Equation

Suppose the vapour to condense on the outside of a vertical tube of diameter d_o . Then the hydraulic mean diameter for the film

$$= 4 \times \frac{\text{flow area}}{\text{wetted perimeter}} = \frac{4S}{b} \quad (\text{say})$$

If G is the mass rate of flow of condensate, the mass rate of flow per unit area G' is $\frac{G}{S}$.

The Reynolds Number for the condensate film is then given by

$$Re. = \frac{\frac{4S}{b} \cdot \frac{G}{S}}{\mu} = \frac{4G}{\mu b} = \frac{4M}{\mu}$$

where M is the mass rate of flow of condensate per unit length of perimeter, i.e. $M = \frac{G}{\pi d_o}$.

Then for streamline conditions in the film, $\frac{4M}{\mu} \gg 2100$. Thus

$$h_m = \frac{Q}{A \Delta T_f} = \frac{G \cdot \lambda}{b l \Delta T_f} = \frac{\lambda \cdot M}{l \Delta T_f}$$

From equation 6.67,

$$h_m = 0.943 \left[\frac{k^3 \rho^2 g}{\mu} \cdot \frac{\lambda}{l \Delta T_f} \right]^{\frac{1}{4}} = 0.943 \left[\frac{k^3 \rho^2 g}{\mu} \cdot \frac{h_m}{M} \right]^{\frac{1}{4}}$$

whence
$$h_m \left[\frac{\mu^2}{k^3 \rho^2 g} \right]^{\frac{1}{4}} = 1.47 \left[\frac{4M}{\mu} \right]^{-\frac{1}{4}} \dots (6.68)$$

Horizontal Tubes. For horizontal tubes, NUSSELT gives

$$h_m = 0.72 \left[\frac{k^3 \rho^2 g \lambda}{d_o \mu \Delta T_f} \right]^{\frac{1}{4}} \dots (6.69)$$

This can be rearranged to give

$$h_m \left(\frac{\mu^2}{k^3 \rho^2 g} \right)^{\frac{1}{4}} = 1.50 \left(\frac{4M}{\mu} \right)^{-\frac{1}{4}} \dots (6.70)$$

where M is the mass rate of flow per unit length of tube.

This is approximately the same as equation 6.68 for vertical tubes and is a universal equation for condensation if it is remembered that for vertical tubes $M = G/\pi d_o$ and for horizontal tubes $M = G/l$, where l is the length of the tube. Comparison of the two equations shows that, provided the length is more than three times the diameter, the horizontal tube will give a higher transfer coefficient for the same temperature conditions.

For j vertical rows of horizontal tubes, equation 6.69 is modified to give

$$h_m = 0.72 \left[\frac{k^3 \rho^2 g \lambda}{j d_o \mu \Delta T_f} \right]^{\frac{1}{4}} \dots (6.69a)$$

Experimental Values

In testing NUSSELT's equation it is important to see that the conditions comply with the requirements of the theory. In particular, it is necessary for the condensate to form a uniform film on the tubes, for the drainage of this film to be

by gravity and the flow streamline. Although some of these requirements have probably not been entirely fulfilled, results for pure vapours such as steam, benzene, toluene, diphenyl, ethanol, etc., are sufficiently near to give support to the theory. Some recent results by HASELDEN and PROSAD⁽⁵³⁾ for condensing oxygen and nitrogen vapours on a vertical surface, where precautions were taken to see that the conditions were met, are in very good agreement with NUSSELT's theory. The results for most of the workers are within 15% for horizontal tubes but tend to be substantially higher than the theoretical for vertical tubes. Some typical values are given in Table 6.7 taken from MCADAMS⁽³⁾ and elsewhere.

TABLE 6.7. *Average values of film coefficients h_m for condensation of pure saturated vapours on horizontal tubes*

Vapour	Value of h_m (lb-cal/hr-ft ² -°C)	Range of ΔT_f (°C)
Steam .	1700–5000	1–11
Steam .	3200–6500	4–37
Benzene .	242–381	23–37
Diphenyl .	225–400	4–15
Toluene .	193–241	31–40
Methanol .	500–600	8–16
Ethanol .	320–450	6–22
Propanol .	250–300	13–20
Oxygen .	573–1400	0.08–2.5
Nitrogen .	400–1009	0.15–3.5
Ammonia .	1000	—
Freon-12 .	200–400	—

When considering commercial equipment, there are several factors which prevent the true conditions of NUSSELT's theory being met. Thus the temperature of the tube wall will not be constant, and for a vertical condenser with a ratio of ΔT at the bottom to ΔT at the top of five, the film coefficient should be increased by about 15%.

Influence of Vapour Velocity

A high vapour velocity upwards tends to increase the thickness of the film and thus reduce h though the film may sometimes be disrupted mechanically as a result of the formation of small waves. For the downward flow of vapour TEN BOSCH^(2a) has shown that h increases considerably at high vapour velocities and may increase to two or three times the value given by the Nusselt equation. It must be remembered that, when a large fraction of the vapour is condensed, there may be a considerable change in velocity over the surface.

Under conditions of high vapour velocity CARPENTER and COLBURN⁽⁵⁷⁾ have shown that turbulence may occur with low values of the Reynolds number, e.g. 200–400. When the vapour velocity is high there will be an appreciable drag on the condensate film and equation 6.61 becomes:

$$(s - y) \cdot dx \cdot \rho \cdot g \cdot \sin \phi = \mu \frac{\partial u_y}{\partial y} dx + R' dx \quad \dots (6.71)$$

(where R' is the shear stress produced by the vapour at the surface of the condensate). The expression obtained for the heat transfer coefficient by using equation 6.71 is difficult to manage.

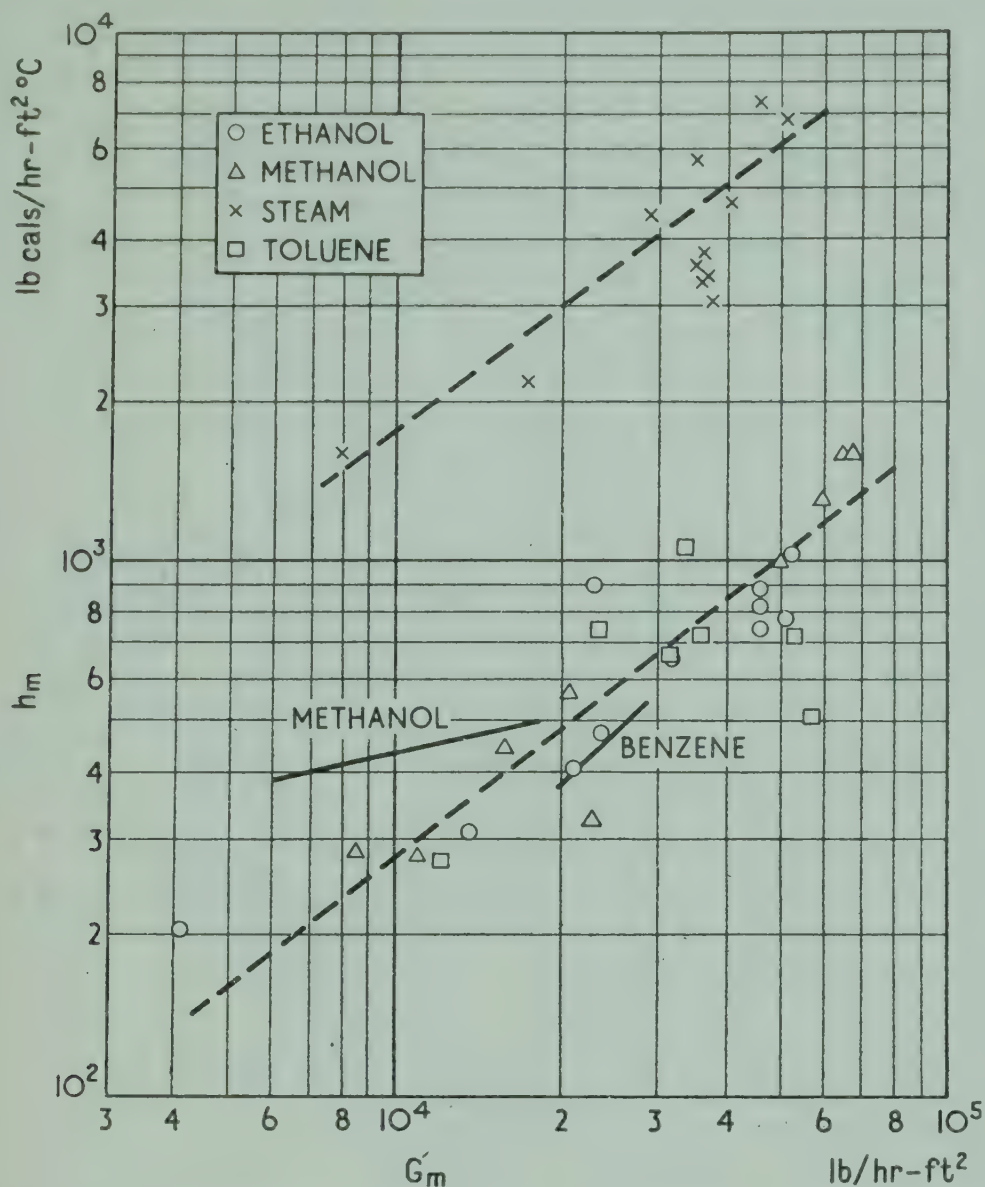


Fig. 6.22. Average heat transfer data of CARPENTER (shown as points) compared with those of TEPE and MUELLER^(50a) (shown as solid lines). The "dashed" lines represent equation 6.71a

CARPENTER and COLBURN have put forward a simple experimental correlation of their results for condensation at varying vapour velocities on the inner surface of a vertical tube

$$h_m = 0.065 G'_m \sqrt{\frac{C_p \rho k \left(\frac{R'}{\rho_v u^2} \right)}{\mu \rho_v}} \quad \dots (6.71a)$$

where

$$G'_m = \sqrt{\frac{G_1'^2 + G_1' G_2' + G_2'^2}{3}}$$

and u is the velocity calculated from G'_m .

In this equation C_p , k , ρ , and μ are properties of the condensate and ρ_v refers to the vapour. G_1' is the mass rate of flow per unit area at the top of the tube, and G_2' the corresponding value at the bottom.

As pointed out by COLBURN⁽⁵⁸⁾ the group $\frac{C_p \rho k}{\mu \rho_v}$ does not vary much for a

number of organic vapours so that a plot of h_m and G_m' will provide a simple approximate correlation with separate lines for steam and for organic vapours, as shown in Fig. 6.22. Whilst this must be regarded as an empirical approximation it is very useful for obtaining a good idea of the effect of vapour velocity.

Turbulence in the Film

If $Re.$ is greater than 2100, during condensation on a vertical tube, the mean coefficient h_m will increase. Data of KIRKBRIDE⁽²⁸⁾ and BADGER^(19, 35) for the condensation of diphenyl vapour and Dowtherm on nickel tubes were expressed in the form,

$$h_m \left[\frac{\mu^2}{k^3 \cdot \rho^2 \cdot g} \right]^{\frac{1}{3}} = 0.0077 \left[\frac{4M}{\mu} \right]^{0.4} \quad \dots (6.72)$$

Comparing equation 6.68 for streamline flow of condensate and equation 6.72 for turbulent flow, it is seen that, with increasing Reynolds number, h decreases

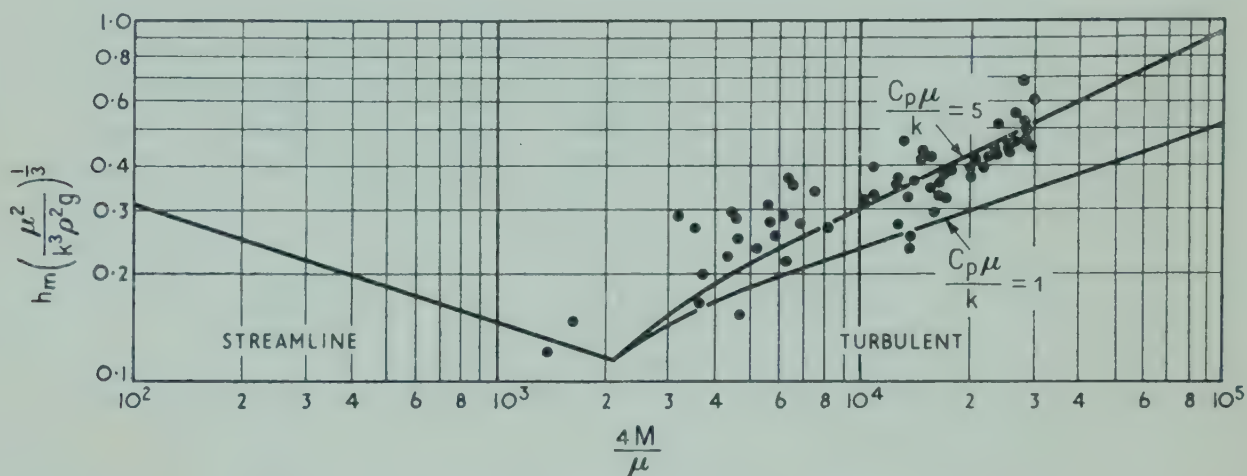


Fig. 6.23. Effect of turbulence in condensate film

with streamline flow but increases with turbulent flow. These results are indicated in Fig. 6.23 but they are as yet incomplete.

Dropwise Condensation

In the discussion so far it has been assumed that the condensing vapour, on coming into contact with the cold surface, wets the tube so that a continuous film of condensate is formed. If the droplets initially formed do not wet the surface, after growing slightly they will fall from the tube exposing fresh condensing surface. This gives what is known as dropwise condensation and, since the heat does not have to flow through a film by conduction, much higher transfer coefficients are obtained. Steam is the only pure vapour for which definite dropwise condensation has been obtained, and values of h from 7000 to 20,000 lb-cal/hr-ft²-°C have been obtained, with much higher values on occasions. This question has been discussed by DREW, NAGLE, and SMITH⁽³⁰⁾, who have shown that there are many materials which make the surface non-wettable; of these only those which are firmly held to the surface are of any practical use. Mercaptans and oleic acid have been used to promote dropwise condensation, but at present there is little practical application of this technique.

Exceptionally high values of h will not give a corresponding increase in the overall coefficient, since for a condenser with steam, a value of about 2000 lb-cal/hr-ft²-°C can be obtained with film condensation. On the other hand, it may be helpful in experimental work to reduce the thermal resistance on one side of a surface to negligible value.

Condensation of Mixed Vapours

In the previous discussion it has been assumed that the vapour was a pure material, such as steam or organic vapour. If it contains a proportion of non-condensable gas and is cooled below its dew point, a layer of condensate is formed on the surface with a mixture of non-condensable gas and vapour above it. The heat flow from the vapour to the surface then takes place by two methods. First sensible heat is passed to the surface because of the temperature difference. Secondly, since the concentration of vapour in the main stream is greater than that in the gas film at the condensate surface, vapour molecules diffuse to the surface and there condense, giving up their latent heat. The actual rate of condensation is then determined by the combination of these two effects and its calculation requires a knowledge of mass transfer by diffusion discussed in Chapter 7.

A general equation for the rate of heat transfer per unit area at a point on a water-cooled surface can then be written as,

$$h_g(T_s - T_c) + k_G \cdot \lambda(P_g - P_s) = h(T_c - T_w) = U\Delta T \quad \dots(6.73)$$

where the first term represents the sensible heat transferred to the condensing surface, the second term the latent heat transferred by the diffusing molecules, and the third term the heat transferred from the condensing surface to the water.

h_g = heat transfer coefficient over the gas film,

h = heat transfer coefficient for condensate film,

U = overall heat transfer coefficient,

T_s = vapour temperature,

T_c = surface temperature of condensate,

T_w = temperature of condensation surface,

ΔT = overall temperature difference,

P_g = partial pressure of vapour,

P_s = vapour pressure at T_c ,

λ = latent heat of vaporisation per unit mass and

k_G = mass transfer coefficient in mass per unit time . unit area . unit partial pressure difference

This equation must be solved by assuming a temperature T_c , and calculating the first three terms in equation 6.73 to see if the first equality is satisfied. This calculation must then be made for a number of points and the whole problem therefore becomes complex. An approximate method for treating this subject

has been outlined by COLBURN⁽⁶⁰⁾ and this gives a good picture of the required area; BRAS⁽⁶⁰⁾ has given a graphical method illustrated with an example.

If a non-condensable gas, such as air, is present in appreciable amounts it should be vented, since it will otherwise blanket the cooling surface.

HEAT TRANSFER TO BOILING LIQUIDS

In industrial equipment boiling usually occurs either on a submerged surface or on the inside of vertical tubes and it is convenient to consider these two conditions separately. Much of the fundamental work on heat transfer to boiling liquids has been done by JAKOB⁽³²⁾ and BOSNJAKOVIC⁽¹⁸⁾, and it is helpful to consider some of their ideas before discussing industrial equipment.

For a bubble to be formed in a liquid, for instance, steam in water, it is necessary for a surface of separation to be produced. KELVIN has shown that, as a result of the surface tension between the liquid and vapour, the vapour pressure on the inside of a concave surface will be less than that at a plane surface. Thus a concave fluid surface is not at the saturation pressure P_s but at some lower pressure P_r , given approximately by

$$P_r = P_s - \frac{2\tau}{r} \cdot \frac{\rho_v}{\rho} \quad \dots (6.74)$$

where r is the radius of curvature of the bubble

τ is the surface tension

ρ is the density of the liquid

ρ_v is the density of the saturated vapour.

Hence the liquid must be superheated near the surface of the bubble, the extent of the superheat increasing with decrease in the radius of the bubble. On this basis it follows that very small bubbles are difficult to form without excessive superheat. The formation of bubbles is made much easier by the fact that they will form on curved surfaces or on irregularities on the heating surface, so that only a small degree of superheat is normally required. It follows that for boiling to occur a small difference in temperature must exist between the liquid and the vapour. JAKOB and FRITZ⁽²³⁾ have measured the temperature distribution for water boiling above an electrically-heated hot plate. The temperature dropped very steeply from about 110°C on the actual surface of the plate to 101°C about 0.1 mm from it. Beyond this the temperature was reasonably constant till the water surface was reached. The mean superheat of the water above the temperature in the vapour space was about 0.5°C and this changed very little with the rate of evaporation. At higher pressures this superheating became smaller, becoming 0.2°C at 50 atmospheres and 0.05°C at 100 atmospheres. The temperature drop from the heating surface, however, depends very much on the rate of heat transfer and on the nature of the surface. Thus in order to maintain a heat flux of about 8000 lb-cal/hr-ft² a temperature difference of only 6°C was required with a rough surface as against 10.6°C with a

smooth surface. The heat transfer coefficient on the boiling side is therefore dependent on the nature of the surface and on the difference in temperature available. For water boiling on copper plates JAKOB and FRITZ give the following coefficients for a constant temperature difference of 5.6°C , with different surfaces.

- | | |
|--|---|
| (1) Surface after 8 hr use and 48 hr immersion
in water | $h = 1400 \text{ lb-cal/hr-ft}^2\text{-}^{\circ}\text{C}$ |
| (2) Freshly sandblasted | $h = 680 \text{ lb-cal/hr-ft}^2\text{-}^{\circ}\text{C}$ |
| (3) Sandblasted surface after long use | $h = 450 \text{ lb-cal/hr-ft}^2\text{-}^{\circ}\text{C}$ |
| (4) Chromium plated | $h = 350 \text{ lb-cal/hr-ft}^2\text{-}^{\circ}\text{C}$ |

The initial surface, with freshly cut grooves, gave much higher figures than case (1) above.

The various forms of surface can broadly be classified as:

(a) Non-wettable surface. Here the vapour bubbles spread out, thus reducing the area available for heat transfer from the hot surface to the liquid.

(b) Partially wettable surface such as chromium plate. The bubbles rise from a larger number of spots and the transfer is increased.

(c) Entirely wetted surface like that formed by a screen. This gives the maximum area of transfer to the liquid and the bubbles leave the plate when still very small.

With submerged boiling, heat is transferred directly to the liquid and the bubbles of vapour are formed subsequently. The high rate of heat transfer is largely due to the agitation of the liquid by the bubbles. ROHSENOW and CLARK^(55, 59) have shown that, as the agitation becomes more vigorous, the effect of any forced convection becomes progressively less. This is shown in Fig. 6.24, taken from their work, where it is seen that at high rates of boiling the heat flux becomes constant over a range of fluid velocities.

Types of Boiling

In boiling liquids it is found that the heat transfer coefficient depends very much on the temperature difference between the heating surface and the bulk of the liquid. Three separate conditions may be distinguished as the temperature difference is increased.

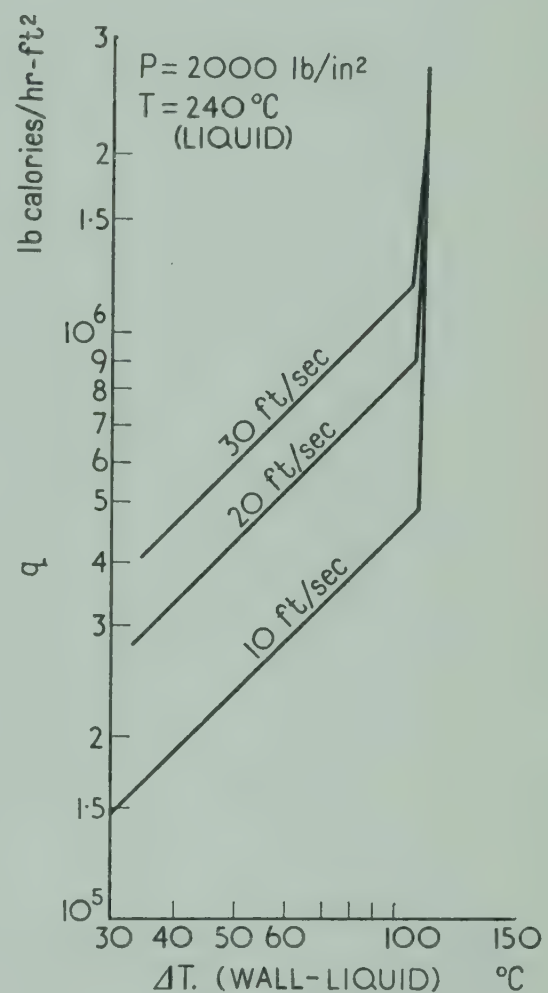


Fig. 6.24. Effect of velocity on heat transfer to boiling liquid

(a) With very low temperature differences the bubbles form slowly and exert very little stirring action in the liquid.

(b) With greater temperature differences the bubbles form more rapidly and exert an appreciable stirring action on the liquid. This is known as nucleate boiling and is the normal form of boiling.

(c) At very high temperature differences the bubbles form so rapidly that they cannot get away and a film of vapour forms over the surface. The rate of heat transfer falls since the vapour film has a low conductivity.

The general form of the curve for boiling liquids is shown in Fig. 6.25, which is taken from the work of NUKIYAMA, who boiled water at atmospheric pressure by

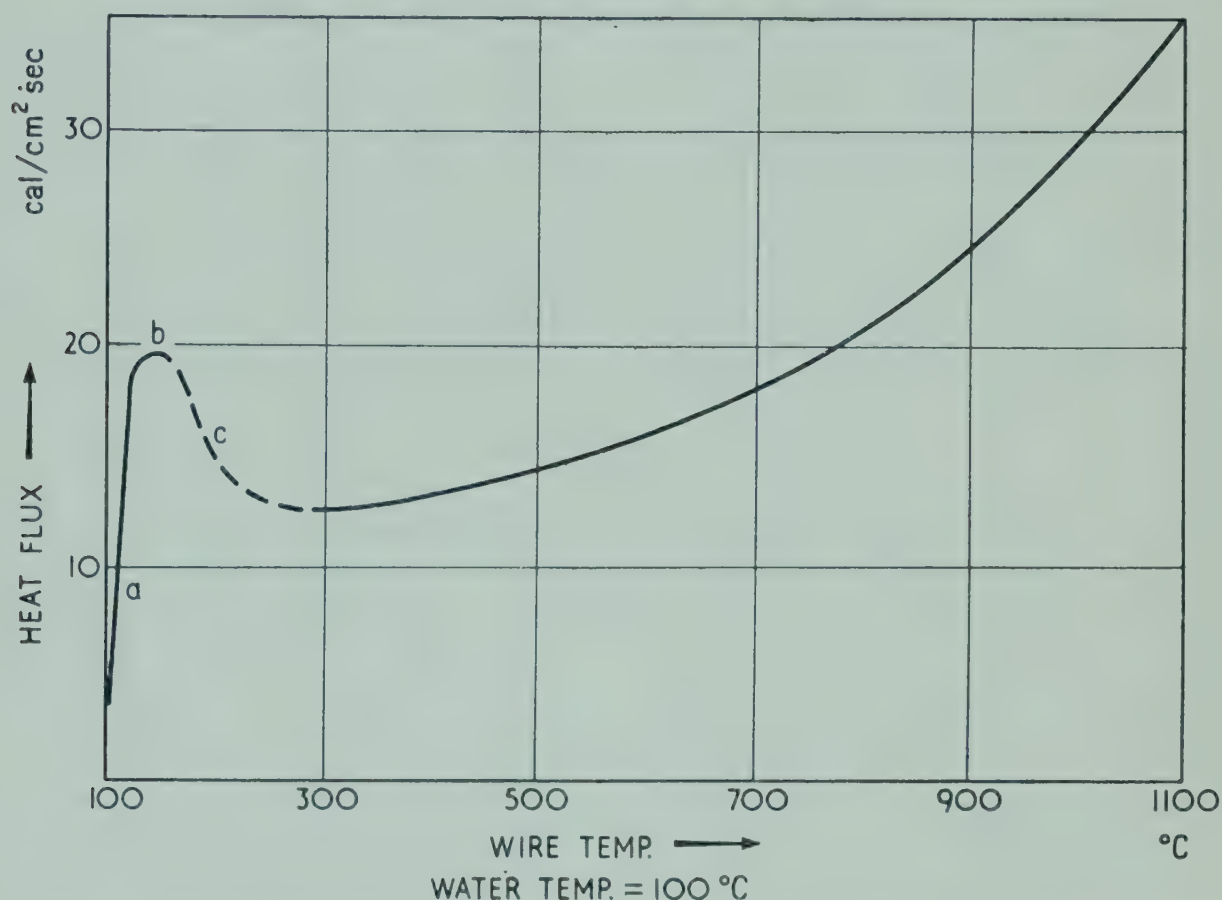


Fig. 6.25. Effect of temperature difference on heat flux to boiling liquid (NUKIYAMA)

means of electrically heated nichrome, platinum, and other wires of various diameters. By measuring the resistance of the wire he found its temperature and thus the temperature difference between the heating surface and the boiling water. His curve shows a steady rise in heat flux with increasing temperature difference corresponding to stages *a* and *b*, followed by a fall in heat flux for stage *c*. With very high temperature differences he was able to get an increase in the heat flux but such conditions are rarely obtained in chemical plant.

Heat Transfer Coefficients and Heat Flux

For very small temperature differences JAKOB⁽²³⁾ found that the laws for natural convection could be applied to boiling liquids. Thus for values of $[Gr.Pr.]$ less than 10^9 , he could express his results for boiling on a vertical surface as

$$Nu. = 0.61 \cdot [Gr.Pr.]^{\frac{1}{4}} \quad \dots(6.75)$$

which is practically the same as that for natural convection under streamline conditions, where the constant was 0.56 (equation 6.46). For horizontal surfaces he obtained the relation,

$$Nu. = 0.16 \cdot [Gr.Pr.]^{\frac{1}{3}} \quad \dots(6.76)$$

which is similar to that for natural convection under turbulent conditions. Such low values of heat flux are of little practical value but serve to confirm the

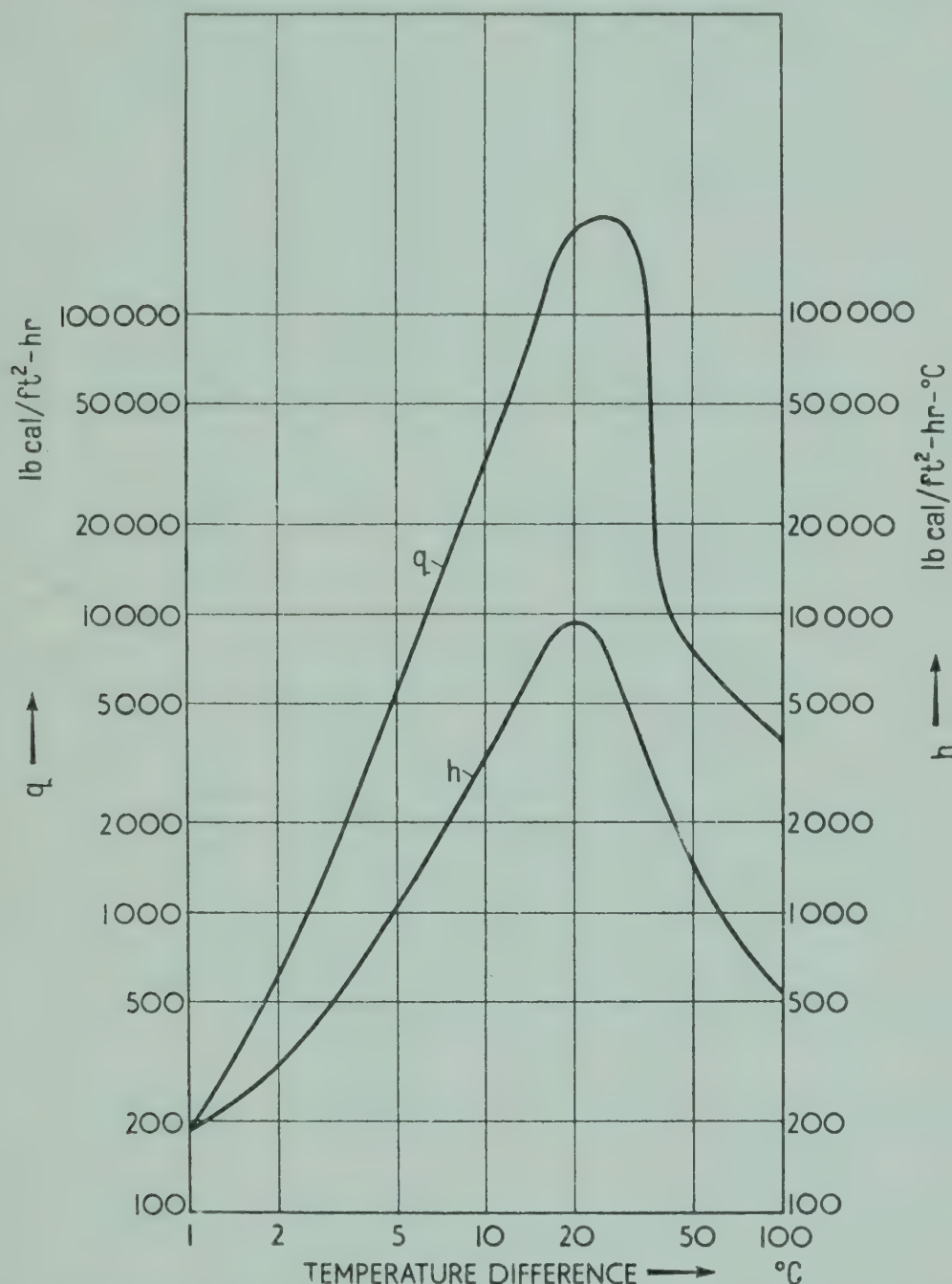


Fig. 6.26. Effect of temperature difference on heat flux and heat transfer coefficient to water boiling at 100°C on a submerged surface

nature of the process. For higher values of $(Gr.Pr.)$, the Nusselt Number increases much more rapidly.

Boiling on Submerged Surfaces. Fig. 6.26 shows the effect of increasing the temperature difference between the surface and the liquid for water boiling at atmospheric pressure on a submerged heating surface. Whilst the actual values of the transfer coefficient h and the heat flux Q/A for the boiling liquid vary with different observers, the important feature is that they all show a maximum value at about 22°C. The maximum value of h is about 9000 lb-cal/hr-ft²-°C and corresponds to a heat flux of about 190,000 lb-cal/hr-ft².

Similar results have been obtained by BONILLA and PERRY⁽⁴⁵⁾, INSINGER and BLISS⁽⁴³⁾, and others for a number of organic liquids such as benzene, alcohols, acetone and carbon tetrachloride. The data in Table 6.8 for liquids boiling at atmospheric pressure show that the maximum heat flux is much smaller with organic liquids than with water and the temperature difference at this condition is rather higher. In practice the critical value of ΔT may be exceeded. SAUER and COOPER⁽⁴¹⁾ found that the overall transfer coefficient U for boiling ethyl

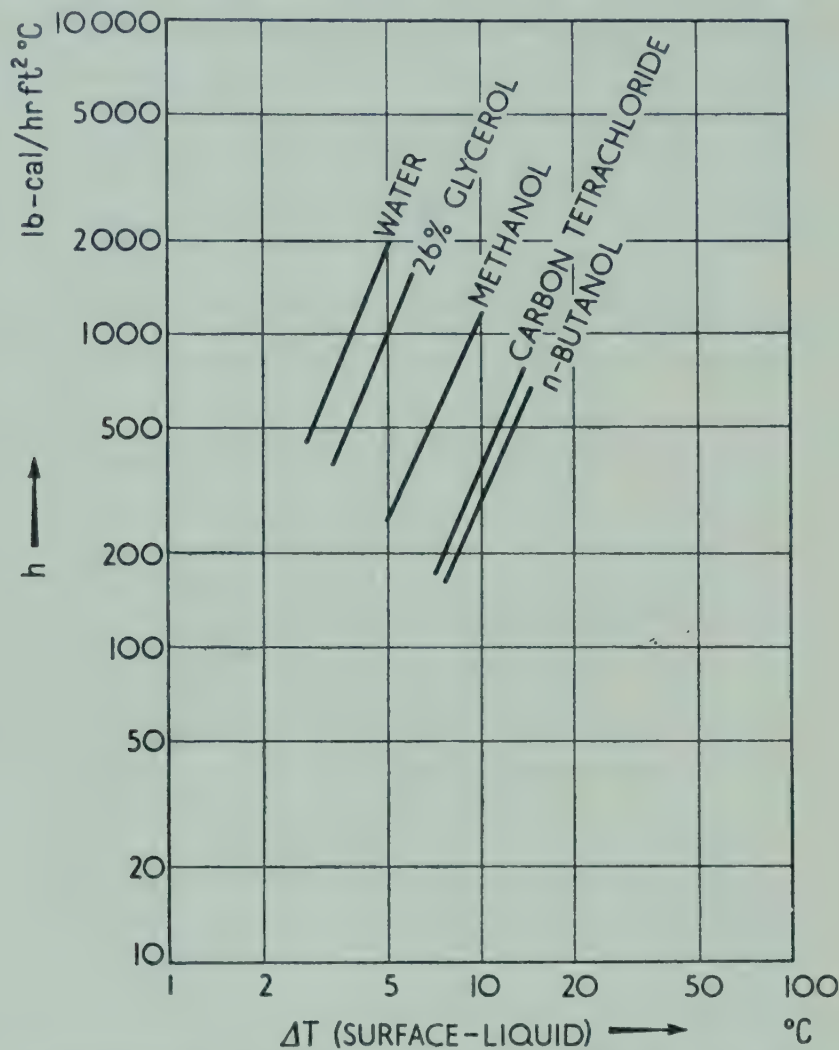


Fig. 6.27. Effect of temperature difference on heat transfer coefficient for boiling liquids (CRYDER and FINALBORGO)

acetate with steam at 40 lb/sq in. was only 14% of that when the steam pressure was reduced to 2 lb/sq in.

TABLE 6.8. Maximum heat flux for various liquids boiling at atmospheric pressure

Liquid	Surface	Critical ΔT (°C)	Maximum flux (lb-cal/hr-ft ²)
Water	Chromium	25	160,000
50 Mol % ethanol-water	Chromium	29	105,000
Ethanol	Chromium	33	80,000
n-Butanol	Chromium	44	80,000
iso-Butanol	Nickel	44	65,000
Acetone	Chromium	25	80,000
iso-Propanol	Chromium	33	60,000
Carbon tetrachloride	Copper	—	32,000
Benzene	Copper	—	30,000–40,000

In considering the problem of nucleate boiling therefore the nature of the surface, the pressure and the temperature difference must be taken into consideration as well as the actual physical properties of the liquid.

Effect of Surface. Apart from the question of scale, the nature of the clean surface has a pronounced influence on the rate of boiling. Thus BONILLA and PERRY boiled ethanol at atmospheric pressure and found the heat flux at atmospheric pressure to be 150,000 lb-cal/hr-ft²-°C for polished copper, 80,000 for gold plate, and 65,000 for fresh chromium plate, and only 25,000 for old chromium plate. This wide fluctuation means that care must be taken in judging the anticipated heat flux, since the high values that may be obtained initially will not be obtained in practice because of tarnishing of the surface.

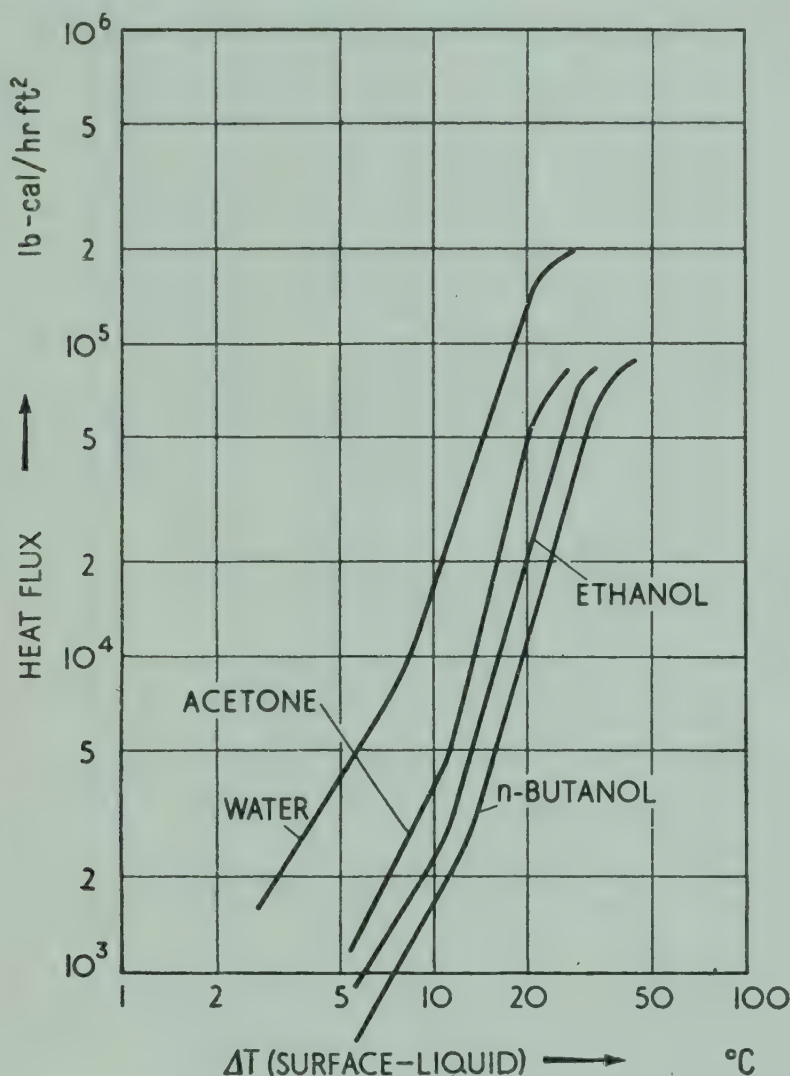


Fig. 6.28. Effect of temperature difference on heat flux to boiling liquids (BONILLA and PERRY)

Effect of Temperature Difference. CRYDER and FINALBORGO⁽³⁹⁾, boiled a number of liquids on a horizontal brass surface, both at atmospheric and at reduced pressure. Some of their results are shown in Fig. 6.27, where the coefficient for the boiling liquid h is plotted against the temperature difference between the hot surface and the liquid. The points for the various liquids in Fig. 6.27 lie on nearly parallel straight lines, which can be represented by

$$h = \text{constant} \times \Delta T^{2.5} \quad \dots(6.77)$$

This value for the index of ΔT has been found by other workers (GILLILAND, WILLIAMS) but JAKOB and LINKE found values as high as 4 for some of their work. It is important to note that this value of 2.5 is true only for temperature differences up to 19°C.

In some ways it is more convenient to show the results in the form of heat flux versus temperature difference. This is shown in Fig. 6.28, where some results

from a number of workers are given together and the maximum flux is shown from the work of BONILLA and PERRY⁽⁴⁵⁾.

Effect of Pressure. CRYDER and FINALBORGO found that h fell off regularly as the pressure and hence the boiling point were reduced, according to the relation $h = \text{constant} \times B^{T''}$, where T'' is numerically equal to the temperature in $^{\circ}\text{C}$ and B is a constant. Combining this with equation 6.77, they gave their results for h in the empirical form

$$h = \text{constant} \times \Delta T^{2.5} B^{T''}$$

or

$$\log h = a' + 2.5 \log \Delta T + b'T'' \quad \dots (6.78)$$

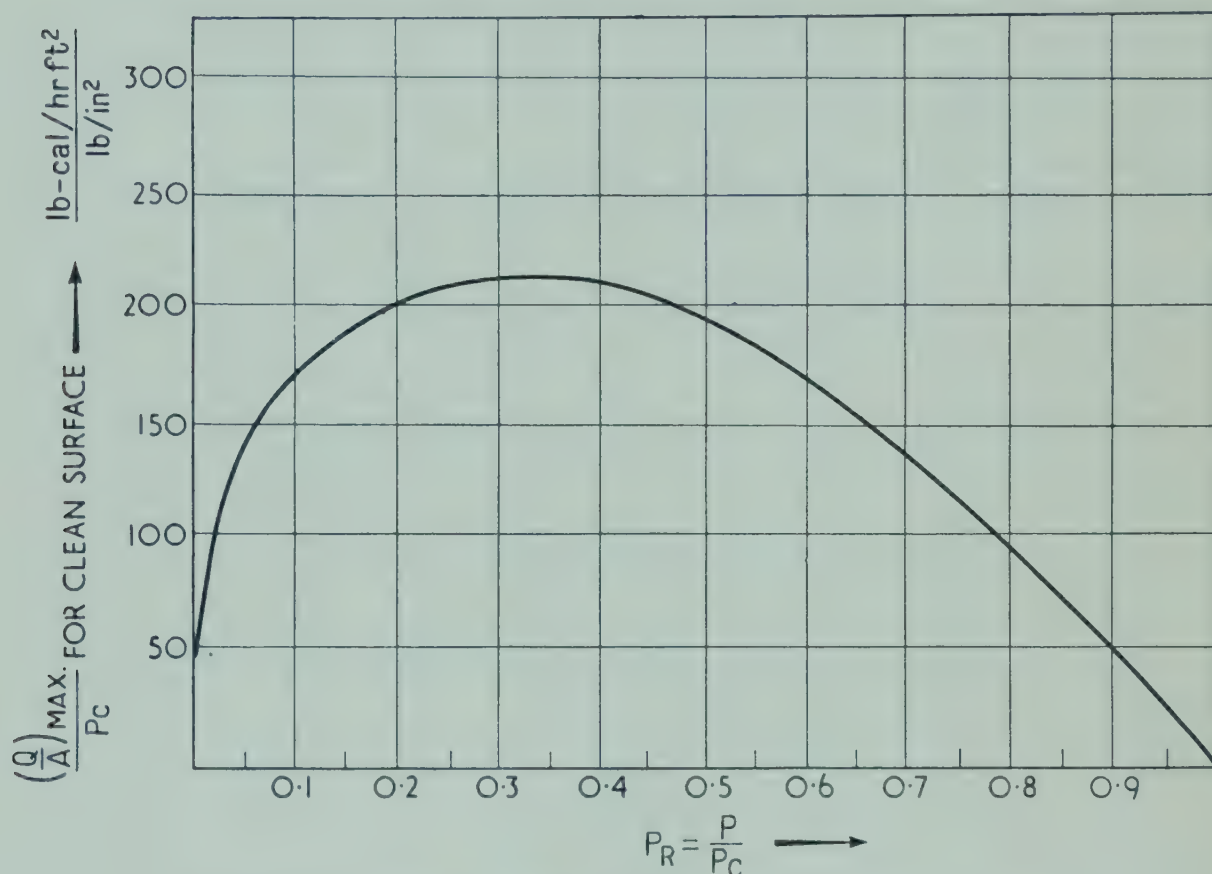


Fig. 6.29. Effect of pressure on heat flux to boiling liquid

If a' and b' are given the following values, h is expressed in $\text{lb-cal/hr-ft}^2\text{-}^{\circ}\text{C}$.

	a'	b'		a'	b'
Water . . .	-0.96	0.025	Kerosene . . .	-4.13	0.022
Methanol . . .	-1.11	0.027	10% Na_2SO_4 . . .	-1.47	0.029
CCl_4 . . .	-1.55	0.022	24% NaCl . . .	-2.43	0.031

The values of a' will apply only to a given apparatus but a value of b' of 0.025 is of more general application. If h_n is the coefficient at some standard boiling point T_n , and h at some other temperature T , equation 6.78 can be rearranged to give

$$\log h/h_n = 0.025(T'' - T_n'') \quad \dots (6.79)$$

for a given material and temperature difference.

Boiling at High Pressures. As the temperature is raised above the normal boiling point, the film coefficient increases for a constant temperature difference. CICHELLI and BONILLA⁽⁴⁸⁾ have examined this problem for pressures up to the

critical value for the vapour, and have shown that ΔT for maximum rate of boiling decreases with the pressure. They obtained a single curve (Fig. 6.29) by plotting $\frac{(Q/A)_{\max}}{P_c}$ against P_R , where P_c is the critical pressure and P_R the reduced pressure $= P/P_c$. This curve represented the data for water, ethanol, benzene, propane, n-pentane, n-heptane, and several mixtures with water. For water their results cover only a small range of P/P_c because of the high value of P_c . For the organic liquids investigated, they were able to show that the maximum value of heat flux Q/A occurs at a pressure P of about one-third of the critical pressure P_c . The range of physical properties of the organic liquids is not wide and further data are required to substantiate the above equation.

TABLE 6.9. Heat transfer coefficients, h , for boiling liquids

Liquid	Boiling Point (°C)	ΔT (°C)	h (lb-cal/hr-ft ² -°C)
Water . . .	99	4.7	1614
	99	2.9	480
	53	8.8	830
	53	6.1	235
Methanol . . .	64	8.9	845
	64	5.6	260
	33	14.4	515
	33	9.3	158
Carbon tetrachloride .	76	12.6	613
	76	7.2	191
	42	20.1	370
	42	11.8	125

Equations for Estimating h for Boiling Liquids. For low rates of boiling we have seen that JAKOB's equation is satisfactory and an indication of the complexity of the process has been given by considering some of the published work. INSINGER and BLISS⁽⁴³⁾ have boiled water, carbon tetrachloride, isopropyl alcohol, and sucrose on vertical chromium plated tubes electrically heated, and have proposed a complex equation to correlate their data. Their results show that

$$h \propto C_p^{0.5} \cdot k^{0.5} \cdot q^{0.7} \cdot \tau^{-0.5} \cdot \rho_v^{0.5} \cdot \rho_L^{0.8} \cdot \lambda^{-0.3}$$

It will be noticed that the relation includes the surface tension τ but not the viscosity, and that h is related to the heat flux q . It is tolerably consistent with the data of ATKIN and MCADAMS and with those of JAKOB and LINKE, but it gives substantially lower values than those obtained by CRYDER and FINALBORGO, whose results are now believed to be a little high. It must at present be accepted that there are no satisfactory relations for determining film coefficients for boiling liquids but the experimental work has indicated the main features

involved and does enable a judicious estimate to be made of the likely effect of altering the conditions of operation in a given piece of equipment.

Boiling occurs outside tubes in coil heaters for evaporators and distillation units and also in the kettle heater which is commonly used in the petroleum industry. Boiling takes place on the outside of a surface in the standard jacketed pan. It seems safest at present to design the units on a basis of anticipated heat flux rather than on the transfer coefficient.

Kettle Boilers

A typical boiler as used for vaporising the liquid at the bottom of a distillation column is shown in Fig. 6.30. In this unit steam is condensed inside the tubes: the liquid boils outside. The whole tube bundle is easily removed from the shell for cleaning, a vital requirement for apparatus where scaling or carbon

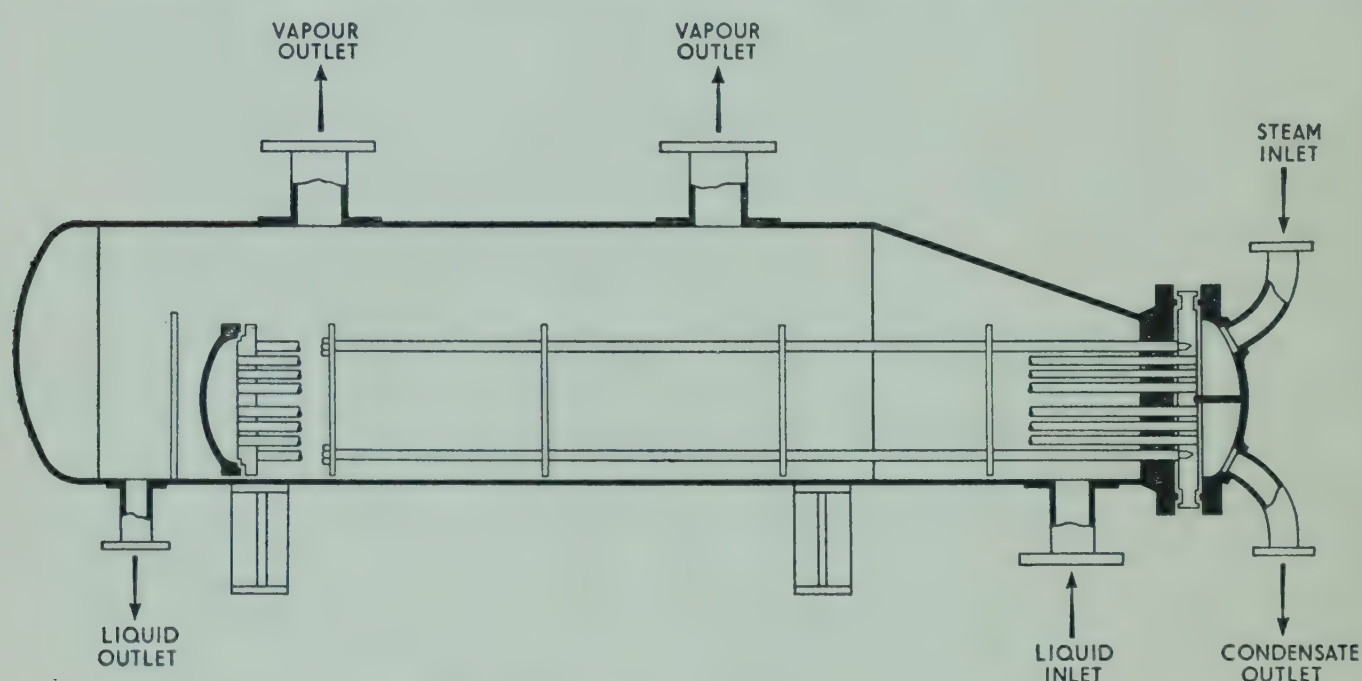


Fig. 6.30. Kettle boiler for distillation column

deposition is likely to occur. The tube bundle is free to move on the bottom of the shell so that differential expansion can be accommodated. The liquid to be boiled enters at the bottom and the vapour leaves at the top.

With this type of unit KERN⁽⁷⁾ suggests that it is undesirable to vaporise more than 80% of the feed, but this figure must be reduced if the liquor is liable to deposit carbon or scale. A common arrangement is to use natural circulation of the fluid over the tubes, but with very viscous liquids an external pump is fitted to give forced circulation, although this will usually demand considerable maintenance.

As with other equipment, for design purposes the maximum safe flux to assume for boiling organic materials is about 20,000 lb-cal/hr-ft²-°C with forced circulation and only 12,000 with natural circulation.

HEAT TRANSFER IN REACTION VESSELS

A simple jacketed pan or kettle is very commonly used in the chemical industries as a reaction vessel. In many cases, such as in nitration or sulphonation reactions,

heat has to be removed or added to the mixture in order, either to control the rate of reaction, or to bring it to completion. The addition or removal of heat is conveniently arranged by passing steam or water through a jacket fitted to the outside of the vessel, or through a helical coil fitted to the inside. In either case some form of agitator is used to obtain even distribution in the vessel. This may be of the anchor type for very thick mixes, or a propeller or turbine if the contents are not too viscous.

Reaction Vessel with Helical Cooling Coil

In this case the thermal resistances to heat transfer arise from the water film on the inside of the coil, the wall of the tube, the film on the outside of the coil, and any scale that may be present on either surface. We can express the overall transfer coefficient by the relation,

$$\frac{1}{U \cdot A} = \frac{1}{h_i \cdot A_i} + \frac{x_w}{k_w \cdot A_w} + \frac{1}{h_o \cdot A_o} + \frac{R_o}{A_o} + \frac{R_i}{A_i} \quad \dots (6.80)$$

where R_o and R_i are the scale resistances and the other terms have the usual meaning.

Inside Film Coefficient for a Coil. The value of h_i can be found from equation 6.31,

$$\frac{h_i \cdot d}{k} = 0.023 \left[\frac{d \cdot u \cdot \rho}{\mu} \right]^{0.8} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{0.4} \quad \dots (6.31)$$

if water is used in the coil, and the SIEDER and TATE equation (6.32) if a viscous brine is used for cooling.

These equations have been obtained for straight tubes; with a coil somewhat greater transfer is obtained for the same physical conditions. JESCHKE⁽¹⁵⁾ cooled air in a $1\frac{1}{4}$ in. steel tube wound in the form of a helix and expressed his results in the form

$$h_i(\text{coil}) = h_i(\text{straight pipe}) \left(1 + 3.5 \frac{d}{d_c} \right) \quad \dots (6.81)$$

where d is the inside diameter of the tube and d_c the diameter of the helix. PRATT⁽⁵¹⁾ has examined this problem in greater detail for liquids and has given almost the same result. Combining equations 6.31 and 6.81 we can therefore calculate the inside film coefficient h_i for the coil.

Outside Film Coefficient. The value of h_o is determined by the physical properties of the liquor and by the degree of agitation achieved. This latter quantity is difficult to express in a quantitative manner and the group $\frac{L^2 \cdot N \cdot \rho}{\mu}$

has been used both for this problem and for the allied one of power used in agitation. In this group L is the length of the paddle and N the revolutions per unit time. CHILTON DREW and JEBENS⁽⁴⁷⁾, working with a small tank, only 1 ft in diameter d_v , expressed their results in the form,

$$\frac{h_o \cdot d_v}{k} \left(\frac{\mu_s}{\mu} \right)^{0.14} = 0.87 \left[\frac{C_p \cdot \mu}{k} \right]^{\frac{1}{3}} \left[\frac{L^2 \cdot N \cdot \rho}{\mu} \right]^{0.62} \quad \dots (6.82)$$

where the factor $\left(\frac{\mu_s}{\mu}\right)^{0.14}$ was introduced to cover the difference between the viscosity adjacent to the coil (μ_s) and that in the bulk of the liquor. They obtained a range in physical properties by using water, two oils, and glycerol.

PRATT⁽⁵¹⁾ used both circular and square tanks up to 2 ft across and a series of different arrangements of a simple paddle (Fig. 6.31). He also examined the influence of alteration in the arrangement of the coil and varied the tube diameter d , the gap between the turns d_g , the diameter of the helix d_c , the height of the coil d_p , and the width of the stirrer W . His final equations were,

$$\frac{h_o \cdot d_v}{k} = 34 \left[\frac{L^2 \cdot N \cdot \rho}{\mu} \right]^{0.5} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{0.3} \cdot \left[\frac{d_g}{d_p} \right]^{0.8} \cdot \left[\frac{W}{d_c} \right]^{0.25} \cdot \left[\frac{L^2 \cdot d_v}{d_e^3} \right]^{0.1} \dots (6.83)$$

for cylindrical tanks and

$$\frac{h_o \cdot l_v}{k} = 39 \left[\frac{L^2 \cdot N \cdot \rho}{\mu} \right]^{0.5} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{0.3} \cdot \left[\frac{d_g}{d_p} \right]^{0.8} \cdot \left[\frac{W}{d_c} \right]^{0.25} \cdot \left[\frac{L^2 \cdot l_v}{d_e^3} \right]^{0.1} \dots (6.84)$$

for square tanks (where l_v is the length of the side of the vessel).

These give almost the same results as the earlier equations over a wide range in conditions. CUMMINGS and WEST⁽⁵⁴⁾ have tested these results with a much larger tank of 100 gal capacity and have given an expression similar to equation 6.82, but with a constant of 1.01 instead of 0.87. They used a retreating blade turbine impeller and in many cases mounted a second impeller above the first; this agitation was probably more intense than that given by the other workers. A constant of 0.9 seems a reasonable average from existing work.

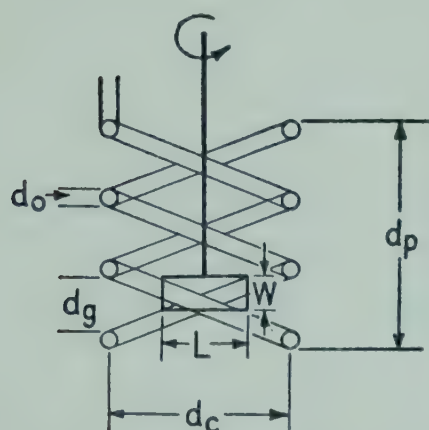


Fig. 6.31. Arrangement of coil in Pratt's work

Example. Toluene is continuously nitrated to mononitrotoluene in a cast iron vessel of 3 ft diameter fitted with a propeller agitator of 1 ft diameter driven at 150 r.p.m. The temperature is maintained at 35°C by circulating cooling water at 3500 lb/hr through a stainless steel coil of 1 in. o.d. and 0.87 in. i.d. wound in the form of a helix of 32 in. diameter. The conditions are such that the reacting material may be considered to have the same physical properties as 75% sulphuric acid. If the mean water temperature is 17°C what is the overall heat transfer coefficient?

Solution. The overall coefficient U_o based on the outside area of the coil is given by,

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{x_w \cdot d_o}{k_w \cdot d_w} + \frac{1 \cdot d_o}{h_i \cdot d} + R_o + \frac{R_i \cdot d_o}{d}$$

(where d_w is the mean diameter of the pipe)

(from equation 6.80)

The inside film coefficient for the water is given by,

$$h_i = \frac{k}{d} \left[1 + 3.5 \times \frac{d}{d_c} \right] \times 0.023 \left(\frac{d \cdot u \cdot \rho}{\mu} \right)^{0.8} \times \left(\frac{C_p \cdot \mu}{k} \right)^{0.4}$$

(from equations 6.31 and 6.81)

The cross sectional area of the coil is 0.00413 ft^2

$$\text{Hence } u \cdot \rho = \frac{3500}{0.00413} \text{ lb/hr-ft}^2$$

$$d = 0.87 \text{ in.} = 0.0725 \text{ ft} \quad d_c = 32 \text{ in.}$$

$$\text{Taking } k = 0.34 \text{ lb-cal/hr-ft-}^\circ\text{C,}$$

$$\mu = 2.61 \text{ lb/ft-hr and}$$

$$C_p = 1$$

$$\begin{aligned} h_i &= \frac{0.34}{0.0725} \left(1 + 3.5 \times \frac{0.87}{32} \right) \times 0.023 \left(\frac{0.0725 \times 3500}{2.61 \times 0.00413} \right)^{0.8} \left(\frac{2.61}{0.34} \right)^{0.4} \\ &= 0.118 \times 23,600^{0.8} \times 7.68^{0.4} \\ &= 840 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C} \end{aligned}$$

The external film coefficient is given by equation 6.82.

$$\frac{h_o \cdot d_v}{k} \left(\frac{\mu_s}{\mu} \right)^{0.14} = 0.87 \left(\frac{C_p \mu}{k} \right)^{0.33} \cdot \left(\frac{L^2 \cdot N \cdot \rho}{\mu} \right)^{0.62}$$

For 75% sulphuric acid,

$$k = 0.23 \text{ lb-cal/hr-ft-}^\circ\text{C}$$

$$\mu_s \text{ (taken at } 24^\circ\text{C)} = 8.6 \text{ centipoises}$$

$$\mu \text{ (taken at } 35^\circ\text{C)} = 6.5 \text{ centipoises} = 15.7 \text{ lb/ft-hr}$$

$$C_p = 0.45 \text{ lb-cal/lb-}^\circ\text{C and}$$

$$\rho = 104 \text{ lb/ft}^3$$

$$N = 150 \text{ r.p.m.} = 9000 \text{ r.p.hr}$$

$$\text{Hence } \frac{h_o \times 3}{0.23} \times \left(\frac{8.6}{6.5} \right)^{0.14} = 0.87 \left(\frac{0.45 \times 15.7}{0.23} \right)^{0.33} \left(\frac{1 \times 9000 \times 104}{15.7} \right)^{0.62}$$

$$\therefore 13.1 h_o \times 1.04 = 0.87 \times 3.09 \times 915$$

$$\therefore h_o = 180 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C}$$

Then, taking k_w as $9.2 \text{ lb-cal/hr-ft-}^\circ\text{C}$, and R_o and R_i as 0.002 and $0.001 \text{ hr-ft}^2\text{-}^\circ\text{C/lb-cal}$,

$$\begin{aligned} \frac{1}{U_o} &= \frac{1}{180} + \frac{0.065 \times 1.0}{12 \times 9.2 \times 0.935} + \frac{1 \times 1.0}{840 \times 0.87} + 0.002 + \frac{0.001 \times 1.0}{0.87} \\ &= 0.00556 + 0.00063 + 0.00137 + 0.002 + 0.00115 \end{aligned}$$

$$\therefore U_o = \underline{\underline{93 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C}}}$$

In this calculation a mean area of surface might have been used with sufficient accuracy. It is important to note the great importance of the scale terms which together form a major part of the thermal resistance.

Reaction Vessel with Jacket

For many purposes heating or cooling of a reaction mixture is most satisfactorily achieved by condensing steam in a jacket or passing water through it; this is commonly done in organic reactions where the mixture is too viscous for the use of coils and a high speed agitator. CHILTON *et al.*⁽⁴⁷⁾ and CUMMINGS and WEST⁽⁵⁴⁾ have measured the transfer coefficients for this case by using an arrangement as shown in Fig. 6.32, where heat is applied to the jacket and simultaneously removed by passing water through the coil. CHILTON measured

the temperatures of the inside of the vessel wall, of the bulk liquid and of the surface of the coil by means of thermocouples and thus obtained the film heat transfer coefficients directly. CUMMINGS and WEST used an indirect method to give the film coefficient from measurements of the overall coefficients.

CHILTON expressed his results in the form,

$$\frac{h_b \cdot d_v}{k} \cdot \left(\frac{\mu_s}{\mu} \right)^{0.14} = 0.36 \left[\frac{L^2 \cdot N \cdot \rho}{\mu} \right]^{\frac{2}{3}} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{\frac{1}{4}} \quad \dots (6.85)$$

where h_b is the film coefficient for the liquor adjacent to the wall of the vessel. CUMMINGS and WEST used the same equation but gave the value of the constant as 0.40. Considering that CHILTON's vessel was only 1 ft in diameter and was fitted with a single paddle of 6 in. length, and that CUMMINGS used a 100 gal

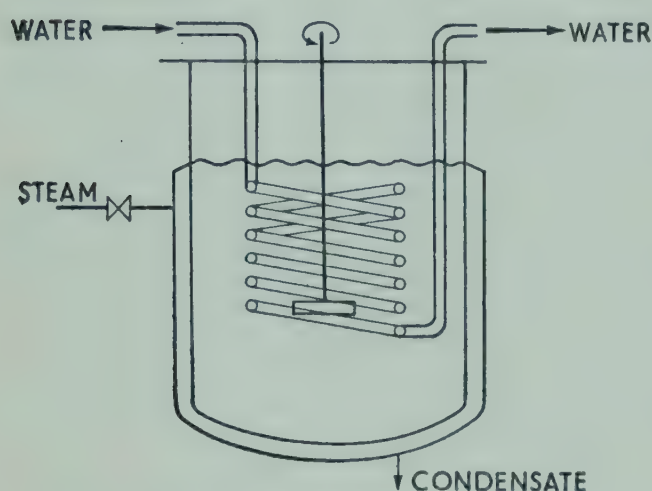


Fig. 6.32. Reaction vessel with jacket and coil

vessel with two turbine impellers, agreement between their results is remarkably good. The group $\left(\frac{\mu_s}{\mu} \right)^{0.14}$ is again used to cover the difference in the viscosities at the surface and in the bulk of the fluid.

BROWN *et al.*⁽⁵⁰⁾ have given results of measurements on 5 ft diameter sulphonators and nitrators of 750 gal capacity as used in the dyestuffs industry. The sulphonators were of cast iron and had a wall thickness of 1 in; the annular space in the jacket was also 1 in. The agitator of the sulphonator was of the anchor type with a 5 in. clearance at the walls and was driven at 40 r.p.m. The nitrators were fitted with four bladed propellers of 2 ft diameter driven at 120 r.p.m. For cooling, they expressed the film coefficient, h_b , for the inside of the vessel by the relation,

$$\frac{h_b \cdot d_v}{k} \cdot \left(\frac{\mu_s}{\mu} \right)^{0.14} = 0.55 \left[\frac{L^2 \cdot N \cdot \rho}{\mu} \right]^{\frac{2}{3}} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{\frac{1}{4}} \quad \dots (6.86)$$

which is very similar to that given by equation 6.85.

They also measured the film resistance in the water jacket and gave values of 112 to 206 lb-cal/hr-ft²-°C for water rates of 1140 to 7310 gal/hr, respectively, in the sulphonator. It should be noted that 6000 gal/hr corresponds to a vertical velocity of only 0.15 ft/sec. and to a Reynolds number in the annulus of 5350. The thermal resistance of the wall of the pan was important, since with the sulphonator it accounted for 13 % of the total resistance at 50°C and 31 % at 130°C. The change in viscosity with temperature is important when considering these processes, since, for example, the viscosity of the sulphonation liquors ranged from 340 centipoises at 50°C to 22 centipoises at 130°C.

Heating Liquids in Tanks

It is frequently necessary to heat or cool the contents of a large batch reactor or storage tank. In this case the physical constants of the liquor may alter and

the overall transfer coefficient change during the process. In practice, however, it is often possible to assume an average value of the transfer coefficient so that the time of heating or cooling may then be evaluated without using the complex techniques necessary for calculations involving unsteady states. The heating of the contents of a storage tank is commonly effected by condensing steam, either in a coil or in some form of hairpin tube heater.

For the case of a storage tank with liquor of mass m and specific heat C_p , heated by steam condensing in an helical coil, suppose that the overall transfer coefficient U is constant. Let T_s be the temperature of the condensing steam, T_1 and T_2 the initial and final temperatures of the liquor, and A the area of heat transfer surface. If T is the temperature of the liquor at any time t , the rate of transfer of heat is given by

$$\frac{dQ}{dt} = m \cdot C_p \cdot \frac{dT}{dt} = U \cdot A \cdot (T_s - T)$$

$$\therefore \frac{dT}{dt} = \frac{U \cdot A}{m \cdot C_p} \cdot (T_s - T)$$

$$\therefore \int_{T_1}^{T_2} \frac{dT}{T_s - T} = \frac{U \cdot A}{m \cdot C_p} \cdot \int_0^t dt$$

$$\therefore \ln \frac{T_s - T_1}{T_s - T_2} = \frac{U \cdot A}{m \cdot C_p} \cdot t \quad \dots (6.87)$$

From this, t the time of heating from T_1 to T_2 , can be found. The same analysis can be used if the steam condenses in a jacket of a reaction vessel.

SHELL AND TUBE EXCHANGERS

These units form the most important class of heat transfer equipment because they can be constructed with very large surfaces in a relatively small volume, can be fabricated from alloy steels to resist corrosion and can be used for heating and condensing all kinds of fluids. Figs. 6.33 to 6.36 show various forms of construction of varying complexity and Fig. 6.37 shows an outside view. The simplest type, shown in Fig. 6.33, has fixed tube plates at each end and the tubes are expanded into the plates. The tubes are so connected that the water or internal fluid makes several passes up and down the exchanger, thus enabling a high velocity of flow to be obtained for a given heat transfer area and given throughput of liquid. The fluid flowing in the shell is made to flow first in one sense and then in the opposite sense across the tube bundle by fitting a series of baffles along the length. These baffles are frequently of the segmental form with about 25% cut for the free space and increase the velocity of flow across the tubes, thus giving better heat transfer. The difficulty with this type of construction is that the tube bundle cannot be removed for cleaning and no provision is made for differential expansion between the tubes and the shell, although an expansion joint is often fitted in the shell.

In order to make removal of the tube bundle possible and to allow for considerable expansion of the tube, a floating head exchanger is used, as shown in Fig. 6.34. In this arrangement one tube plate is fixed as before but the second is bolted to a floating head cover so that the tube bundle can move relative to the shell. This floating tube sheet is clamped between the floating head and a split

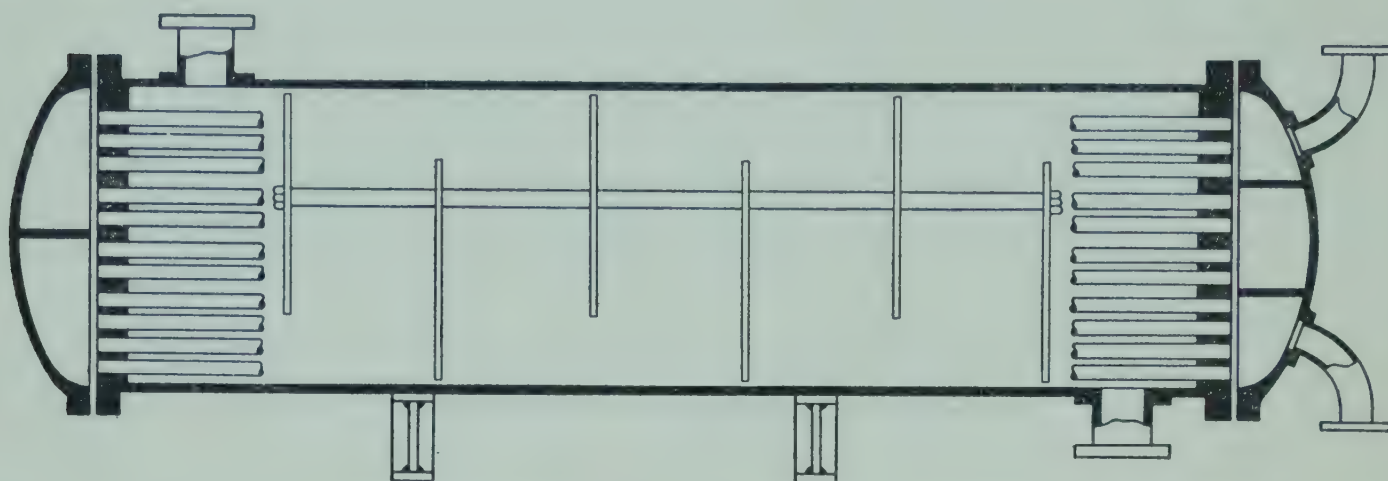


Fig. 6.33. Heat exchanger with fixed tube plates (4 tube pass 1 shell pass)

backing flange in such a way that it is relatively easy to break the flanges at both ends and pull out the tube bundle. It will be seen that the shell cover at the floating head end is larger than that at the other end; this is to enable the tubes to be placed as near as possible to the edge of the fixed tube plate and leave very little unused space between the outer ring of tubes and the shell.

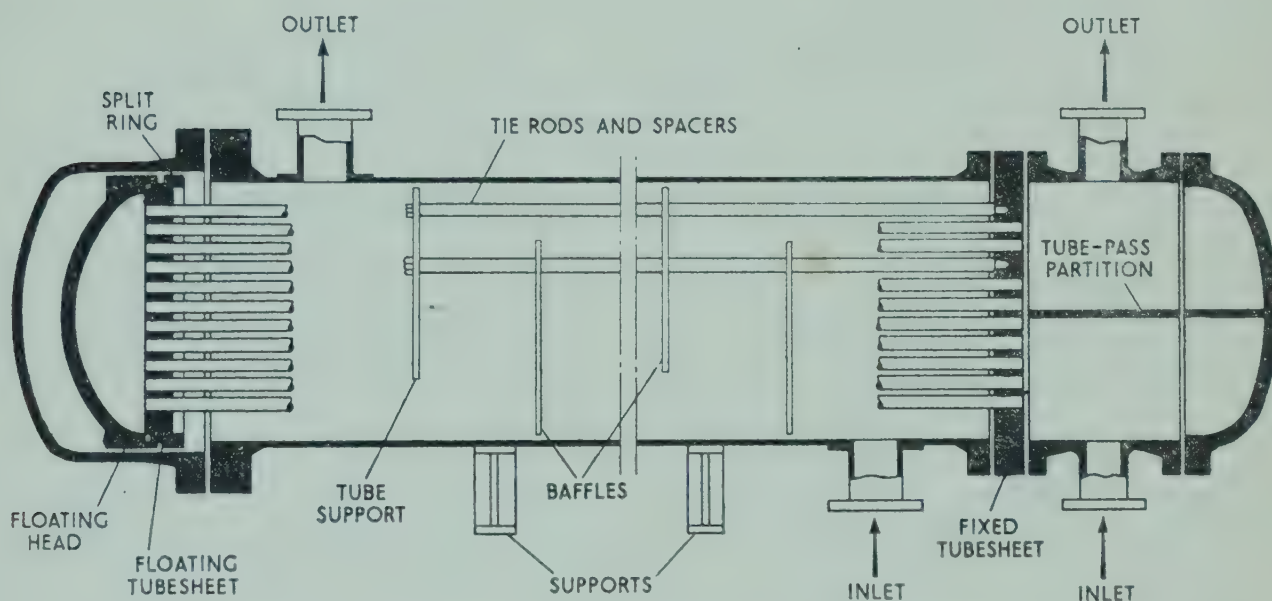


Fig. 6.34. Heat exchanger with floating head (2 tube pass 1 shell pass)

Other arrangements which provide for expansion consists of an externally packed floating head, as shown in Fig. 6.35, and hairpin tubes, as shown in Fig. 6.36. The latter design is very commonly used for the reboilers on large fractionating columns where steam is condensed inside the tubes.

In these designs there is one pass for the fluid on the shell side and a number of passes on the tube side. As will be shown later it is often desirable to have two or more shell side passes, but they considerably increase the difficulty of

construction and therefore several smaller exchangers are often connected together to obtain the same effect.

The essential requirements in the design of a heat exchanger are first the provision of a unit which is reliable and has the desired capacity, and secondly

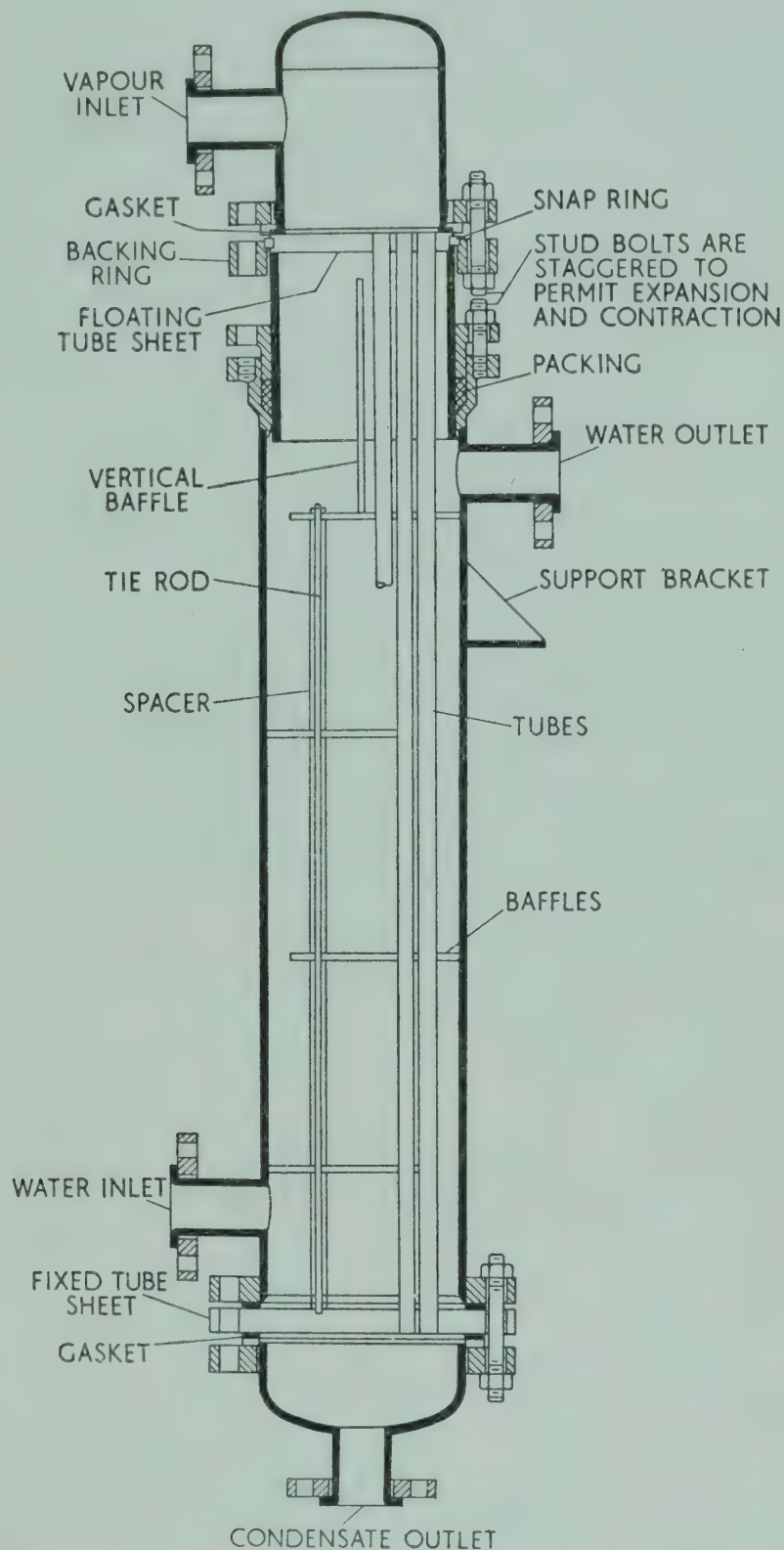


Fig. 6.35. Heat exchanger with externally packed floating head (vertical condenser)

the economic consideration of making an exchanger at the minimum overall cost. This will generally involve using standard parts and fittings and making the design as simple as possible. In most cases, it will be necessary to balance the capital cost, as measured by depreciation, against the operating cost. Thus, for instance, in a condenser, a high heat transfer coefficient and hence a smaller exchanger is required, if a high water velocity is used through the tubes; on the other hand, the cost of pumping increases rapidly with increase in velocity and an economic balance must be struck. A typical graph showing the operating

cost, depreciation, and total overall cost plotted against water velocity through the tubes for a condenser is given in Fig. 6.38.

Tubes. These are usually $\frac{3}{4}$ or 1 in. outside diameter d_o , and 14 or 16 B.W.G. for admiralty metal or other alloy and 10 or 12 B.W.G. for carbon steel. The

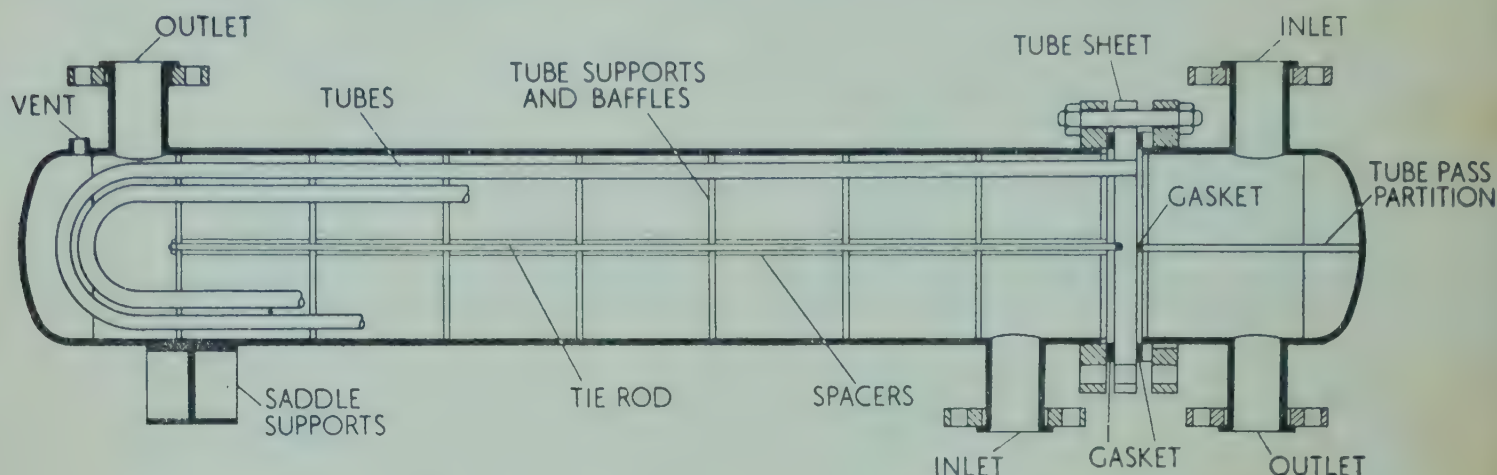


Fig. 6.36. Heat exchanger with hairpin tubes

pitch of the tubes is dictated largely by the necessity for cleaning the outside of the bundle if the fluid is likely to give rise to considerable scale. A square pitch is preferred as this permits cleaning round the tubes; the minimum spacing is $1.25d_o$ between centres. If space is at a premium, or if the fluids are

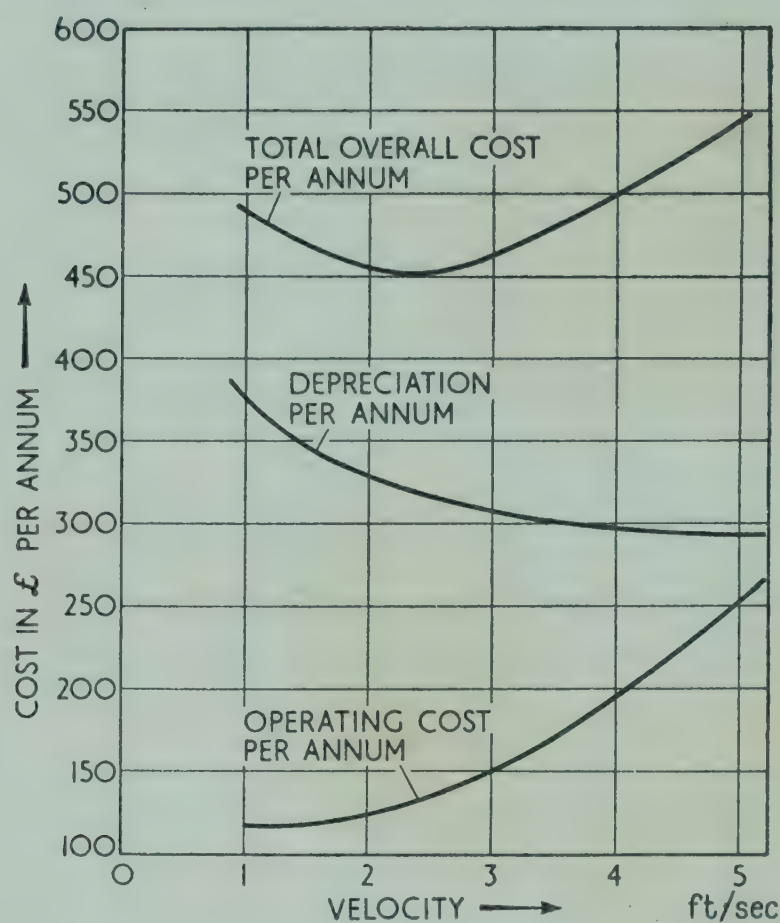


Fig. 6.38. Effect of water velocity on annual operating cost of condenser

very clean, a triangular pitch is used with a $\frac{1}{4}$ in. space between the tubes, as this enables considerably more tubes to be put into a given shell.

Shells. These are commonly made of carbon steel and standard pipes are used for the smaller sizes and rolled welded plate for larger sizes (say 15 to 40 in.). The thickness of the shell can be calculated from the formula for thin

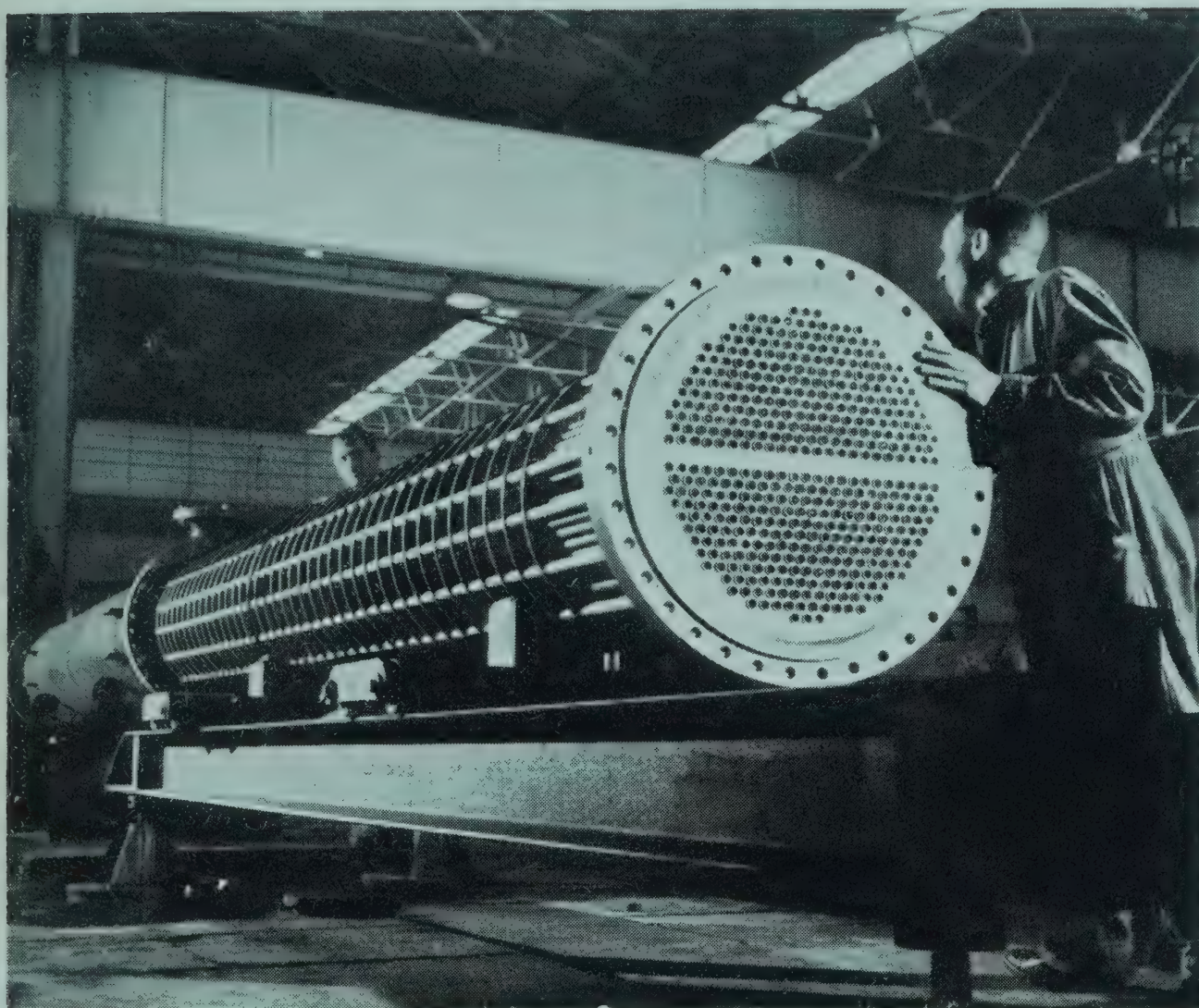


Fig. 6.37. Heat exchanger being assembled
(Courtesy of *W. J. Fraser & Co. Ltd.*)

walled cylinders but a minimum of $\frac{3}{8}$ in. is used for shells over 13 in. outside diameter and $\frac{7}{16}$ in. for shells over 31 in. outside diameter. Unless the exchanger works at very high pressure the calculated thickness will usually be less than these figures but a corrosion allowance of $\frac{1}{8}$ in. is commonly added to all carbon steel parts; the thickness may often be determined more by questions of rigidity than of simple internal pressure.

Baffles and Support Plates for Tubes. As mentioned earlier these are fitted to increase the rate of flow over the tube bundle. The commonest type is the segmental baffle with about 25% cut, as shown in Fig. 6.13. The diameter is governed by the fit of the bundle in the shell but for good work the clearance between the baffle and the inside of the shell should be not more than $\frac{3}{16}$ to $\frac{3}{8}$ in. for vapours, and $\frac{3}{32}$ to $\frac{3}{16}$ in. for liquids. Baffles are not normally spaced closer than one-fifth of the shell diameter; very close spacing is avoided because the increase in heat transfer is then small compared with the increase in pressure drop. Support plates to hold the tubes in position are similar to the baffles but the holes for the tubes are only $\frac{1}{64}$ in. greater than the tube diameter, whereas for baffles the clearance may be twice this. These support plates are fitted at least every 40 in. for $\frac{3}{4}$ in. tubes and every 60 tube diameters for larger tubes.

Tie Rods. In order to keep the tube bundle straight tie rods are fitted to the fixed tube sheet and to the baffle nearest the floating tube sheet. Usually between four and six rods of thickness $\frac{3}{8}$ to $\frac{1}{2}$ in. are necessary. These can be used to hold the baffles and support plates in position if sleeves are fitted over the rods between the baffles.

Tube Sheets. The thickness of the fixed tube sheet is frequently calculated from the relation

$$d_t = d_G \sqrt{\frac{0.25P}{f}}$$

where d_G is the diameter of the gasket on the tube sheet, P the design pressure, f the allowable working stress, and d_t the thickness of the sheet measured at the bottom of the partition baffle grooves. The floating tube sheet may be made $\sqrt{2}$ times as thick.

Flanges and Covers. Where dished ends are used for the cover plates, these are made to the designs supplied by the maker, but details are given in the "Code of the Tubular Exchangers Manufacturers' Association of America" (T.E.M.A.) and in B.S. 1500. Flanges are also designed from one of these codes. Extensive practical details are given in the T.E.M.A. Code which applies particularly to the petroleum industry.

Mean Temperature Difference in Multipass Exchangers

In an exchanger with one shell pass and several tube passes, the fluids in the tubes and shell will flow co-currently in some of the passes and counter-currently in the others. The mean temperature difference for countercurrent flow is

greater than that for parallel flow, so that with the multipass exchanger there is no easy way of finding the true mean temperature difference. This problem has been examined by BOWMAN MUELLER and NAGLE⁽⁴⁴⁾, who have presented a graphical method for finding a correction factor to apply to the simple logarithmic mean temperature difference for these units. Using the nomenclature shown in Figs. 6.39 and 6.40, let

$$X = \frac{\theta_2 - \theta_1}{T_1 - \theta_1} \text{ and } Y = \frac{T_1 - T_2}{\theta_2 - \theta_1}$$

Then for a unit with one shell side pass and an even number of passes on the tube side, the correction factor F is found from Fig. 6.39, where F is given by

$$\Delta T_{\text{true mean}} = \Delta T_{\text{log mean}} \times F \quad \dots (6.88)$$

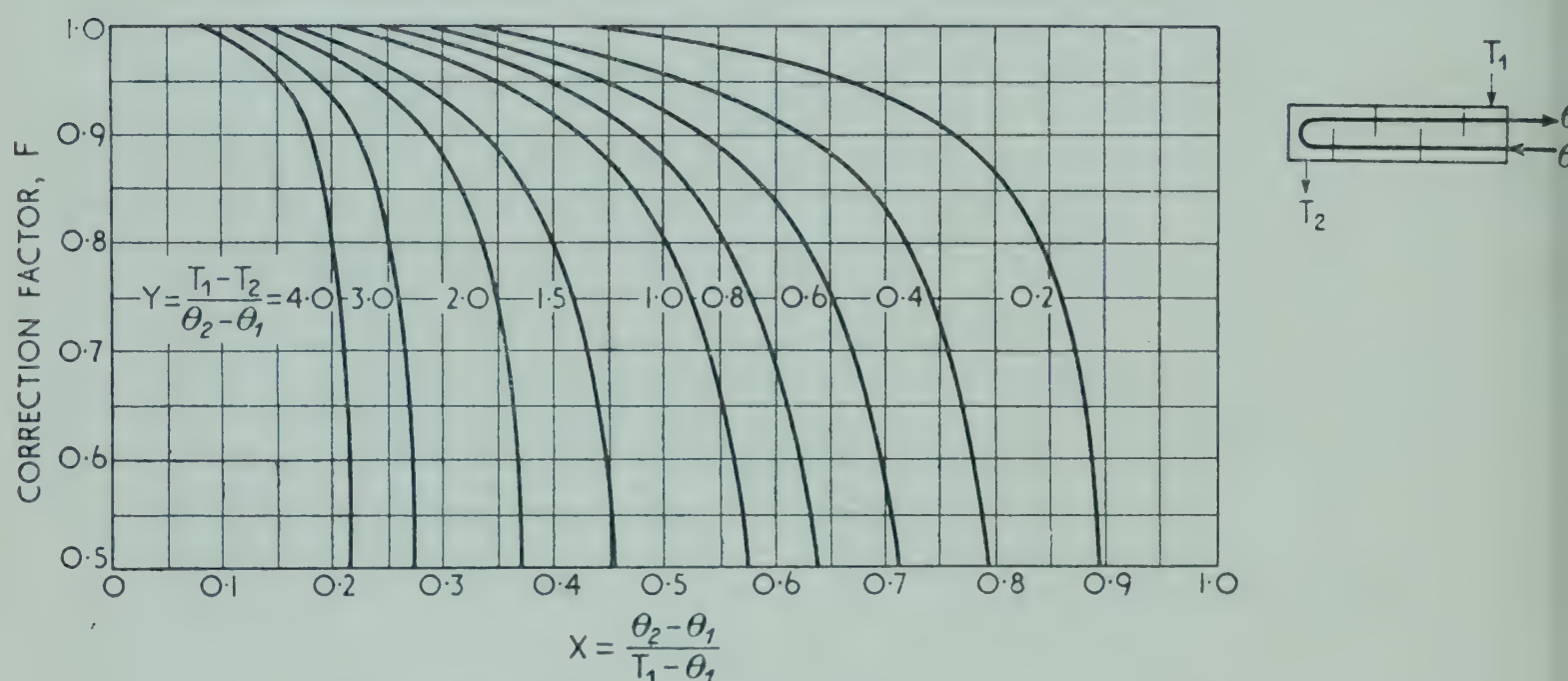


Fig. 6.39. Correction for logarithmic mean temperature difference for single shell pass exchanger

This chart is developed on the assumption that an average temperature can be taken for the shell fluid at any cross section, that there is an equal transfer surface in each pass, that the overall transfer coefficient is constant, that the specific heat of each fluid is constant throughout the exchanger and that no phase change occurs in either fluid: these conditions are not exactly met in commercial units. If F has a value less than 0.75 it is generally desirable to use a different arrangement because the heat transfer area is not being used in the most effective manner. If two shell side passes are provided F is obtained from Fig. 6.40.

Suppose an exchanger is required to operate over the temperature ranges,

$$T_1 = 182^\circ\text{C} \quad T_2 = 99^\circ\text{C}$$

$$\theta_1 = 10^\circ\text{C} \quad \theta_2 = 115^\circ\text{C}$$

Then

$$X = \frac{\theta_2 - \theta_1}{T_1 - \theta_1} = \frac{115 - 10}{182 - 10} = 0.6$$

$$Y = \frac{T_1 - T_2}{\theta_2 - \theta_1} = \frac{182 - 99}{115 - 10} = 0.8$$

If a single shell pass is used, F is found from Fig. 6.39 to be 0.65, which is regarded as too low. If two shell passes are used F is obtained from Fig. 6.40 as 0.95 and a much better heat recovery is obtained.

To obtain maximum heat recovery from the hot fluid, θ_2 must be as high as possible. When θ_2 exceeds T_2 , a temperature cross is said to occur and, with a single shell pass, any further increase in θ_2 will result in a rapid fall in F . With more shell passes higher values of θ_2 can be used and a better recovery is obtained. It will be seen from the curves that if the temperature of one fluid remains constant, as, for instance, with a simple condenser, the correction factor F becomes unity and the logarithmic temperature difference is used.

Shell side baffles are rather difficult to fit and the same effect may be obtained by mounting two exchangers one below the other. This is par-

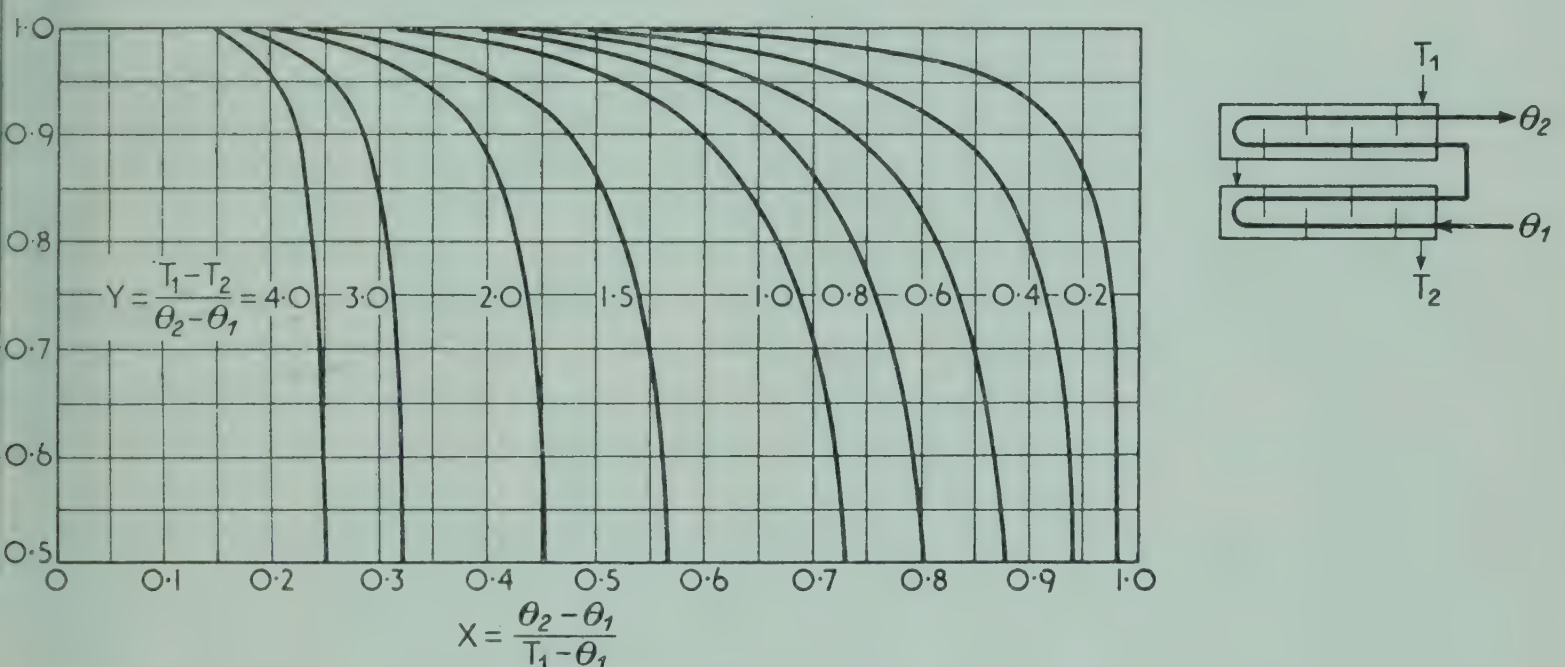


Fig. 6.40. Correction for logarithmic mean temperature difference for double shell pass exchanger

ticularly useful where each unit is large and a single exchanger would be unwieldy.

DETERMINATION OF FILM COEFFICIENTS

If in any heat transfer apparatus the overall temperature difference, the area of surface and the heat transferred are measured directly, the overall transfer coefficient U is obtained from the simple relation, $Q = U \cdot A \cdot \Delta T$. The determination of the individual film coefficients has proved difficult even for the simplest cases, and it is quite common for equipment to be designed on the basis of practical values of U , rather than from a series of film coefficients. For the important case of the transfer of heat from one fluid to another across a metal surface two methods have been developed for measuring the film coefficients. The first requires a knowledge of the temperature difference across each film and therefore involves measuring the temperatures of both fluids and the surface of separation. With a concentric tube system it is very difficult to insert a thermocouple into the thin tube and to prevent the thermocouple wires

from interfering with the flow of the fluid. Nevertheless this method is commonly adopted, particularly when electric heating is used. It must be remembered that, when the heat flux is very high, as with boiling liquids, there will be an appreciable temperature drop across the tube wall and the position of the thermocouple is then important. For this reason working with stainless steel, which has a relatively low value of thermal conductivity, is difficult.

The second method uses a technique put forward by WILSON⁽⁹⁾. Suppose steam to be condensing on the outside of a horizontal tube through which water is passed at various velocities. Then the overall and film transfer coefficients can be related by the equation

$$\frac{1}{U} = \frac{1}{h_o} + \frac{x_w}{k_w} + R_i + \frac{1}{h_i} \quad \text{(from equation 6.80)}$$

if the transfer area on each side of the tube is approximately the same.

For conditions of turbulent flow the transfer coefficient for the water side, $h = \epsilon \cdot u^{0.8}$; R_i the scale resistance is constant, and h_o the coefficient for the condensate film is almost independent of the water velocity. Thus the above equation reduces to

$$\frac{1}{U} = [\text{const.}] + \frac{1}{\epsilon \cdot u^{0.8}} \quad \dots(6.89)$$

and if $1/U$ is plotted against $1/u^{0.8}$ a straight line is obtained with slope $1/\epsilon$ and intercept equal to the value of the constant in equation 6.89. ϵ represents the value of the film coefficient h_i for unit water velocity. For a clean tube R_i should be nil and hence h_o can be found from the value of the intercept.

This technique has been applied by RHODES and YOUNGER^(29a) to obtain the values of h_o for condensation of a number of organic vapours, by PRATT⁽⁵¹⁾ to obtain the inside coefficient for a coiled tube, and by COULSON and MEHTA⁽⁶¹⁾ to obtain the coefficient for an annulus. If the results are repeated over a period of time, R_i can also be obtained by this method.

Typical values of thermal resistances and individual and overall heat transfer coefficients are given in Tables 6.10, 6.11, 6.12 and 6.13.

TABLE 6.10. Thermal resistance of heat exchanger tubes

		Values of x_w/k_w ft ² -hr-°C/lb-cal				
Gauge S.W.G.	Thickness (in.)	Copper	Steel	Stainless steel	Admiralty metal	Aluminium
18	0.049	0.000018	0.00011	0.00047	0.000065	0.000031
16	0.065	0.000024	0.00014	0.00062	0.000086	0.000042
14	0.083	0.000031	0.00018	0.0008	0.00011	0.000053
12	0.109	0.000041	0.00024	0.001	0.00026	0.000070

TABLE 6.11. *Thermal resistances of scale deposits from water, etc.*

	ft ² -hr-°C/ lb-cal		ft ² -hr-°C/ lb-cal
<i>Water</i> (3 ft/sec. velocity, temperatures less than 50°C)		<i>Steam</i>	
Distilled	0.0005	Good quality—oil free .	0.0003
Sea	0.0005	Poor quality—oil free .	0.0005
Clear river	0.0012	Exhaust from reciprocating engines	0.001
Untreated cooling tower .	0.0033	<i>Liquids</i>	
Treated cooling tower .	0.0015	Treated brine	0.0015
Treated boiler feed . . .	0.0015	Organics	0.001
Hard well	0.0033	Fuel oils	0.006
		Tars	0.01
		<i>Gases</i>	
		Air	0.0015 to 0.003
		Solvent vapours	0.0008

Table 6.12. *Approximate overall heat transfer coefficients U for shell and tube equipment*
(lb-cal/hr-ft²-°C)*Condensers*

Hot side	Cold side	Overall U
Steam (pressure)	Water	350 to 750
Steam (vacuum)	Water	300 to 600
Saturated organic solvents (atmospheric)	Water	100 to 200
Saturated organic solvents (vacuum some non-condensable) .	Water-brine	50 to 120
Organic solvents (atmospheric and high non-condensable) .	Water-brine	20 to 80
Organic solvents (vacuum and high non-condensable) . . .	Water-brine	10 to 50
Low boiling hydrocarbons (atmospheric)	Water	80 to 200
High boiling hydrocarbons (vacuum)	Water	10 to 30

Heaters

Hot side	Cold side	Overall U
Steam	Water	250 to 750
Steam	Light oils	50 to 150
Steam	Heavy oils	10 to 80
Steam	Organic solvents	100 to 200
Steam	Gases	5 to 50
Dowtherm	Gases	4 to 40
Dowtherm	Heavy oils	8 to 60

Evaporators

Hot side	Cold side	Overall U
Steam	Water	350 to 750
Steam	Organic solvents	100 to 200
Steam	Light oils	80 to 180
Steam	Heavy oils (vacuum)	25 to 75
Water	Refrigerants	75 to 150
Organic solvents	Refrigerants	30 to 100

Table 6.12. (contd.)

Heat Exchangers (no change of state)

Hot side	Cold side	Overall <i>U</i>
Water	Water	150 to 300
Organic solvents	Water	50 to 150
Gases	Water	3 to 50
Light oils	Water	60 to 160
Heavy oils	Water	10 to 50
Organic solvents	Light oil	20 to 70
Water	Brine	100 to 200
Organic solvents	Brine	30 to 90
Gases	Brine	3 to 50
Organic solvents	Organic solvents	20 to 60
Heavy oils	Heavy oils	8 to 50

Table 6.13. Approximate film coefficients
(*h_i* or *h_o*)

	Approximate film coefficients (lb-cal/hr-ft ² -°C)
<i>No Change of State</i>	
Water	300 to 2000
Gases	3 to 50
Organic solvents	60 to 500
Oils	10 to 120
<i>Condensing</i>	
Steam	1000 to 3000
Organic solvents	150 to 500
Light oils	200 to 400
Heavy oils (vacuum)	20 to 50
Ammonia	500 to 1000
<i>Evaporation</i>	
Water	800 to 2000
Organic solvents	100 to 300
Ammonia	200 to 400
Light oils	150 to 300
Heavy oils	10 to 50

Example. 60,000 lb/hr of pure isobutane are to be condensed at a temperature of 58·5°C, in a horizontal tubular exchanger using water at an inlet temperature of 28°C. It is proposed to

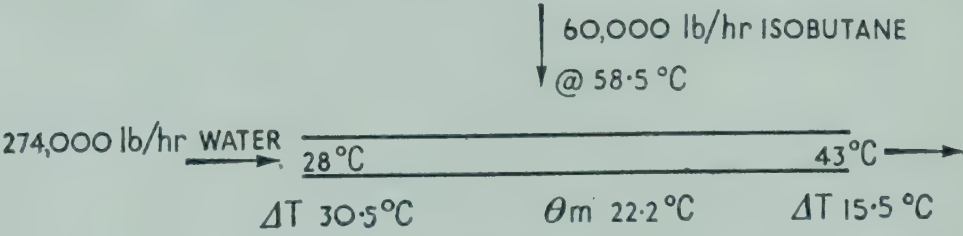


Fig. 6.41

use $\frac{3}{4}$ in. outside diameter tubes of 16 B.W.G. arranged on a 1 in. triangular pitch. Under the conditions the resistance of the scale may be taken as 0·003 ft²-hr-°C/lb-cal. It is required to determine the number and arrangement of the tubes in the shell (Fig. 6.41).

Solution. The latent heat of vaporization of the isobutane is 68·5 lb-cal/lb, so that the heat load on the exchanger is

$$Q = 60,000 \times 68\cdot5 = 4,100,000 \text{ lb-cal/hr}$$

It is undesirable for the outlet temperature of the cooling water to exceed 50°C and a figure of 43°C will be used. The water required is then $4,100,000 \div 15$ or 273,300 lb/hr. We shall take a figure of 274,000 lb/hr.

In the first place we shall obtain an idea of the size of the unit by assuming that the overall transfer coefficient based on the outside area of the tubes will be 90 lb-cal/hr-ft²-°C.

The temperatures as shown in Fig. 6.41 are,

Shell side	In 58.5°C	Out 58.5°C	
Tube side	In 28°C	Out 43°C	Mean 35.5°C
ΔT	30.5°C	15.5°C	
θ_m	22.2°C		

The approximate area of surface required is then given by,

$$A = \frac{4,100,000}{90 \times 22.2} = 2050 \text{ ft}^2$$

The tubes have an outside surface of 0.1963 ft²/ft length, so the total length of tube is $2050/0.1963 = 10,500$ ft.

Using 16-ft tubes the number of tubes required is 660. With this large quantity of water it will be possible to use a four pass exchanger and still have a reasonable water velocity. In a shell of 31 in. inner diameter, 678 tubes can be arranged in four passes using a 1-in. triangular pitch and this size will be selected. (Tables showing the number of tubes that can be fitted to a given size shell are usually made up in a design office.) This calculation has given a good indication of the probable size of the unit and it is now possible to check the result more closely by calculating the individual film coefficients.

Calculation of Inside and Outside Film Coefficients

(a) Inside coefficient h_i .

$$\text{Water flow through each tube} = \frac{274,000}{678 \div 4} = 1615 \text{ lb/hr.}$$

The cross sectional area of the tube is 0.3019 in.² and $d = 0.62$ in.

$$\text{The water velocity is then, } \frac{1615 \times 144}{0.3019 \times 62.3 \times 3600} = 3.45 \text{ ft/sec.}$$

$$\begin{aligned} h_i &= 200 (1 + 0.015 \times 35.5) \frac{3.45^{0.8}}{0.62^{0.2}} \text{ (from equation 6.33)} \\ &= 200 \times \frac{1.53 \times 2.70}{0.908} \\ &= 920 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C} \\ &= 920 \times \frac{0.62}{0.75} = 760 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C based on outside diameter} \end{aligned}$$

(b) Outside coefficient h_o .

To find the temperature drop ΔT_f across the condensate film, we have,

$$\begin{aligned} \frac{1/760 + 0.003}{1/90} &= \frac{\text{thermal resistance, water film + scale}}{\text{total thermal resistance}} \\ &= \frac{\theta_m - \Delta T_f}{\theta_m} \end{aligned}$$

whence

$$\begin{aligned}\Delta T_f &= 22.2 - 22.2 \left[\frac{1/760 + 1/330}{1/90} \right] \\ &= 22.2 - 22.2 \left(\frac{1090 \times 90}{760 \times 330} \right) \\ &= 22.2 - 22.2 \times 0.39 \\ &= 22.2 - 8.7 = 13.5^\circ\text{C}\end{aligned}$$

The condensate film temperature will then be $58.5 - 13.5 = 45^\circ\text{C}$.

From equation 6.69a

$$h_o = 0.72 \left[\frac{k^3 \rho^2 g \lambda}{j d_o \mu \Delta T_f} \right]^{0.25}$$

Taking $k = 0.075$ lb-cal/hr-ft²-°C, ρ as 31.7 lb/ft³, j as $\sqrt{678} = 26$, and μ as 0.136 centipoises

$$\begin{aligned}h_o &= 0.72 \left[\frac{0.075^3 \times 31.7^2 \times 4.17 \times 10^8 \times 68.5}{26 \times 0.0625 \times 0.136 \times 2.42 \times 13.5} \right]^{0.25} \\ &= 0.72 \times 202 \\ &= \text{approx. } 150 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C}\end{aligned}$$

Hence the overall coefficient based on the outside area can be found as,

$$\begin{aligned}\frac{1}{U} &= \frac{1}{760} + \frac{1}{150} + \frac{1}{330} \\ U &= 91 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C}.\end{aligned}$$

This is sufficiently near the assumed value. The total heat capacity of the condenser under these conditions will be $U \cdot A \cdot \theta_m = 91 \times 16 \times 678 \times 0.1963 \times 22.2$ or 4,300,000 lb-cal/hr.

It should be noticed that this is not a complete solution, since the exit water temperature and the optimum velocity should be determined on economic grounds but it serves to show the general method of approach.

Example. 300,000 lb/hr of a crude oil are to be heated from 70 to 136°F by heat exchange with the bottom product from a distillation unit. The product at 257,500 lb/hr is to be cooled

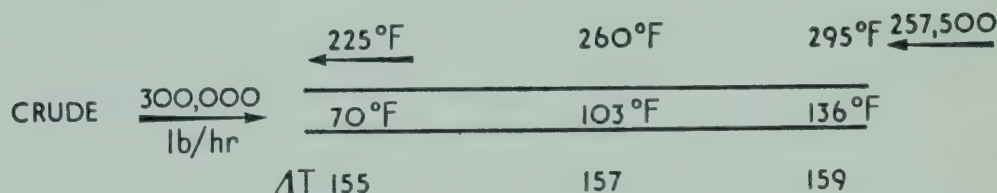


Fig. 6.42

from 295 to 225°F. There is available a tubular exchanger with an inside shell diameter of 23½ in., having one pass on the shell side and two passes on the tube side. It has 324 tubes ¾ in. outside diameter by 14 B.W.G. and 12 ft long, arranged on a 1 in. square pitch and supported by baffles with a 25 per cent cut, spaced at 9 in. intervals (Fig. 6.42).

Would this exchanger be suitable?

Solution. The flow arrangement, terminal and mean temperatures of the streams are as shown in the diagram, Fig. 6.42.

For the crude at 103°F, $C_p = 0.475$ B.Th.U./lb-°F

$\mu = 2.9$ centipoises

$k = 0.0789$ B.Th.U./ft-hr-°F

$\rho = 51.5$ lb/ft³

For the product at 260°F, $C_p = 0.525$ B.Th.U./lb-°F

$$\begin{aligned}\text{Heat transferred on the tube side } Q &= 300,000 \times 66 \times 0.475 \\ &= 9,400,000 \text{ B.Th.U./hr}\end{aligned}$$

$$\begin{aligned}\text{Heat transferred on the shell side } Q &= 257,500 \times 70 \times 0.525 \\ &= 9,400,000 \text{ B.Th.U./hr}\end{aligned}$$

Shell Side Coefficient h_o

$$\text{Temperature of pipe wall} \approx \frac{260 + 103}{2} = 181.5^\circ\text{F}$$

$$\text{Film temperature, } T_f = \frac{260 + 181.5}{2} = \frac{441.5}{2} = (\text{say}) 220^\circ\text{F}$$

$$\text{At } T_f, \rho = 54.1 \text{ lb/ft}^3$$

$$\mu = 5.2 \text{ centipoises} = 12.6 \text{ lb/ft-hr}$$

$$k = 0.069 \text{ B.Th.U./hr-ft-}^\circ\text{F}$$

$$\begin{aligned}\text{Available cross flow area} &= \frac{\text{shell inner diameter}}{\text{pitch}} \times \text{clearance} \times \text{baffle spacing} \\ &= \frac{23.25}{1} \times \frac{\frac{1}{4} \times 9}{144} \\ &= 0.363 \text{ ft}^2\end{aligned}$$

$$\text{Hence, } G'_{\max} = \frac{257,500}{0.363} = 709,000 \text{ lb/hr-ft}^2$$

$$\text{and } Re_{\max} = \frac{3}{48} \times \frac{709,000}{12.6} = 3510$$

$$\therefore (Re_{\max})^{0.6} = 3510^{0.6} = 134$$

$$Pr^{0.3} = \left(\frac{0.525 \times 12.6}{0.069} \right)^{0.3} = (95.8)^{0.3} = 3.93$$

From equation 6.41 and taking C_h equal to unity,

$$h_o \cdot \frac{3}{48} \times \frac{1}{0.069} = 0.33 \times 134 \times 3.93$$

$$\therefore h_o = 174 \text{ B.Th.U./hr-ft}^2\text{-}^\circ\text{F}$$

Tube Side Coefficient h_i

$$\text{Number of tubes per pass} = 162$$

$$\text{Cross sectional area of tube} = 0.2678 \text{ in.}^2$$

$$\text{Mass flow } G' = \frac{300,000 \times 12^2}{162 \times 0.2678} = 995,000 \text{ lb/hr-ft}^2$$

$$Re = \frac{d \cdot G'}{\mu} = \frac{0.584 \times 995,000}{12 \times 2.9 \times 2.42} = 6880 \text{ (taking } d = 0.584 \text{ in.)}$$

$$\therefore Re^{0.8} = 1175$$

$$Pr^{0.4} = \left[\frac{0.475 \times 2.9 \times 2.42}{0.0789} \right]^{0.4} = 42.2^{0.4} = 4.47$$

From equation (6.31),

$$h_i \times \frac{0.584}{12 \times 0.0789} = 0.023 \times 1175 \times 4.47$$

$$\therefore h_i = 197 \text{ B.Th.U./hr-ft}^2\text{-}^\circ\text{F}$$

$$\text{Based on outside area, } h_i = \frac{0.584}{0.75} \times 197 = 154 \text{ B.Th.U./hr-ft}^2\text{-}^\circ\text{F}$$

Neglecting the thermal resistance of the wall and scale, the clean overall coefficient based on the outside area of the tubes is

$$\begin{aligned}\frac{1}{U} &= \frac{1}{154} + \frac{1}{174} \\ &= 0.00650 + 0.00580 = 0.0123\end{aligned}$$

The outside surface area available $A = 324 \times 12 \times 0.1963 = 763 \text{ ft}^2$.

When in operation scale would form and the operating overall transfer coefficient would have to be not less than U_d , given by

$$\frac{1}{U_d} = \frac{A \cdot \Delta T}{Q} = \frac{763 \times 157}{9,400,000} = 0.0127$$

The permissible thermal resistance of the scale is then given by

$$R = \frac{1}{U_d} - \frac{1}{U} = 0.0127 - 0.0123 = 0.0004 \text{ hr-ft}^2\text{-}^\circ\text{F/B.Th.U.}$$

This value of scale resistance is very low, as seen from Table 6.11, and the exchanger, if installed, would not give the required temperatures without frequent cleaning.

Note. With $\frac{3}{4}$ in. tubes 14 B.W.G. the surface per foot length on the outside is 0.1963 ft^2 , and the inside area is 0.2678 in.^2

SPECIAL FORMS OF EQUIPMENT

A number of special types of equipment have been developed to give improved heat transfer under conditions where the standard forms of tubular units are unsatisfactory. Of these the finned tube units and the plate type exchangers will be taken as illustrations.

Finned Tube Units

When viscous liquids are heated in a concentric tube or tubular exchanger by condensing steam or by a hot liquid of low viscosity, the film coefficient for the viscous liquid will be much smaller than that on the other side and will therefore control the rate of heat transfer. Examples where one fluid offers a much greater heat transfer resistance than the other occur in the heating of oils in storage or in pipes and in air heating. With low flow rates it may be impossible to reach turbulent conditions in a simple unit whatever the properties of the fluid, and low rates of heat transfer will be obtained.

It is often possible to obtain a better performance by increasing the surface exposed to the fluid offering the high resistance. This may be done by fitting fins to the outside of a tube, as shown in Figs. 6.43 and 6.44, which show two typical forms of finned tubes, one with longitudinal fins along the length of the tube and the other with transverse fins.

The calculation of film coefficients on the outside of these surfaces is complex because each unit of surface on the fin is less effective than a unit of surface on the outside of the tube. When the fluid on the fin side is being cooled the heat must be transferred first to the fin and then along the fin to the tube wall. There will therefore be a temperature gradient along the fin from the tip to the tube and the temperature difference between the fluid and the fin will therefore

increase from tip to base. The calculation of the true mean temperature of the surface either for cooling or heating the viscous fluid is therefore very complex, and although some equations have been obtained for these conditions, it is generally more convenient and reliable to make use of empirical relations obtained from measurements on the particular form of surface proposed.

A neat form of construction by the Brown Fintube Company of America is illustrated in Fig. 6.43. On both prongs of a hairpin tube are fitted horizontal fins which fit inside concentric tubes, joined at the base of the hairpin as shown. Units of this form can conveniently be arranged in banks to give large heat transfer surfaces. It is usual for the extended surface to be at least five times greater than the inside surface, so that the low coefficient on the fin side is balanced by the increase in surface. An indication of the surface obtained is given in Table 6.14.

Table 6.14. Data on surface of finned tube units*

Pipe size outside diameter (in.)	Outside surface of pipe (ft ² /ft length)	Surface of finned pipe (ft ² /ft)		
		Number of fins	Height of fin	
			½ in.	1 in.
1	0.262	12	1.262	2.262
		16	1.595	2.929
		20	1.927	3.595
1.9	0.497	20	2.164	3.830
		24	2.497	4.497
		28	2.830	5.164
		36	3.498	6.497

* Brown Fintube Company.

A typical hairpin unit, with an effective surface on the fin side of 101 ft², has an overall length of 21½ ft, height of 13½ in. and width of 8 in. The free area for flow on the fin side is 4.1 in.², against 2.04 in.² on the inside; the ratio of the transfer surface on the fin side to that inside the tubes is 5.93 : 1.

The fin side film coefficient h_f has been expressed by plotting

$$\frac{h_f}{C_p G'} \cdot \left(\frac{C_p \mu}{k}\right)^{2/3} \cdot \left(\frac{\mu}{\mu_s}\right)^{-0.14} \text{ against } \frac{d_e \cdot G'}{\mu}$$

where h_f is based on the total finside surface area (fin and tube),

G' is the mass rate of flow per unit area,

and d_e is the equivalent diameter

$$d_e = \frac{4 \times \text{cross sectional area for flow on fin side}}{\text{total wetted perimeter for flow (fin + outside of tube + inner surface of shell tube)}}$$

Experimental work has been carried out with exchangers in which the inside tube was 1.9 in. outside diameter and was fitted with 24, 28 or 36 fins (½ in. by 0.035 in.) in a 20 ft length; the finned tubes were inserted inside tubes

3½ in. outside diameter. With steam on the tube side, and lube oils and kerosene on the fin side, the experimental data were well correlated by plotting $\frac{h_f}{C_p G'} \left(\frac{C_p \mu}{k} \right)^{2/3} \left(\frac{\mu}{\mu_s} \right)^{-0.14}$ against $\frac{d_e G'}{\mu}$ (see page 254); typical values were:

$$\frac{h_f}{C_p G'} \left(\frac{C_p \mu}{k} \right)^{2/3} \left(\frac{\mu}{\mu_s} \right)^{-0.14} = 0.25 \quad 0.055 \quad 0.012 \quad 0.004$$

$$\frac{d_e G'}{\mu} = 1 \quad 10 \quad 100 \quad 1000$$

Transverse Fins. Some indication of the performance obtained with transverse finned tubes is given in Table 6.15. The figures show the heat transferred (lb-cal) per unit length of pipe when heating air on the fin side with steam or hot water on the tube side, using a temperature difference of 100°C. The results are given for three different spacings of the fins.

Table 6.15. Data on finned tubes*

Inside diam of tube	¾ in.	1 in.	1½ in.	2 in.	3 in.
Outside diam of fin	2½ in.	2¾ in.	3⅞ in.	4 ⅝ in.	5½ in.
No. of fins/ft run					
20	270	360	560	620	800
24	280	370	590	650	830
30	315	400	660	720	920

* Data taken from catalogue of G. A. Harvey and Co. Ltd. of London.

Plate Type Exchangers

A series of plate type heat exchangers which present some special features has been developed by the A.P.V. Company of London. The general construction is shown in Fig. 6.45, from which it is seen that the exchangers consist of a series of parallel plates held firmly together between two substantial head frames. The plates are one piece pressings, frequently of stainless steel, and are spaced by rubber sealing gaskets cemented into a channel around the edge of each plate. Each plate has a number of troughs pressed out at right angles to the direction of flow and arranged so that they interlink with each other to form a channel of constantly changing direction and section. With normal construction the gap between the plates is 0.05 to 0.06 in. Each liquid flows in alternate spaces and a large surface can be obtained in a small volume.

Because of the shape of the plates, the developed area of surface is appreciably greater than the projected area; this is shown below for the four common sizes of plate.

Plate type	Projected area	Developed area
	ft ²	ft ²
H.T.	1.00	1.35
H.X.	1.45	1.81
H.M.	2.88	3.73
H.F.	3.85	4.60

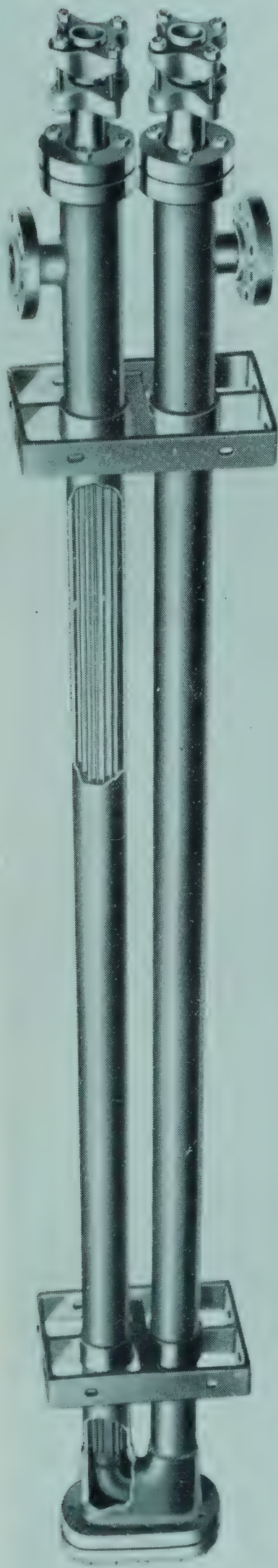


Fig. 6.43. Heat exchanger showing tubes with longitudinal fins
(Courtesy of *The Brown Fintube Co.*)

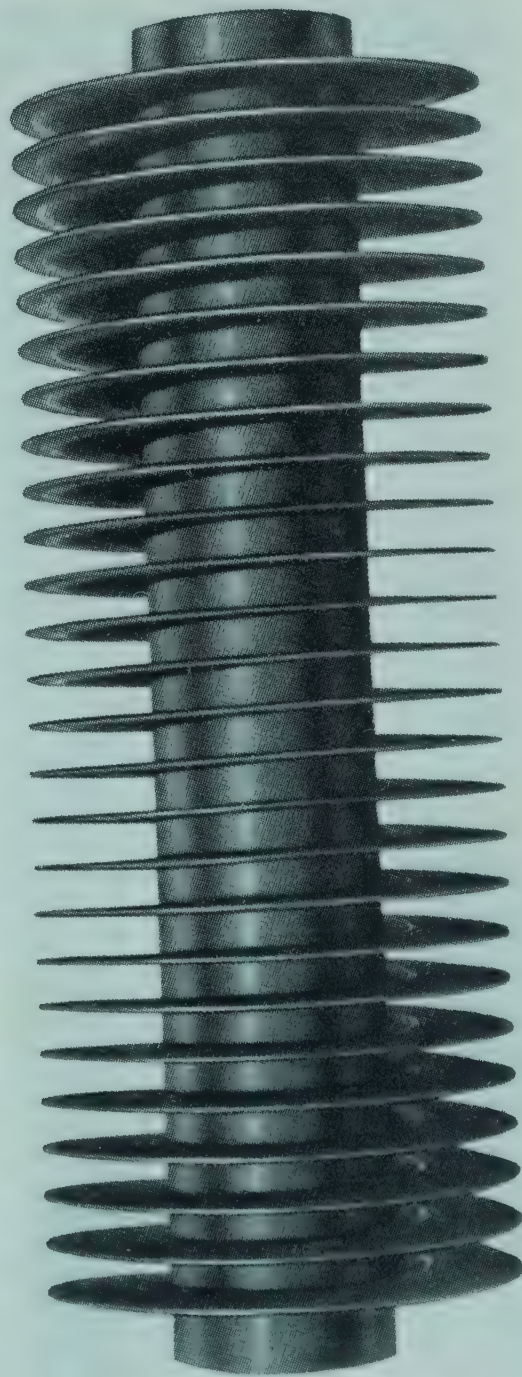
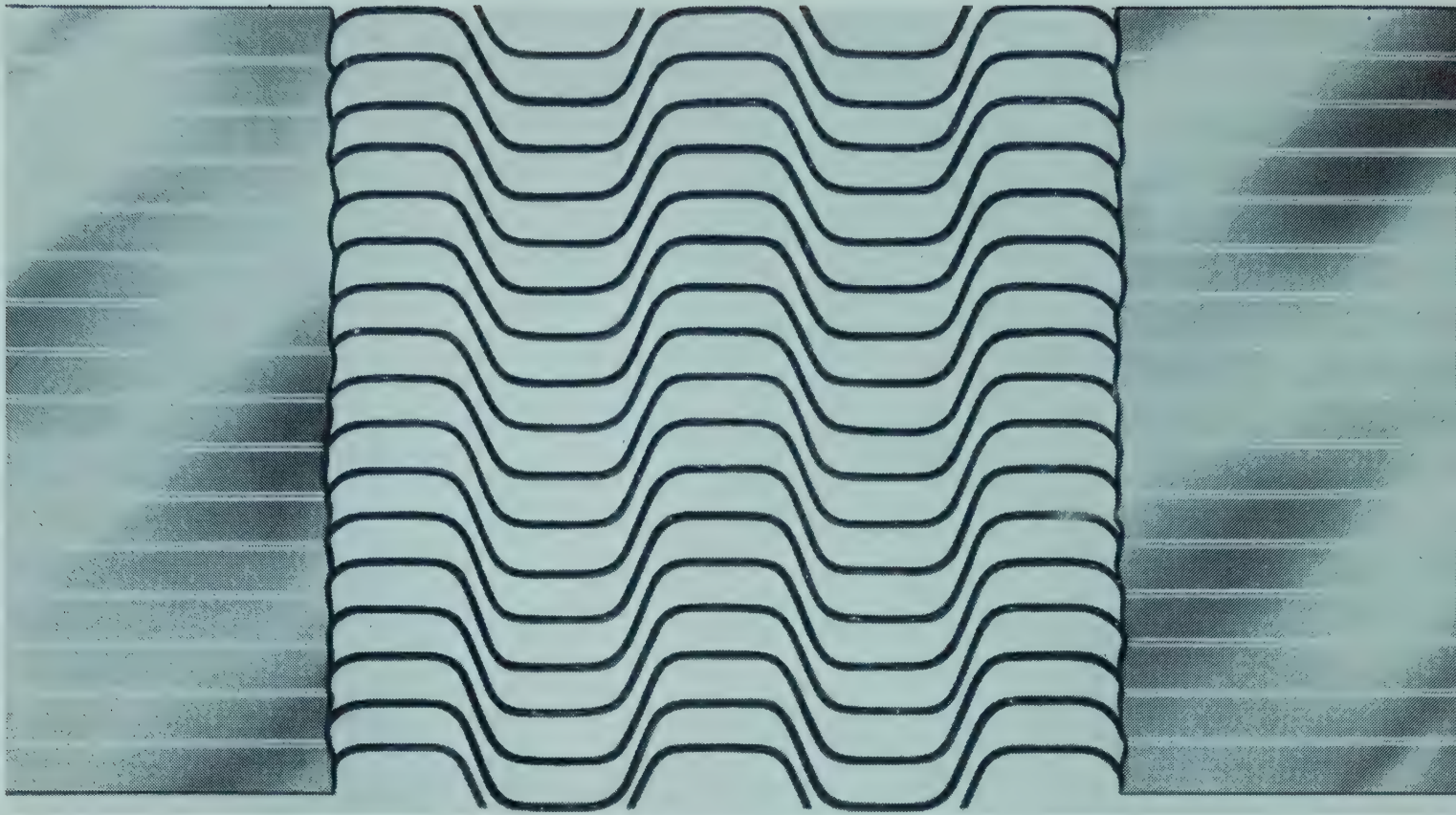
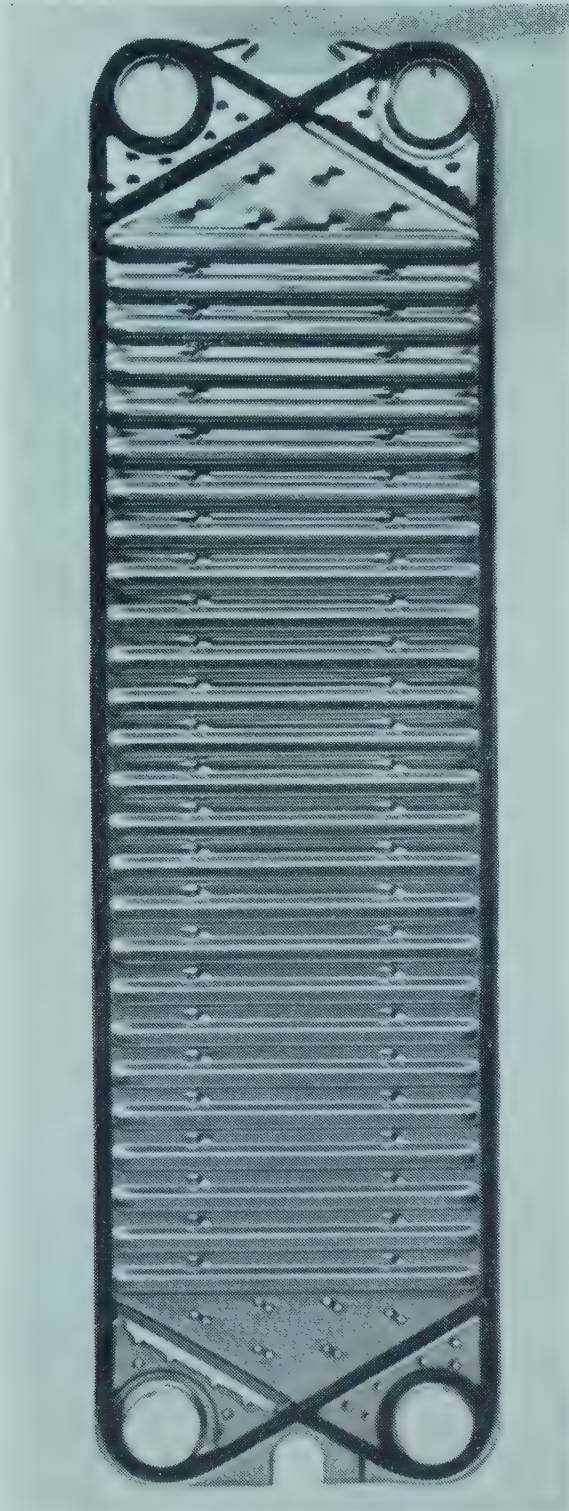


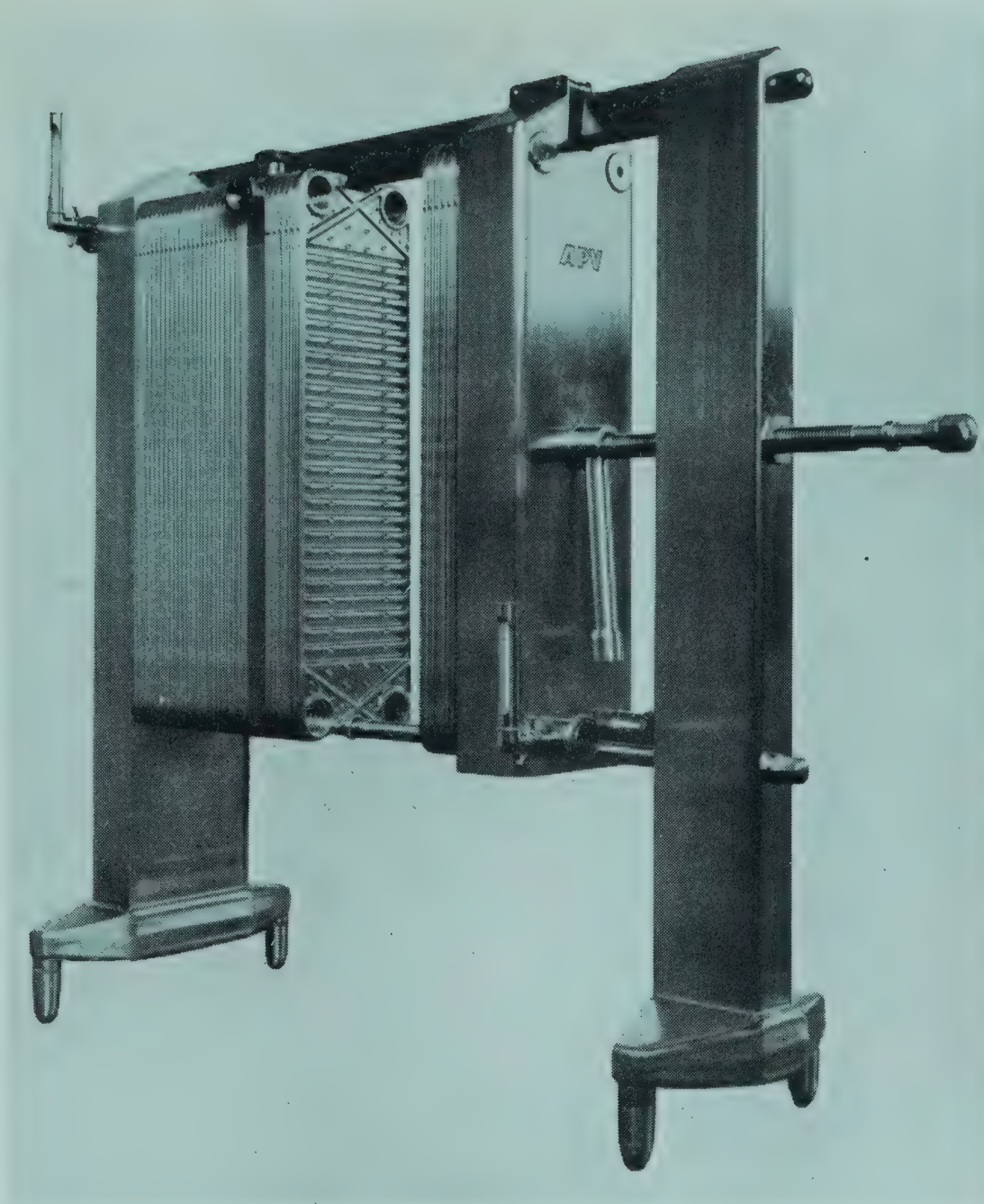
Fig. 6.44. Tube with radial fins
(Courtesy of *G. A. Harvey & Co. Ltd. London*)



(a)



(b)



(c)

Fig. 6.45. Plate heat exchanger

- (a) Shape of flow spaces.
- (b) Plate.
- (c) Assembly view with head withdrawn.

(Courtesy of *The A.P.V. Co. Ltd.*)

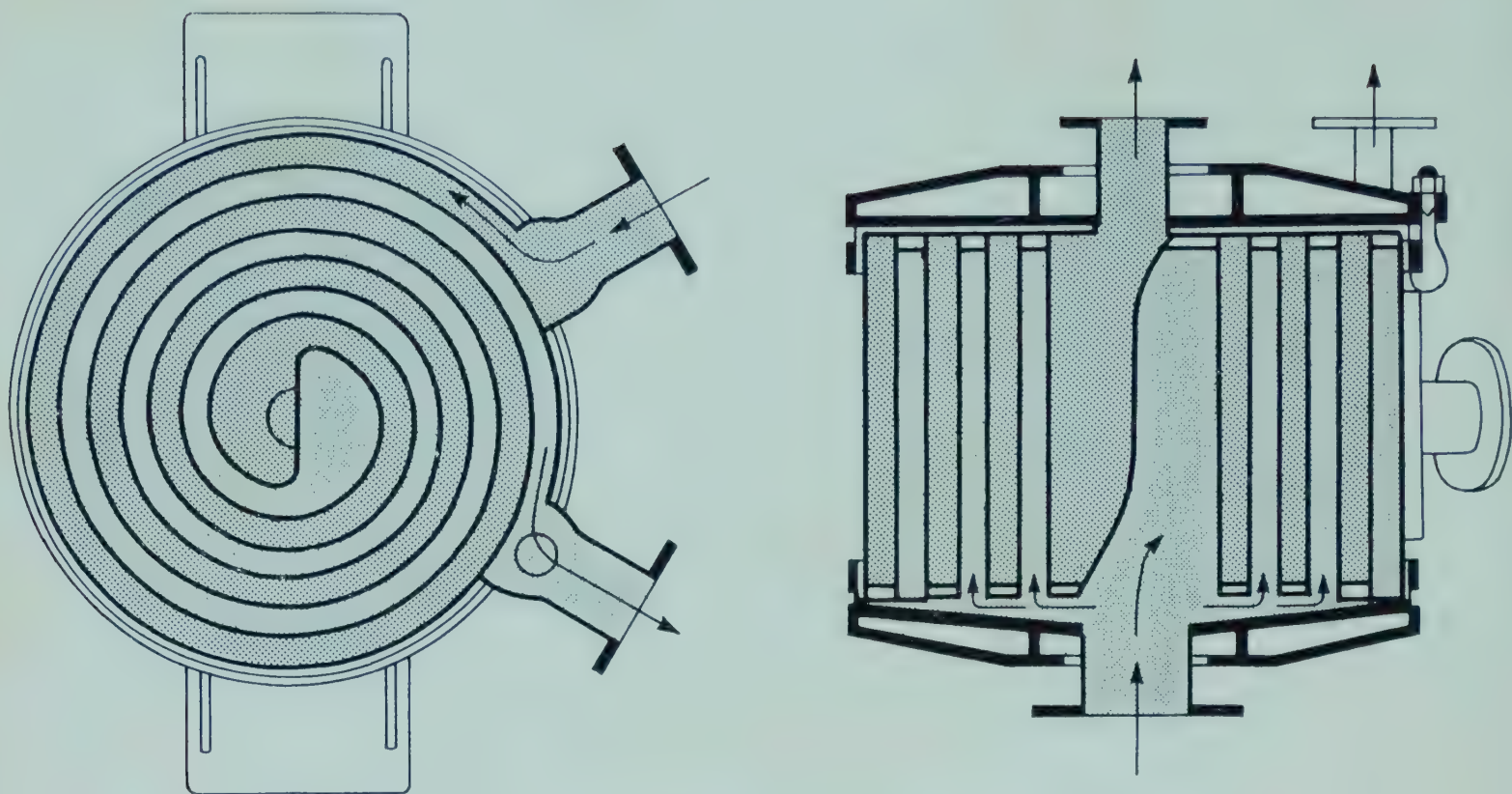


Fig. 6.46. Spiral plate heat exchanger

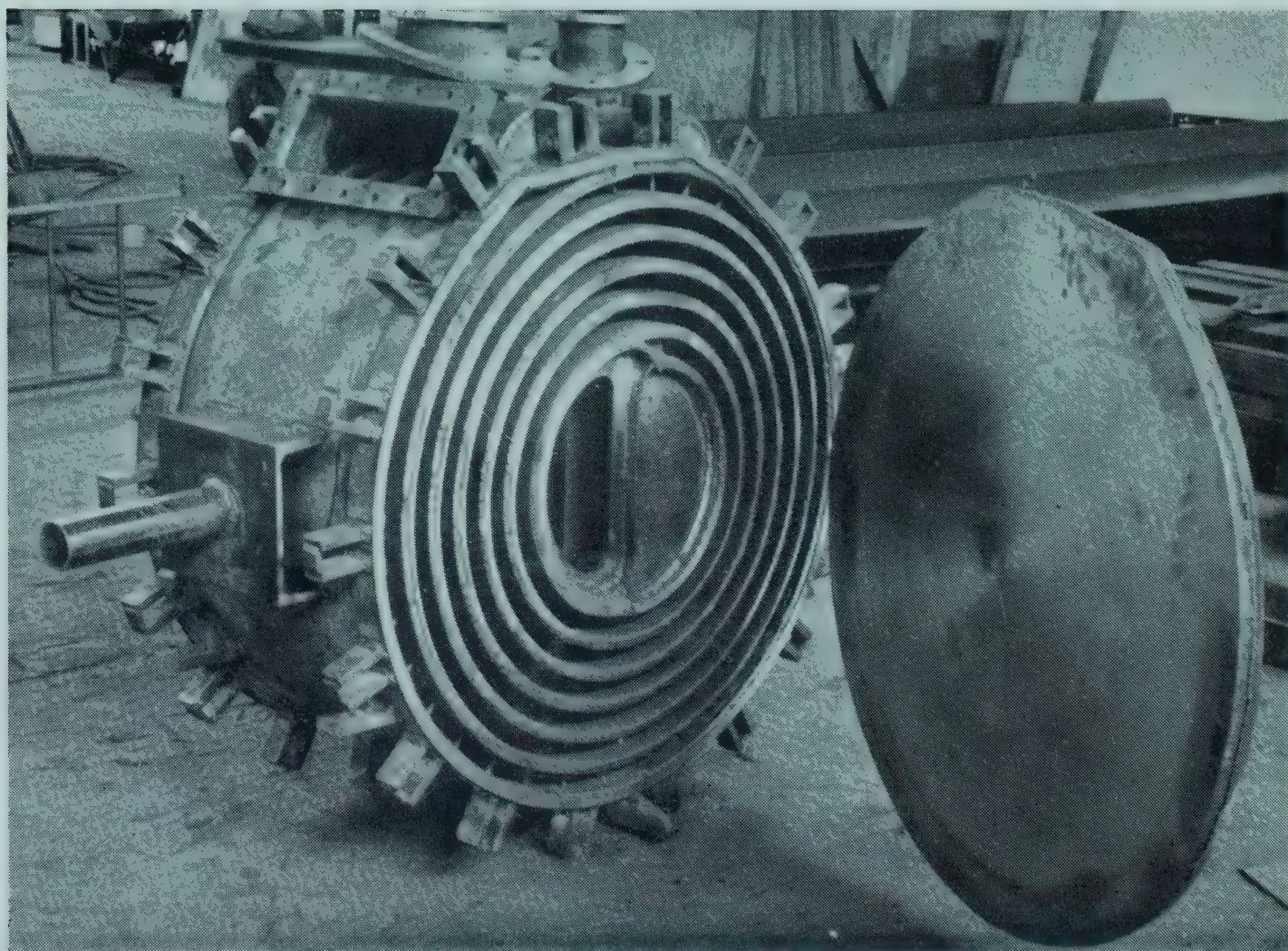


Fig. 6.47. Spiral plate exchanger with cover removed
(Courtesy of Ashmore, Benson, Pease and Co. Ltd.)

A high degree of turbulence is obtained even at low flow rates and the high heat transfer coefficients obtained are illustrated by the data in Table 6.16: these refer to the heating of cold water by an equal flow of hot water in an H.F. type exchanger (aluminium or copper), at an average temperature of 100°F.

Table 6.16. *Performance of plate type exchanger.*
*Type H.F.**

Heat transferred per plate (lb-cal/hr-°C)	Water flow (gal/hr)	<i>U</i> based on developed area (lb-cal/hr-ft ² -°C)
3000	550	650
4000	850	870
5000	1250	1080

* Courtesy of Mr. GOODMAN of the A.P.V. Company.

Using a stainless steel plate with a flow of 900 gal/hr, the heat transferred is 3000 lb-cal per degree centigrade temperature difference for each plate.

The high transfer coefficient enables these exchangers to be operated with very small temperature difference, so that a high heat recovery is obtained. The ratio of temperature rise to temperature difference may be as high as 0.85. These units have been particularly successful in the dairying and brewing industries, where the low liquid capacity, and the close control of temperature have been valuable features. A further advantage is that they are easily dismantled for inspection of the whole plate. The necessity for the long gasket is an inherent weakness but the exchangers have been worked successfully up to 150°C and at pressures of 120 lb/in.² They are now being used in the chemical and gas industries, with solvents, sugar, acetic acid, ammoniacal liquor, etc.

The Rosenblad Spiral plate type exchanger is illustrated in Figs. 6.46 and 6.47. Here two fluids flow countercurrently through the channels formed between the spiral plates. With this form of construction the velocity may be as high as 7 ft per sec. and overall transfer coefficients of 500 lb-cal/hr-ft²-°C are frequently obtained. The size can therefore be kept relatively small and the cost becomes comparable or even less than that of tubular units, particularly when they are fabricated from alloy steels.

THERMAL INSULATION

A hot reaction or storage vessel or a steam pipe will lose heat to the atmosphere by radiation, conduction, and convection. The loss by radiation is a function of the fourth power of the absolute temperatures of the body and surroundings, and will be small for low temperature differences but will increase rapidly as the temperature difference increases. Air is a very poor conductor and heat loss by conduction will therefore be small. On the other hand, since convection currents form very easily, the heat loss from an unlagged surface is considerable. The

conservation of heat, and hence usually of steam and coal, is an economic necessity and some form of lagging should normally be applied to hot surfaces. In furnaces, as we have already seen, the surface temperature is reduced substantially by using a series of insulating bricks which are poor conductors.

The two main requirements of a good lagging material are that it should have a low thermal conductivity and that it should suppress convection currents. The materials that are frequently used are cork, asbestos, 85 per cent magnesia, glass wool and vermiculite. Cork is a very good insulator but it becomes charred at moderate temperatures; it is mainly used in refrigerating plants. 85 per cent magnesia with asbestos, and asbestos itself, are very widely used for lagging steam pipes. 85 per cent magnesia is probably the most widely used and may be applied either as a hot plastic material or in preformed sections. The preformed sections are quickly fitted and can frequently be dismantled and re-used whereas the plastic material must be applied to a hot surface and cannot be re-used. Thin sheeting is often used to protect the lagging.

The rate of heat loss per unit area is given by $\frac{\text{total temperature difference}}{\text{total thermal resistance}}$.

For the case of heat loss to the atmosphere from a lagged steam pipe, the thermal resistance is due to that of the condensate film and dirt on the inside of the pipe, that of the pipe wall, that of the lagging, and that of the air film outside the lagging. Thus for unit length of a lagged pipe we can write

$$\frac{Q}{l} = \frac{\Sigma \Delta T}{\frac{1}{h_i \cdot \pi \cdot d} + \frac{x_w}{k_w \pi d_w} + \frac{x_l}{k_l \cdot \pi \cdot d_m} + \frac{1}{(h_r + h_c) \pi \cdot d_s}} \quad \dots (6.90)$$

where d is the inside diameter of pipe,

d_w is the mean diameter of pipe wall,

d_m is the logarithmic mean diameter of lagging,

d_s is the outside diameter of lagging,

x_w, x_l are the pipe wall and lagging thickness respectively,

k_w, k_l are the thermal conductivity of the pipe wall and lagging,

and h_i, h_r, h_c are the inside film, radiation, and convection coefficients.

Example. A 6 in. internal diameter steam pipe is carrying steam at 171°C and is lagged with 2 in. of 85 per cent magnesia. What will be the heat loss to the air at 21°C?

Solution. Here $d = 6$ in.

$$d_o = 6.625 \text{ in.}$$

$$d_w = 6.312 \text{ in.}$$

$$d_s = 10.625 \text{ in.}$$

$$d_m = \log \text{ mean of } d_o \text{ and } d_s = 8.47 \text{ in.}$$

The coefficient for condensing steam together with that for any scale will be taken as 1500 lb-cal/hr-ft²-°C, k_w as 26, and k_l as 0.042 lb-cal/hr-ft-°C. The temperature on the outside of the lagging will be estimated as 40°C and $h_r + h_c$ will be taken as 1.8 lb-cal/hr-ft²-°C.

The thermal resistances are then obtained as,

$$\begin{aligned}\frac{1}{h_i \cdot \pi \cdot d} &= \frac{1}{1500 \times \pi \times 6/12} = 0.00042 \\ \frac{x_w}{k_w \cdot \pi \cdot d_w} &= \frac{0.3125/12}{26 \times \pi \times 6.312/12} = 0.00060 \\ \frac{x_l}{k_l \cdot \pi \cdot d_m} &= \frac{2/12}{0.042 \times \pi \times 8.47/12} = 1.78 \\ \frac{1}{(h_r + h_c) \cdot \pi \cdot d_s} &= \frac{1}{1.8 \times \pi \times 10.62/12} = 0.20\end{aligned}$$

We can see that the first two terms can be neglected and the total thermal resistance is 1.98.

The heat loss per foot length is then, $\frac{150}{1.98} = 76$ lb-cal/hr (from equation 6.90)

We can now check the temperature at the outside of the lagging by the relation

$$\frac{\Delta T (\text{lagging})}{\Sigma \Delta T} = \frac{1.78}{1.98}$$

whence ΔT (lagging) is 135°C and the temperature on the outside of the lagging is $171 - 135$ or 36° . This is sufficiently near the assumed value. The value of h_r and h_c can also be checked; thus, taking an emissivity of 0.9 and assuming streamline natural convection,

$$h_r = 1.01 \times 10^{-8} \times 0.9 \cdot \left[\frac{309^4 - 294^4}{36 - 21} \right] = 1.01 \text{ lb-cal/hr-ft}^2\text{-}^\circ\text{C}$$

and
$$h_c = 0.28 \cdot \left[\frac{36 - 21}{10.62/12} \right]^{1/4} = 0.57 \text{ lb-cal-ft}^2\text{-}^\circ\text{C}$$

Thus $h_r + h_c = 1.6$, which is a little less than the value taken above. In practice the heat loss will probably be rather higher than this value as it is very rare for forced convection currents to be absent.

If the pipe were unlagged, the value of $h_r + h_c$ for the temperature difference of $171 - 21^\circ\text{C}$ would be about 3, and the heat loss per foot length would then be

$$\begin{aligned}\frac{Q}{l} &= (h_r + h_c) \pi \cdot d_s \cdot \Delta T \\ &= 3 \times \pi \times \frac{10.62}{12} \times 150 \\ &= \underline{\underline{1250 \text{ lb-cal/hr}}}\end{aligned}$$

It is clear that for these conditions there has been a considerable saving in heat by applying the 2 in. of lagging.

Economic Thickness of Lagging

Increasing the thickness of the lagging will reduce the loss of heat and thus give a saving in the operating costs. The cost of the lagging will increase with thickness and there will be an optimum thickness when further increase does not save sufficient heat to justify the cost. In general the smaller the pipe the smaller the thickness used, but it cannot be too strongly stressed that some lagging everywhere is better than excellent lagging in some places and none in others. For temperatures of 100 to 150°C , and for pipes up to 6 in. diameter, LYLE⁽⁸⁾ recommends a 1 in. thickness of 85 per cent magnesia lagging, and 2 in.

for pipes over 9 in. diameter. With temperatures of 200 to 250°C he suggests 1½ in. for pipes less than 3 in. diameter and 2 in. for pipes up to 9 in. diameter.

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LIST OF SYMBOLS USED IN CHAPTER 6

A	Area available for heat transfer or area of radiating surface	L^2
a	Absorptivity	—
a_b	Absorptivity of black body	—
a_g	Absorptivity of a gas	—
a'	A constant	
B	A constant	
B_1, B_2	Constants of integration	$T\theta$
b	Wetted perimeter of condensation surface	L
b'	A coefficient	
C	Constant in convection equation	—
C_f	Constant for friction in flow past a tube bundle	—
C_h	Constant for heat transfer in flow past a tube bundle	—
C'	Constant in equation for natural convection	—
C_p	Specific heat at constant pressure	$(QM^{-1}\theta^{-1})$ —
D_H	Thermal diffusivity (k/C_p)	L^2T^{-1}
d	Diameter (internal)	L
d_1, d_2	Inner and outer diameters of annulus	L
d_c	Diameter of helix	L
d_e	Hydraulic mean diameter	L
d_g	Diameter of gasket	L
d_g	Gap between turns in coil	L
d_m	Logarithmic mean diameter of lagging	L
d_o	Outside diameter of tube	L
d_p	Height of coil	L

d_s	Outside diameter of lagging	L
d_t	Thickness of fixed tube sheet	L
d_v	Internal diameter of vessel	L
d_w	Mean diameter of pipe wall	L
E	Emissive power	(or MT^{-3}) $ML^{-2}T^{-1}\theta$
E_b	Emissive power of black body	(or MT^{-3}) $ML^{-2}T^{-1}\theta$
E_z'	Energy emitted per unit area and unit time at wavelengths between Z and $Z + dZ$	(or $ML^{-1}T^{-3}$) $ML^{-3}T^{-1}\theta$
e	Emissivity	—
e_g	Emissivity of a gas	—
e_s	Emissivity of a surface	—
e'	$\frac{1}{2}(e_s + 1)$	—
F	Geometric factor for radiation or correction factor for logarithmic mean temperature difference	—
f	Working stress	$ML^{-1}T^{-2}$
G	Mass rate of flow	MT^{-1}
G'	Mass rate of flow per unit area	$ML^{-2}T^{-1}$
g	Acceleration due to gravity	LT^{-2}
h	Heat transfer coefficient	($QL^{-2}T^{-1}\theta^{-1}$) $ML^{-2}T^{-1}$
h_b	Film coefficient for liquid adjacent to vessel	$ML^{-2}T^{-1}$
h_c	Heat transfer coefficient for convection	$ML^{-2}T^{-1}$
h_f	Fin side film coefficient	$ML^{-2}T^{-1}$
h_g	Heat transfer coefficient for gas film	$ML^{-2}T^{-1}$
h_m	Mean value of h over whole surface	$ML^{-2}T^{-1}$
h_n	Heat transfer coefficient for liquid boiling at T_n	$ML^{-2}T^{-1}$
h_r	Heat transfer coefficient for radiation	$ML^{-2}T^{-1}$
I	Intensity of radiation	(or MT^{-3}) $ML^{-2}T^{-1}\theta$
I'	Intensity of radiation falling on body	(or MT^{-3}) $ML^{-2}T^{-1}\theta$
j	Number of vertical rows of tubes	—
K	Quantity independent of position on surface	$ML^{-1.75}T^{-1}$
k	Thermal conductivity	($QL^{-1}T^{-1}\theta^{-1}$) $ML^{-1}T^{-1}$
k_G	Mass transfer coefficient (mass/unit area . unit time . unit partial pressure difference)	$L^{-1}T$
L	Length of paddle or characteristic dimension	L
l	Length of tube or plate or distance apart of faces or thickness of gas stream	L
l_v	Length of side of vessel	L
M	Mass rate of flow of condensate per unit length of perimeter for vertical pipe and per unit length of pipe for horizontal pipe	$ML^{-1}T^{-1}$
m	Mass of liquid	M
m'	Quantity governing absorptivity of gas	L^{-1}
N	Number of revolutions in unit time	T^{-1}
N	Number of general term in series	—
n	An index	—
P	Pressure	$ML^{-1}T^{-2}$
P_c	Critical pressure	$ML^{-1}T^{-2}$
P_g	Partial pressure of gas or vapour	$ML^{-1}T^{-2}$
P_R	Reduced pressure (P/P_c)	—
P_r	Vapour pressure at surface of radius, r	$ML^{-1}T^{-2}$
P_s	Saturation vapour pressure	$ML^{-1}T^{-2}$
p	Parameter in Laplace transform	T^{-1}
Q	Heat flow per unit time	(QT^{-1}) $MT^{-1}\theta$
Q_k	Heat flow per unit time by conduction in fluid	$MT^{-1}\theta$
q	Heat flow per unit time and unit area	($QL^{-2}T^{-1}$) $ML^{-2}T^{-1}\theta$
R	Thermal resistance	$M^{-1}L^2T$

R_i, R_o	Thermal resistance of scale on inside, outside of tubes	$M^{-1}L^2T$
R'	Shear stress at free surface of condensate film	$ML^{-1}T^{-2}$
r	Radius	L
r_1, r_2	Radius (inner, outer) of annulus or tube	L
r_a	Arithmetic mean radius	L
r_m	Logarithmic mean radius	L
S	Flow area for condensate film	L^2
s	Thickness of condensate film at a point	L
T	Temperature	θ
T_c	Temperature of free surface of condensate	θ
T_f	Mean temperature of film	θ
T_m	Mean temperature of fluid	θ
T_n	Standard boiling point	θ
T_s	Temperature of condensing vapour	θ
T_w	Temperature of wall	θ
T''	Numerical value of temperature in $^{\circ}C$	—
ΔT_f	Temperature difference across condensate film	θ
t	Time	T
U	Overall heat transfer coefficient	$ML^{-2}T^{-1}$
u	Velocity	LT^{-1}
u_m	Maximum velocity in condensate film	LT^{-1}
u_t	Velocity through minimum area	LT^{-1}
u_y	Velocity at distance y from surface	LT^{-1}
W	Width of stirrer	L
w_1, w_2, \dots	Indices in equation for heat transfer by convection	—
X	Distance between centres of tubes in direction of flow	L
X	Ratio of temperature differences used in calculation of mean temperature difference	—
x	Distance in direction of transfer or along surface	L
Y	Distance between centres of tubes at right angles to flow direction	L
Y	Ratio of temperature difference used in calculation of mean temperature difference	—
y	Distance perpendicular to surface	L
Z	Wavelength	L
Z_m	Wavelength at which maximum energy is emitted	L
z	Distance in third principal direction	L
α	Angle between normal and direction of radiation	—
β	Coefficient of cubical expansion	θ^{-1}
ϵ	coefficient relating h to $u^{0.8}$	$ML^{-2.8}T^{-0.2}$
λ	Latent heat of vaporisation per unit mass	θ
μ	Viscosity	$ML^{-1}T^{-1}$
μ_s	Viscosity of fluid at surface	$ML^{-1}T^{-1}$
ρ	Density or density of liquid	ML^{-3}
ρ_v	Density of vapour	ML^{-3}
σ	STEFAN-BOLTZMANN constant	(or $MT^{-3}\theta^{-4}$) $ML^{-2}T^{-1}\theta^{-3}$
τ	Surface tension	MT^{-2}
ψ	$\frac{G_1c_1 + G_2c_2}{G_1c_1G_2c_2}$	—
ϕ	Angle between surface and horizontal	—
θ	Temperature or temperature difference	θ
θ_a	Temperature difference in Schmidt method	θ
θ_m	Logarithmic mean temperature difference	θ
θ_{xt}	Temperature at $t = t, x = x$	θ
θ_0	Initial uniform temperature of body	θ

θ'	Temperature of source or surroundings	θ
$\bar{\theta}$	Laplace transform of temperature	$T\theta$
ω	Solid angle	—
$Gr.$	Grashof Number	—
$Nu.$	Nusselt Number	—
$Pr.$	Prandtl Number	—
$Re.$	Reynolds Number	—
Δ	Finite difference in a property	—
Suffix l, w refer to lagging, wall material		
Suffix i, o refer to inside, outside of wall		

Section C

Momentum, Heat and Mass Transfer

THIS section deals with the diffusion of vapours in gases and of solutes in liquids, and with the similarity between these processes and momentum, and heat transfer which have already been discussed in Sections A and B. Quantitative relations are developed for particular conditions, some from considerations of dimensional analysis and others from the nature of the flow near a surface.

CHAPTER 7

Mass Transfer

WHEN a concentration gradient exists within a fluid consisting of two or more components, there is a tendency for each constituent to flow in such a direction as to reduce the concentration gradient; this process is known as mass transfer. In a still fluid, or in a fluid flowing under streamline conditions in a direction at right angles to the concentration gradient, the transfer is effected as a result of the random motion of the molecules. In a turbulent fluid, this mechanism is supplemented by transference of material by eddy currents.

Mass transfer can take place in either a gas phase or a liquid phase, or in both simultaneously. When a liquid evaporates into a still gas, vapour is transferred from the surface to the bulk of the gas as a result of the concentration gradient; the process continues until the whole of the liquid has evaporated, or until the gas is saturated and the concentration gradient reduced to zero. In the absorption of a soluble gas from a mixture with an insoluble gas, mass transfer takes place from the bulk of the gas to the liquid surface and then into the bulk of the liquid. Neither the insoluble gas nor the solvent moves in the direction of mass transfer. In a distillation column, on the other hand, the less volatile component diffuses in the gas phase to the liquid surface and the more volatile material diffuses at an approximately equal molar rate in the opposite direction. In the liquid phase a similar process takes place, with the less volatile material diffusing away from the gas-liquid interface.

The rate of transfer of A in a mixture of two components, A and B , will therefore be determined not only by the rate of diffusion of A , but also by the behaviour of B . The molar rate of transfer of A , per unit area, due to molecular motion is given by Fick's law⁽⁸⁾,

$$N_A = -D_{AB} \cdot \frac{\partial C_A}{\partial y} \quad \dots(7.1a)$$

where N_A is the molar rate of diffusion per unit area,

D_{AB} is the diffusivity of A in B , a physical property of the two vapours,

C_A is the molar concentration of A , and

y is distance in the direction of diffusion.

The corresponding rate of diffusion of B is given by

$$N_B = -D_{BA} \cdot \frac{\partial C_B}{\partial y} \quad \dots(7.1b)$$

where D_{BA} is the diffusivity of B in A , and

C_B is the molar concentration of B .

If the total pressure, and hence the total molar concentration, is everywhere constant, $\frac{\partial C_A}{\partial y}$ and $\frac{\partial C_B}{\partial y}$ must be equal and opposite and therefore A and B tend to diffuse in opposite directions. In a distillation process where the two components have equal molar latent heats, condensation of one mole of the less volatile material releases just sufficient heat for the vaporisation of one mole of the more volatile component, and therefore “equimolecular counterdiffusion” takes place with two components diffusing at equal and opposite rates, as determined by Fick’s law. In an absorption process there is net transfer of only one of the components although there is a concentration difference of the other. It is therefore necessary to study the two cases separately.

In many processes B will neither remain stationary, nor will it diffuse at an equal and opposite molar rate to A . Exact calculations relating to this type of problem are difficult. An example of this nature is the mass transfer in a distillation column when the two components have unequal molar latent heats.

When the fluid is turbulent, eddy diffusion takes place in addition to molecular diffusion and the rate of diffusion is increased and

$$N_A = - (D_{AB} + E_D) \frac{\partial C_A}{\partial y} \quad \dots (7.2)$$

where E_D is known as the eddy diffusivity. E_D will increase as the turbulence is increased and is more difficult to evaluate than the molecular diffusivity. For this reason it is often convenient to regard the whole of the mass transfer as taking place in a region of constant concentration gradient by molecular diffusion alone. This approach to the problem will be adopted in the present chapter, but the influence of turbulence in the fluid on mass transfer will be considered in Chapter 8.

DIFFUSION IN THE GAS PHASE

Equimolecular Counterdiffusion

Suppose two vapours, A and B , are diffusing at equal and opposite rates and P_A and P_B are their partial pressures at any point in the system.

If A and B are ideal gases

$$P_A V = n_A RT \text{ and } P_B V = n_B RT \text{ (from equation 2.2)}$$

where n_A and n_B are the number of moles of A and B in a volume, V , so that

$$P_A = \frac{n_A}{V} RT = C_A RT = \frac{c_A}{M_A} RT \quad \dots (7.3a)$$

and
$$P_B = \frac{n_B}{V} RT = C_B RT = \frac{c_B}{M_B} RT \quad \dots (7.3b)$$

where c_A and c_B are mass concentrations and M_A and M_B molecular weights. If the total pressure, P (Fig. 7.1), is everywhere constant in the system, from equations 7.3a and 7.3b

$$P = P_A + P_B = \mathbf{RT}(C_A + C_B) = \mathbf{RT} \left(\frac{c_A}{M_A} + \frac{c_B}{M_B} \right) \quad \dots(7.4)$$

so that
$$\frac{\partial P_A}{\partial y} = - \frac{\partial P_B}{\partial y} \quad \dots(7.5a)$$

$$\frac{\partial C_A}{\partial y} = - \frac{\partial C_B}{\partial y} \quad \dots(7.5b)$$

and
$$\frac{\partial c_A}{\partial y} = - \frac{\partial c_B}{\partial y} \cdot \frac{M_A}{M_B} \quad \dots(7.5c)$$

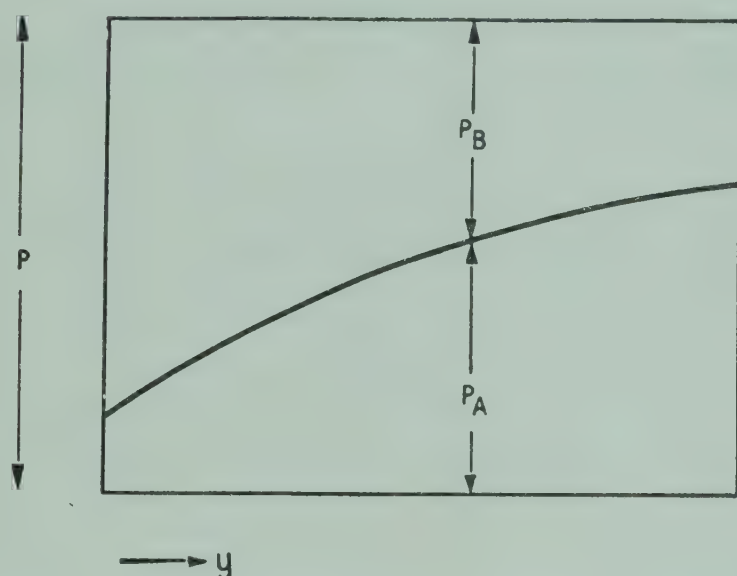


Fig. 7.1. Variation in partial pressures of components—total pressure constant

Thus
$$N_A = - \frac{D_{AB}}{\mathbf{RT}} \cdot \frac{\partial P_A}{\partial y} \text{ (from equations 7.1a and 7.3a)} \quad \dots(7.6a)$$

and
$$N_B = - \frac{D_{BA}}{\mathbf{RT}} \cdot \frac{\partial P_B}{\partial y} = + \frac{D_{BA}}{\mathbf{RT}} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.6b)$$

(from equations 7.1a, and 7.3b, and 7.5a)

In equimolecular counterdiffusion, the total pressure at any point in the system will remain constant as a result of diffusion, and therefore equal numbers of molecules will diffuse in each direction, i.e.

$$N_A = - N_B$$

so that $D_{AB} = D_{BA} = D$ (say), from equation 7.6a and 7.6b

Thus
$$N_A = - D \cdot \frac{\partial C_A}{\partial y} = - \frac{D}{\mathbf{RT}} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.7a)$$

and
$$N_B = + D \cdot \frac{\partial C_A}{\partial y} = + \frac{D}{\mathbf{RT}} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.7b)$$

The above equations can be directly integrated. If the conditions at two different points within the system are denoted by suffixes 1 and 2,

$$N_A = - \frac{D(P_{A_2} - P_{A_1})}{RT(y_2 - y_1)} \quad \dots(7.8)$$

Diffusion through a Stationary Gas

It has just been shown that the rates of diffusion of A and B are given by

$$N_A = - D \cdot \frac{\partial C_A}{\partial y} = - \frac{D}{RT} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.7a)$$

and
$$N_B = + D \cdot \frac{\partial C_A}{\partial y} = + \frac{D}{RT} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.7b)$$

If a surface is introduced on which A is absorbed but B is not absorbed (Fig. 7.2) a partial pressure gradient will be set up, causing A to diffuse towards, and B away from the surface. Imagine this process to continue for a short interval.

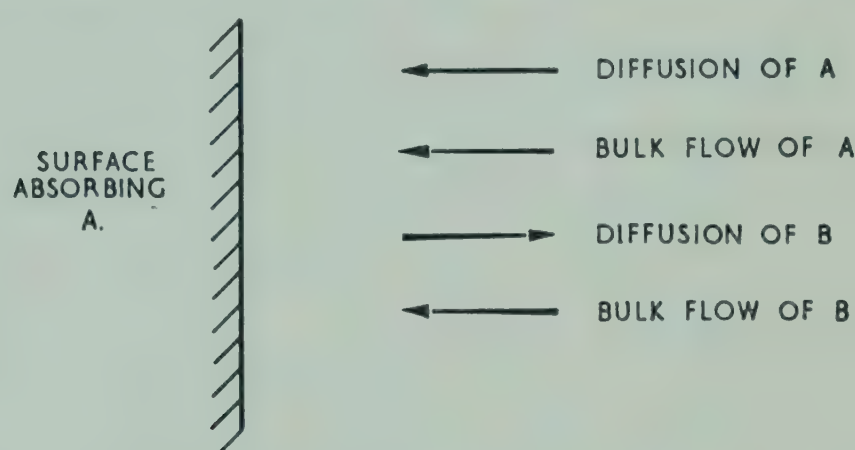


Fig. 7.2. Mass transfer through stationary gas

A will be absorbed at the surface and B will tend to diffuse away and therefore a total pressure gradient will be produced causing a bulk motion of A and B towards the surface, in addition to the transfer by diffusion. Since there is no net motion of B , the bulk rate of flow must exactly balance its transfer by diffusion.

Thus the bulk rate of flow of $B = - N_B$

$$= - \frac{D}{RT} \frac{\partial P_A}{\partial y}$$

The bulk flow of B is accompanied by a bulk flow of A , equal to

$$N_B \cdot \frac{P_A}{P_B} = - \frac{D}{RT} \cdot \frac{P_A}{P - P_A} \cdot \frac{\partial P_A}{\partial y} \quad \text{(from equations 7.4 and 7.7a)} \quad \dots(7.9)$$

This bulk rate of flow produces a velocity u_F , given by

$$u_F \cdot \frac{P_A}{RT} = - \frac{D}{RT} \cdot \frac{P_A}{P - P_A} \cdot \frac{\partial P_A}{\partial y}$$

i.e.

$$u_F = - \frac{D}{P - P_A} \cdot \frac{\partial P_A}{\partial y} \quad \dots(7.10)$$

The total rate of transfer of A , N_A' , is obtained by adding the rates given by equations 7.7a and 7.9,

$$\begin{aligned} \therefore N_A' &= -\frac{D}{RT} \cdot \left(1 + \frac{P_A}{P - P_A}\right) \cdot \frac{\partial P_A}{\partial y} \\ &= -\frac{D}{RT} \cdot \frac{P}{P - P_A} \cdot \frac{\partial P_A}{\partial y} \end{aligned} \quad \dots(7.11)$$

This relation is known as Stefan's Law⁽⁹⁾.

Integration of the above equation between two positions denoted by suffixes 1 and 2, gives

$$\begin{aligned} N_A' &= \frac{D \cdot P}{RT(y_2 - y_1)} \ln \frac{P - P_{A_2}}{P - P_{A_1}} \quad \dots(7.12a) \\ &= \frac{D \cdot P}{RT(y_2 - y_1)} \ln \frac{P_{B_2}}{P_{B_1}} \\ &= \frac{D \cdot P}{RT(y_2 - y_1)P_{Bm}} (P_{B_2} - P_{B_1}) \end{aligned}$$

(where P_{Bm} is the logarithmic mean of P_{B_1} and P_{B_2})

$$= -\frac{D \cdot P}{RT(y_2 - y_1)P_{Bm}} \cdot (P_{A_2} - P_{A_1}) \quad \dots(7.12b)$$

$$= -\frac{D \cdot C_T}{(y_2 - y_1)C_{Bm}} \cdot (C_{A_2} - C_{A_1}) \quad \dots(7.12c)$$

where C_T and C_{Bm} are the total molar concentration and the logarithmic mean value of C_B respectively.

Equation 7.12a can be simplified if P_A is small compared with P , giving

$$\begin{aligned} N_A' &= \frac{D \cdot P}{RT(y_2 - y_1)} \ln \left(1 - \frac{P_{A_2} - P_{A_1}}{P - P_{A_1}}\right) \\ &= \frac{D \cdot P}{RT(y_2 - y_1)} \cdot \left\{-\frac{P_{A_2} - P_{A_1}}{P - P_{A_1}} - \dots\right\} \\ &\approx -\frac{D}{RT(y_2 - y_1)} \cdot (P_{A_2} - P_{A_1}) \end{aligned} \quad \dots(7.13)$$

Comparison of Rates of Mass Transfer in Equimolecular Counter-diffusion and in Diffusion through a Stationary Gas

For equimolecular counter-diffusion, from equation 7.8,

$$N_A = -\frac{D}{RT} \cdot \frac{P_{A_2} - P_{A_1}}{y_2 - y_1} = k_G'(P_{A_1} - P_{A_2}) \quad \dots(7.14)$$

where $k_G' = \frac{D}{RT(y_2 - y_1)}$, a mass transfer coefficient for equimolecular counter-diffusion.

For diffusion through a stationary gas B , from equation 7.12b,

$$N_A' = - \frac{D}{RT} \cdot \frac{P_{A_2} - P_{A_1}}{y_2 - y_1} \cdot \frac{P}{P_{Bm}} = k_G(P_{A_1} - P_{A_2}) \quad \dots(7.15)$$

where $k_G = \frac{D}{RT(y_2 - y_1)} \cdot \frac{P}{P_{Bm}}$, a mass transfer coefficient for diffusion through a stationary gas.

Thus
$$\frac{N_A'}{N_A} = \frac{P}{P_{Bm}} = \frac{k_G}{k_G'} \quad \dots(7.16)$$

Where P_A is small compared with P , it is seen by comparing equations 7.8 and 7.13 that the rate of transfer of A is the same for equimolecular counter-diffusion as it is for diffusion through a stationary gas.

Maxwell's Law of Diffusion

MAXWELL postulated that the partial pressure gradient in the direction of diffusion of a given constituent of a gaseous mixture was proportional to:

- (a) The relative velocity of the molecules in the direction of diffusion.
- (b) The products of the molar concentrations of the two materials.

Thus
$$\begin{aligned} - \frac{\partial P_A}{\partial y} &= F \cdot C_A \cdot C_B (u_A - u_B) \quad \dots(7.17) \\ &= F \cdot \frac{P_A}{RT} \cdot \frac{P_B}{RT} \cdot (u_A - u_B) \\ &= \frac{F \cdot P_A P_B}{(RT)^2} \cdot (u_A - u_B) \end{aligned}$$

where u_A, u_B are the mean molecular velocities of A and B in direction of mass transfer, and F is a coefficient.

The molar rate of transfer of A per unit area N_A' is given by

$$\begin{aligned} N_A' &= \frac{u_A P_A}{RT} \\ &= \frac{P_A}{RT} \left\{ u_B - \frac{RT^2}{F \cdot P_A P_B} \cdot \frac{\partial P_A}{\partial y} \right\} \quad \dots(7.18) \end{aligned}$$

If B remains stationary, $u_B = 0$ and

$$\begin{aligned} N_A' &= \frac{RT}{F \cdot P_B} \cdot - \frac{\partial P_A}{\partial y} \\ &= \frac{RT}{F \cdot P_B} \cdot \frac{\partial P_B}{\partial y} \\ &= \frac{RT}{F(y_2 - y_1)} \ln \frac{P_{B_2}}{P_{B_1}} \\ &= \frac{RT}{F(y_2 - y_1)} \ln \frac{P - P_{A_2}}{P - P_{A_1}} \quad \dots(7.19) \end{aligned}$$

By comparison of the two equations (7.12*a* and 7.19) for N_A' it is seen that

$$\frac{RT}{F} = \frac{DP}{RT}$$
$$D = \frac{R^2T^2}{FP}$$

i.e.

.....(7.20)

For equimolecular counter-diffusion,

$$N_A' = - N_B' = - \frac{u_B P_B}{RT}$$

$$\therefore u_B = - \frac{RT}{P_B} N_A'$$

Substituting in equation 7.18

$$N_A' = - \frac{P_A}{P_B} N_A' - \frac{RT}{FP_B} \cdot \frac{\partial P_A}{\partial y}$$

$$\therefore N_A' = - \frac{RT}{FP} \cdot \frac{\partial P_A}{\partial y} = - \frac{RT}{FP} \cdot \frac{P_{A_2} - P_{A_1}}{y_2 - y_1}$$

.....(7.21)

This equation reduces to equation 7.8 by substituting for F from equation 7.20.

Diffusivities of Various Vapours

Diffusivities for a number of vapours in air are given in Tables 7.1⁽⁴⁾ and 7.2⁽⁴⁾, in which the ratio of the kinematic viscosity of a dilute mixture to the diffusivity $\left(\frac{\mu}{\rho D}\right)$ is also listed. This ratio is known as the Schmidt Number and will be discussed further in Chapter 8.

TABLE 7.1. Diffusion coefficients of gases and vapours in air at 25°C, 1 atm.

Substance	D (cm ² /sec.)	$\frac{\mu}{\rho D}$	Substance	D (cm ² /sec.)	$\frac{\mu}{\rho D}$
Ammonia	0.236	0.66	Valeric acid	0.067	2.31
Carbon dioxide	0.164	0.94	i-Caproic acid	0.060	2.58
Hydrogen	0.410	0.22	Diethyl amine	0.105	1.47
Oxygen	0.206	0.75	Butyl amine	0.101	1.53
Water	0.256	0.60	Aniline	0.072	2.14
Carbon disulphide	0.107	1.45	Chlorobenzene	0.073	2.12
Ethyl ether	0.093	1.66	Chlorotoluene	0.065	2.38
Methanol	0.159	0.97	Propyl bromide	0.105	1.47
Ethanol	0.119	1.30	Propyl iodide	0.096	1.61
Propanol	0.100	1.55	Benzene	0.088	1.76
Butanol	0.090	1.72	Toluene	0.084	1.84
Pentanol	0.070	2.21	Xylene	0.071	2.18
Hexanol	0.059	2.60	Ethyl benzene	0.077	2.01
Formic acid	0.159	0.97	Propyl benzene	0.059	2.62
Acetic acid	0.133	1.16	Diphenyl	0.068	2.28
Propionic acid	0.099	1.56	n-Octane	0.060	2.58
i-Butyric acid	0.081	1.91	Mesitylene	0.067	2.31

Note: the group $(\mu/\rho D)$ in the above table is evaluated for mixtures composed largely of air.

TABLE 7.2. Diffusion coefficients of organic esters in air

Number of carbon atoms	D (cm ² /sec. at 25°C)	$\mu/\rho D$
2*	0.117	1.32
3†	0.097	1.59
4‡	0.086	1.79
5	0.078	1.97
6	0.069	2.23
7	0.065	2.37
8	0.057	2.70
9	0.049	3.14

Note. The group $(\mu/\rho D)$ in the above table is evaluated for mixtures composed largely of air.

- * Methyl formate.
- † Ethyl formate, methyl acetate.
- ‡ n-Propyl formate, i-propyl formate, ethyl acetate, methyl propionate.

Diffusivities of vapours are most conveniently determined by the method developed by WINKELMANN⁽¹¹⁾. Liquid is allowed to evaporate in a vertical glass

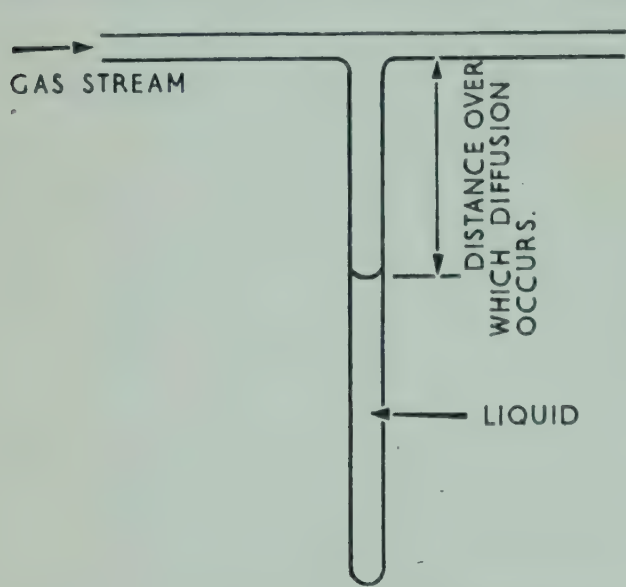


Fig. 7.3. Determination of diffusivities of vapours

tube over the top of which a stream of vapour free gas is passed, sufficiently rapidly for the vapour pressure to be maintained almost at zero (Fig. 7.3). If the apparatus is maintained at a steady temperature, there will be no eddy currents in the vertical tube and mass transfer will take place from the surface by molecular diffusion alone. The rate of evaporation can be followed by the rate of fall of the liquid surface, and since the concentration gradient is known, the diffusivity can then be calculated.

In many cases, the diffusivity D for the transfer of one gas in another may not be known and experimental determination may not be practicable. Many attempts have been made to express D in terms of other physical properties and the following empirical equation of MAXWELL's, modified by GILLILAND⁽²⁷⁾, gives satisfactory agreement with the practical figures,

$$D = \frac{0.0043 \cdot T^{1.5} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}}{P(V_A^{\frac{1}{3}} + V_B^{\frac{1}{3}})^2} \dots (7.22)$$

- where D is the diffusivity in cm²/sec.,
- T is the absolute temperature (°K),
- M_A, M_B are the molecular weights of A and B ,
- P is the total pressure in atmospheres,

V_A, V_B are the molecular volumes of A and B . The molecular volume is the volume in c.c.s. of 1 gram molecule of the material in the form of liquid at its boiling point, and is a measure of the volume occupied by the molecules themselves. It may not always be known but an approximate value can be obtained by application of Kopp's Law of Additive Volumes. Kopp gave a particular value for the equivalent atomic volume of each element. When the atomic volumes of the elements of the molecule in question are added in the appropriate proportions, the equivalent molecular volume is obtained approximately. There are certain exceptions to this rule and additions or subtractions have to be made if the elements are combined in particular manners.

TABLE 7.3. Atomic volumes⁽⁵⁾

Air	29.9	Oxygen, double bonded	7.4
Antimony	24.2	Coupled to two other elements:	
Arsenic	30.5	in aldehydes and ketones	7.4
Bismuth	48.0	in methyl esters	9.1
Bromine	27.0	in ethyl esters	9.9
Carbon	14.8	in higher esters and ethers	11.0
Chlorine, terminal, as in R-Cl	21.6	in acids	12.0
medial, as in R-CHCl-R'	24.6	in union with S, P, N	8.3
Chromium	27.4	Phosphorus	27.0
Fluorine	8.7	Silicon	32.0
Germanium	34.5	Sulphur	25.6
Hydrogen	3.7	Tin	42.3
Nitrogen, in primary amines	10.5	Titanium	35.7
in secondary amines	12.0	Vanadium	32.0
		Zinc	20.4

Notes

- For three membered ring, as in ethylene oxide, deduct 6.0.
- For four membered ring, as in cyclobutane, deduct 8.5.
- For five membered ring, as in furane, deduct 11.5.
- For six membered ring, as in benzene, pyridine, deduct 15.0.
- For anthracene ring formation, deduct 47.5.

DIFFUSION IN THE LIQUID PHASE

The rate of diffusion of a material in the liquid phase is represented by the same basic equation as for gas phase diffusion, i.e.

$$N_A = - D_L \frac{\partial C_A}{\partial y} \dots (7.23)$$

where D_L is the liquid phase diffusivity.

The integrated form of the equation for the diffusion of a material A through a stagnant liquid B is then of the same form as for gaseous diffusion, viz.,

$$N_A' = - D_L \frac{(C_{A_2} - C_{A_1})}{y_L} \frac{C_T}{C_{Bm}} \dots (7.24)$$

where C_{A_1}, C_{A_2} are the molar concentrations of A at two points,

y_L is the equivalent thickness of the liquid film through which diffusion is taking place,

C_T is the total molar concentration. This may not remain constant throughout the whole of the fluid as was the case for diffusion in the gas phase,

C_{Bm} is the logarithmic mean of the molar concentrations of B on each side of the liquid film.

Values of D_L for diffusion of various materials in water are given in Table 7.4⁽⁴⁾.

TABLE 7.4. Diffusion coefficients in liquids at 20°C

Solute	Solvent	$D_L \times 10^5$ (cm ² /sec.) $\times 10^6$	($\mu/\rho D$)*
O ₂ . . .	Water	1.80	558
CO ₂ . . .	Water	1.50	670
N ₂ O . . .	Water	1.51	665
NH ₃ . . .	Water	1.76	570
Cl ₂ . . .	Water	1.22	824
Br ₂ . . .	Water	1.2	840
H ₂ . . .	Water	5.13	196
N ₂ . . .	Water	1.64	613
HCl . . .	Water	2.64	381
H ₂ S . . .	Water	1.41	712
H ₂ SO ₄ . . .	Water	1.73	580
HNO ₃ . . .	Water	2.6	390
Acetylene . . .	Water	1.56	645
Acetic acid . . .	Water	0.88	1140
Methanol . . .	Water	1.28	785
Ethanol . . .	Water	1.00	1005
Propanol . . .	Water	0.87	1150
Butanol . . .	Water	0.77	1310
Allyl alcohol . . .	Water	0.93	1080
Phenol . . .	Water	0.84	1200
Glycerol . . .	Water	0.72	1400
Pyrogallol . . .	Water	0.70	1440
Hydroquinone . . .	Water	0.77	1300
Urea . . .	Water	1.06	946
Resorcinol . . .	Water	0.80	1260
Urethane . . .	Water	0.92	1090
Lactose . . .	Water	0.43	2340
Maltose . . .	Water	0.43	2340
Glucose . . .	Water	0.60	—
Mannitol . . .	Water	0.58	1730
Raffinose . . .	Water	0.37	2720
Sucrose . . .	Water	0.45	2230
Sodium chloride . . .	Water	1.35	745
Sodium hydroxide . . .	Water	1.51	665
CO ₂ . . .	Ethanol	3.4	445
Phenol . . .	Ethanol	0.8	1900
Chloroform . . .	Ethanol	1.23	1230
Phenol . . .	Benzene	1.54	479
Chloroform . . .	Benzene	2.11	350
Acetic acid . . .	Benzene	1.92	384
Ethylene dichloride . . .	Benzene	2.45	301

* Based on $\mu/\rho = 0.01005$ cm²/sec. for water, 0.00737 for benzene, and 0.01511 for ethanol, all at 20°C; applies only for dilute solutions.

An empirical equation from which liquid phase diffusivities can be calculated approximately is

$$D_L = \frac{7.7 \times 10^{-10} T}{\mu(V^{\frac{1}{3}} - V_0^{\frac{1}{3}})}$$

....(7.25)

where D_L is the diffusivity in $\text{cm}^2/\text{sec.}$,

T is the temperature in $^\circ\text{K}$,

μ is the viscosity in poises,

$V_0 = 8.0, 14.9$, and 22.8 for diffusion in dilute solutions in water, methanol, and benzene respectively, and

V is the molecular volume, as in equation 7.22.

MASS TRANSFER BY NATURAL CONVECTION

When there is no flow of fluid across a surface to which heat or mass transfer is taking place, convection currents arise simply because the density of the fluid is a function of its temperature and of its composition. In heat transfer, it has been shown that the process is affected by the value of the Grashof Number,

$$\frac{g\beta\theta_r y_r^3 \rho^2}{\mu^2}; \text{ mass transfer is dependent on a similar dimensionless group, } \frac{g y_r^3 \rho^2}{\mu^2} \left(\frac{\rho_r}{\rho_w} - 1 \right).$$

In the above expressions,

g is the acceleration due to gravity,

μ is the viscosity of the fluid,

β is the coefficient of thermal expansion of the fluid,

ρ is the mean density of the fluid,

ρ_r is the density of the fluid at a distance, y_r , from the surface,

ρ_w is the density of the fluid at the surface, and

θ_r is the difference in the temperatures at the surface and at a distance y_r from the surface.

MASS TRANSFER BY FORCED CONVECTION

In the previous chapter it was shown that the heat transfer by forced convection could be expressed by an equation of the form,

$$\frac{h \cdot L}{k} = f \left(\frac{uL\rho}{\mu}, \frac{C_p\mu}{k} \right)$$

or

$$Nu. = f(Re., Pr.) \quad \dots (6.30)$$

If h_D is the mass transfer coefficient, defined as the molar rate of transfer of material per unit area for a unit molar concentration difference, the group $\frac{h_D L}{D}$

is analogous to the Nusselt Group for heat transfer. The Prandtl number is the ratio of the kinematic viscosity of the fluid to the thermal diffusivity, and the corresponding group for mass transfer is the Schmidt Group $Sc.$ where

$$Sc. = \frac{\mu}{\rho D} = \frac{\text{kinematic viscosity}}{\text{diffusivity}}$$

This has already been referred to on page 239 and will be discussed in more detail in Chapter 8.

For mass transfer we have therefore

$$\frac{h_D \cdot L}{D} = f\left(\frac{uL\rho}{\mu}, \frac{\mu}{\rho D}\right) \quad \dots(7.26)$$

Mass transfer under conditions of forced convection will be considered further in Chapter 8, where the relations for heat and mass transfer processes are compared.

UNSTEADY MASS TRANSFER

We have confined our attention so far to steady mass transfer, i.e. where conditions at any point in the system are not a function of time. In many cases, however, we are interested in the changes in concentration which occur during a process, for instance, during the formation of a drop. This problem is discussed in Volume 2. The basic equation is of the same form as that for unsteady heat transfer and is derived by an exactly similar method. For equimolecular counter-diffusion,

$$\frac{\partial C_A}{\partial t} = D \left\{ \left(\frac{\partial^2 C_A}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 C_A}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 C_A}{\partial z^2} \right)_{xy} \right\} \quad \dots(7.27)$$

and for diffusion through a stationary gas,

$$\frac{\partial C_A}{\partial t} = \frac{P}{P_{Bm}} D \left\{ \left(\frac{\partial^2 C_A}{\partial x^2} \right)_{yz} + \left(\frac{\partial^2 C_A}{\partial y^2} \right)_{zx} + \left(\frac{\partial^2 C_A}{\partial z^2} \right)_{xy} \right\} \quad \dots(7.28)$$

The equations can be solved by the Laplace Transform method (see p. 158). Solution of equation 7.28 is a little more difficult, since $\frac{P}{P_{Bm}}$ is a function of time. The equation must be solved for a number of separate time intervals over each of which $\frac{P}{P_{Bm}}$ may be taken as constant.

APPLICATIONS OF MASS TRANSFER

The application of the various formulae that have been derived in this chapter, together with those to be obtained in Chapter 8, will be discussed later. The main chemical engineering processes in which mass transfer is involved are:

- (1) Humidification and water cooling (Chapter 10).
- (2) Drying and evaporation.
- (3) Absorption.
- (4) Liquid-liquid extraction.
- (5) Distillation.

All but the first of these processes will be discussed in Volume 2.

Momentum, Heat and Mass Transfer

IN the previous chapters, we have discussed the stresses arising from relative motion within a fluid, the transfer of heat by conduction and convection, and the mechanism of mass transfer. These three major processes of momentum, heat, and mass transfer have, however, been regarded as independent problems. The motion of an element of fluid will result in the transfer of momentum and may, in addition, be responsible for the transfer of heat from one part of the fluid to another. We shall now show, not only that the processes are physically related, but also that quantitative relations between them can be developed.

When a fluid is flowing under streamline conditions over a surface, a forward component of velocity is superimposed on the random distribution of velocities of the molecules and movement at right angles to the surface occurs solely as a result of the random motion of the molecules. Thus if two adjacent layers of fluid are moving at different velocities, there will be a tendency for the faster moving layer to be retarded and the slower moving layer to be accelerated by virtue of the continuous passage of molecules in each direction. There will therefore be a net transfer of momentum from the fast to the slow moving stream. Similarly the molecular motion will tend to reduce any temperature gradient, or any concentration gradient if the fluid consists of a mixture of two or more components. In practice, a steady condition will be reached when the effects of the molecular transfer are balanced by the forces at the boundary surface.

If the motion of the fluid is turbulent, the transfer of fluid by eddy motion is superimposed on the molecular transfer process. In this case, the rate of transfer to the surface will be a function of the degree of turbulence. When the fluid is highly turbulent, the rate of transfer by molecular motion will be negligible compared with that by eddy motion. For small degrees of turbulence the two may be of the same order.

When a fluid flows under turbulent conditions over a surface, three regions are distinguishable:

- (1) At the surface, the laminar sub-layer, in which the only motion at right angles to the surface is due to molecular diffusion.
- (2) Next, the buffer layer, in which molecular diffusion and eddy motion are of comparable magnitude.
- (3) Finally, over the greater part of the fluid, the turbulent region in which eddy motion is large compared with molecular diffusion.

In addition to momentum, both heat and mass can be transferred either by molecular diffusion alone or by molecular diffusion combined with eddy diffusion. Because the effects of eddy diffusion are generally far greater than those of the molecular diffusion, the main resistance to transfer will lie in the regions where only molecular diffusion is occurring. Thus the main resistance to the flow of heat or mass to a surface lies within the laminar sub-layer. It will be shown in Chapter 9 that the thickness of the laminar sub-layer is almost inversely proportional to the Reynolds Number, for fully developed turbulent flow in a pipe. Thus the heat and mass transfer coefficients are much higher at high Reynolds Numbers.

TRANSFER BY MOLECULAR DIFFUSION

Momentum Transfer

From equation 3.11,

$$R_y = -\mu \frac{\partial u_y}{\partial y} = -\frac{\mu}{\rho} \cdot \frac{\partial(\rho u_y)}{\partial y} \quad \dots(8.1)$$

where u_y is the velocity of the fluid parallel to the surface at a distance y from it.

The shear stress R_y within a Newtonian fluid, at a distance y from the boundary surface is a measure of the rate of transfer of momentum per unit area at right angles to the surface.

Since (ρu_y) is the momentum per unit volume of the fluid, the rate of transfer of momentum per unit area is proportional to the gradient in the Y -direction of the momentum per unit volume. The negative sign indicates that momentum is transferred from the fast to the slow moving fluid and the shear stress acts in such a direction as to oppose the motion of the fluid.

Heat Transfer

From the definition of thermal conductivity given by equation 6.2, the heat transferred per unit time through a unit area q_y is given by,

$$q_y = -k \frac{\partial \theta}{\partial y} = -\frac{k}{C_p \rho} \cdot \frac{\partial(C_p \rho \theta)}{\partial y} \quad \dots(8.2)$$

where C_p is the specific heat of the fluid at constant pressure,

θ is the temperature,

k is the thermal conductivity.

The term $(C_p \rho \theta)$ represents the heat content per unit volume of fluid and therefore the flow of heat is proportional to the gradient in the Y -direction, of the heat content per unit volume. The proportionality constant, $k/C_p \rho$, is called the thermal diffusivity, D_H .

Mass Transfer

We have seen in Chapter 7, from FICK's Law of diffusion, that the rate of diffusion of a constituent A in a mixture is proportional to its concentration gradient.

Thus, from equation 7.7a,

$$N_A = -D \cdot \frac{\partial C_A}{\partial y} \quad \dots (8.3)$$

where N_A is the molar rate of diffusion of constituent A per unit area,
 C_A is the molar concentration of constituent A ,
 D is the diffusivity.

The essential similarity between the three processes is that the rates of transfer of momentum, heat, and mass are all proportional to the concentration gradients of these quantities. In the case of gases we shall see the physical significance of the proportionality constants μ/ρ , D_H , and D , all of which have the dimensions $\text{length}^2/\text{time}$. For liquids the constants cannot be interpreted in a similar manner. We shall now consider in turn the viscosity, thermal conductivity, and diffusivity of a gas.

Viscosity

Consider the flow of a gas parallel to a solid surface and the movement of molecules at right angles to this direction through a plane $a - a$ of unit area,

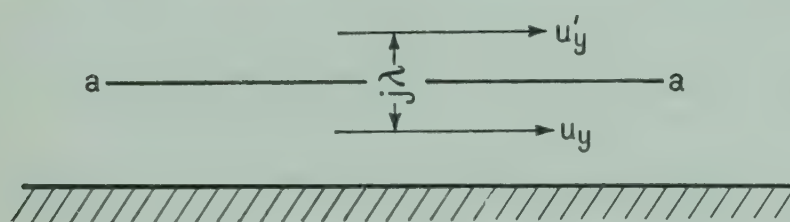


Fig. 8.1. Transfer of momentum near a surface

parallel to the surface and sufficiently close to it to be within the laminar sub-layer (Fig. 8.1). During an interval of time, dt , molecules with an average velocity $i_1 \cdot u_m$ in the Y -direction will pass through the plane (where u_m is the root mean square velocity and i_1 is some fraction of it, depending on the actual distribution of velocities).

If all these molecules can be considered as having the same component of velocity in the Y -direction, molecules from a volume $i_1 \cdot u_m \cdot dt$ will pass through the plane in time dt .

If N is the numerical concentration of molecules close to the surface, the number of molecules passing $= i_1 \cdot u_m \cdot N \cdot dt$.

Thus the rate of passage of molecules $= i_1 \cdot u_m \cdot N$.

These molecules have a mean velocity u_y (say) in the X -direction.

Thus the rate at which momentum is transferred across the plane away from the surface

$$= i_1 \cdot N \cdot u_m \cdot m \cdot u_y$$

where m is the mass of each molecule.

By similar reasoning there must be an equivalent stream of molecules also passing through the plane in the opposite direction; otherwise there would be a resultant flow perpendicular to the surface.

Let this other stream of molecules have originated at a distance $j\lambda$ from the previous ones and let the mean component of their velocities in the X -direction be u_y' (where λ is the mean free path of the molecules and j is some fraction of the order of unity).

The net rate of transfer of momentum away from the surface

$$= i_1 \cdot N \cdot u_m \cdot m(u_y - u_y')$$

The gradient of the velocity with respect to the Y -direction

$$= \frac{\partial u_y}{\partial y} = \frac{u_y' - u_y}{j\lambda}$$

since λ is small.

Thus the rate of transfer of momentum per unit area which can be written as R_y

$$\begin{aligned} &= -i_1 \cdot N \cdot u_m \cdot m \cdot j \cdot \lambda \cdot \frac{\partial u_y}{\partial y} \\ &= -i_1 \cdot j \cdot \rho \cdot u_m \cdot \lambda \cdot \frac{\partial u_y}{\partial y} \end{aligned} \quad \dots (8.4)$$

(since $N \cdot m = \rho$, the density of the fluid)

$$\text{But} \quad R_y = -\mu \frac{\partial u_y}{\partial y} \quad (\text{from equation 8.1})$$

$$\therefore \quad \frac{\mu}{\rho} = i_1 \cdot j \cdot u_m \cdot \lambda \quad \dots (8.5)$$

The value of the product $i_1 \cdot j$ has been variously given by different workers, from statistical treatment of the velocities of the molecules⁽²⁾; we shall take a value of $\frac{1}{2}$.

$$\text{Thus} \quad \frac{\mu}{\rho} = \frac{1}{2} u_m \cdot \lambda \quad \dots (8.6)$$

It is now possible to give a physical interpretation to the Reynolds Number, which is complementary to that given in Chapter 3.

$$Re. = \frac{u \cdot d \cdot \rho}{\mu} = u \cdot d \cdot \frac{2}{u_m \cdot \lambda} = 2 \cdot \frac{u}{u_m} \cdot \frac{d}{\lambda} \quad \dots (8.7)$$

i.e. it is proportional to the product of the ratio of the flow velocity to the molecular velocity, and the ratio of the characteristic linear dimension of the system to the mean free path of the molecules.

From the kinetic theory⁽²⁾, $u_m = \sqrt{\frac{8RT}{\pi}}$ and is independent of pressure and $\rho\lambda$ is a constant.

Thus the viscosity of a gas would be expected to be a function of temperature but not of pressure.

Thermal Conductivity

Consider now the case where there is a temperature gradient in the Y -direction.

The rate of passage of molecules through the unit plane $a - a = i_2 \cdot u_m \cdot N$ (where i_2 is some fraction of the order of unity). Now if the temperature difference between two planes situated a distance $j\lambda$ apart is $\theta - \theta'$, the net heat transferred as one molecule passes in one direction and another molecule passes in the opposite direction $= c_m(\theta - \theta')$, where c_m is the heat capacity per molecule.

The net rate of heat transfer per unit area $= i_2 \cdot u_m \cdot N \cdot c_m(\theta - \theta')$.

The temperature gradient, $\frac{\partial \theta}{\partial y} = \frac{\theta' - \theta}{j\lambda}$, since λ is small.

Thus the net rate of heat transfer per unit area

$$\begin{aligned} &= -i_2 \cdot j \cdot u_m \cdot N \cdot c_m \cdot \lambda \cdot \frac{\partial \theta}{\partial y} \\ &= -i_2 \cdot j \cdot u_m \cdot C_v \cdot \rho \cdot \lambda \cdot \frac{\partial \theta}{\partial y} \end{aligned} \quad \dots (8.8)$$

(since $Nc_m = \rho C_v$, the specific heat per unit volume of fluid)

$$= -k \cdot \frac{\partial \theta}{\partial y} \quad (\text{from equation 8.2})$$

Thus the thermal diffusivity,

$$\frac{k}{C_p \cdot \rho} = i_2 \cdot j \cdot u_m \cdot \lambda \cdot \frac{C_v}{C_p} \quad \dots (8.9)$$

From statistical calculations⁽²⁾, the value of $i_2 \cdot j$ has been given as $\frac{9\gamma - 5}{8}$

(where $\gamma = C_p/C_v$, the ratio of the specific heat at constant pressure to the specific heat at constant volume).

$$\text{Thus} \quad \frac{k}{C_p \cdot \rho} = \frac{9\gamma - 5}{8\gamma} u_m \lambda \quad \dots (8.9a)$$

The Prandtl Number $Pr.$ is defined as the ratio of the kinematic viscosity to the thermal diffusivity.

$$\begin{aligned} \text{Thus} \quad Pr. &= \frac{\frac{\mu}{\rho}}{\frac{k}{C_p \cdot \rho}} = \frac{C_p \cdot \mu}{k} = \frac{\frac{1}{2} u_m \lambda}{\frac{9\gamma - 5}{8\gamma} u_m \lambda} \\ &= \frac{4\gamma}{9\gamma - 5} \end{aligned} \quad \dots (8.10)$$

Values of $Pr.$ calculated from equation 8.10 are in close agreement with practical figures.

Diffusivity

Consider the diffusion, in the Y -direction, of one constituent, A , of a mixture, across the plane $a - a$.

If the numerical concentration is N on one side of the plane and N' on the other side at a distance of $j\lambda$:

Net rate of passage of molecules per unit area

$$= i_3 \cdot u_m \cdot (N - N')$$

where i_3 is an appropriate fraction of the order of unity.

Rate of mass transfer per unit area

$$= i_3 \cdot u_m \cdot (N - N') \cdot m$$

Concentration gradient of A in the Y -direction

$$= \frac{\partial C_A}{\partial y} = \frac{(N' - N)}{j\lambda} m$$

Thus rate of mass transfer per unit area

$$= -i_3 \cdot j \cdot \lambda \cdot u_m \cdot \frac{\partial C_A}{\partial y} \quad \dots (8.11)$$

$$= -D \frac{\partial C_A}{\partial y} \text{ (from equation 8.3)}$$

Thus

$$D = i_3 \cdot j \cdot u_m \cdot \lambda \quad \dots (8.12)$$

There is, however, no satisfactory evaluation of the product $i_3 \cdot j$.

The ratio of the kinematic viscosity to the diffusivity is the Schmidt Number, $Sc.$, i.e.

$$Sc. = \frac{\mu/\rho}{D} = \frac{\mu}{\rho D} \quad \dots (8.13)$$

It is thus seen that the kinematic viscosity, the thermal diffusivity and the diffusivity for mass transfer are all proportional to the product of the mean free path and the root mean square velocity of the molecules, and that the expressions for the transfer of momentum, heat, and mass are of the same form.

EDDY TRANSFER

If a fluid is flowing under turbulent conditions in a fixed direction, the average velocity of flow in each of the other two directions at right angles will be zero. At any instant, however, the fluid may have a finite velocity in one of these two directions due to the presence of circulating currents. Suppose the mean component of velocity due to the eddy currents to be u_E , irrespective of sense, in the X -, Y - and Z -directions. Superimposed on this is the forward velocity in the X -direction; thus the mean deviation in the X -direction will also be u_E . Under these conditions the turbulence is said to be isotropic.

For turbulent motion, PRANDTL⁽¹⁸⁾ and TAYLOR have defined "a mixing length" λ_E as the mean distance which a fluid in an eddy travels at right angles to the direction of flow before it can be considered to have lost its identity and to

be assimilated by the fluid at this new position. It will be seen that the mixing length is analogous to the mean free path for molecular diffusion. By using the same reasoning as was used for molecular diffusion it can be shown that the eddy kinematic viscosity E which controls momentum transfer in a turbulent fluid, is proportional to the product $u_E \lambda_E$.

PRANDTL suggested that the mean eddy velocity u_E would be proportional to the stream velocity and hence to the velocity gradient $\frac{\partial u_y}{\partial y}$. Since u_E will also increase as the mixing length λ_E increases, he assumed that

$$u_E \propto \lambda_E \left| \frac{\partial u_y}{\partial y} \right| \quad \dots(8.14)$$

where $\left| \frac{\partial u_y}{\partial y} \right|$ denotes the positive value of the velocity gradient.

We now have that

$$E \propto \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \quad \dots(8.15)$$

Since λ_E cannot be directly determined, it is convenient to put the proportionality constant arbitrarily equal to unity and write equation 8.15,

$$E = \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \quad \dots(8.16)$$

in which λ_E now has a slightly different meaning.

The rate of transfer due to eddy motion is therefore given by

$$R_y = - E \rho \frac{\partial u_y}{\partial y} \quad \dots(8.17)$$

$$= - \lambda_E^2 \rho \frac{\partial u_y}{\partial y} \left| \frac{\partial u_y}{\partial y} \right| \quad \dots(8.18)$$

In a similar manner, we can define an eddy thermal diffusivity E_H for heat transfer and an eddy diffusivity E_D for mass transfer. Both E_H and E_D will be proportional to the product $u_E \cdot \lambda_E$ and hence to E , i.e.

$$E_D \propto E_H \propto \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \quad \dots(8.19)$$

Thus the rate of transfer of heat and mass by eddy motion are given by the equations,

$$q_y = - E_H \frac{\partial(C_p \rho \theta)}{\partial y} \propto - \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \frac{\partial(C_p \rho \theta)}{\partial y} \quad \dots(8.20)$$

$$N_A = - E_D \frac{\partial C_A}{\partial y} \propto - \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \frac{\partial C_A}{\partial y} \quad \dots(8.21)$$

The ratio of the eddy kinematic viscosity to the eddy thermal diffusivity E/E_H is known as the turbulent Prandtl Number; the ratio of the eddy kinematic

viscosity to the eddy diffusivity for mass transfer E/E_D is known as the turbulent Schmidt Number.

When the transfer rates by molecular and eddy motion are of a comparable magnitude, both processes must be taken into account and the relations for momentum, heat, and mass transfer become,

$$R_y = - \left(\frac{\mu}{\rho} + E \right) \frac{\partial(\rho u_y)}{\partial y} \quad \dots(8.22)$$

$$q_y = - \left(\frac{k}{C_p \rho} + E_H \right) \frac{\partial(C_p \rho \theta)}{\partial y} \quad \dots(8.23)$$

$$N_A = - (D + E_D) \frac{\partial C_A}{\partial y} \quad \dots(8.24)$$

The relations can be tabulated as follows:

	<i>Molecular processes only</i>	<i>Molecular and eddy transfer together</i>	<i>Eddy transfer predominating</i>
Momentum transfer	$R_y = - \frac{\mu}{\rho} \frac{\partial(\rho u_y)}{\partial y}$	$R_y = - \left(\frac{\mu}{\rho} + E \right) \frac{\partial(\rho u_y)}{\partial y}$	$R_y = - E \frac{\partial(\rho u_y)}{\partial y}$
Heat transfer	$q_y = - \frac{k}{C_p \rho} \frac{\partial(C_p \rho \theta)}{\partial y}$	$q_y = - \left(\frac{k}{C_p \rho} + E_H \right) \frac{\partial(C_p \rho \theta)}{\partial y}$	$q_y = - E_H \frac{\partial(C_p \rho \theta)}{\partial y}$
Mass transfer	$N_A = - D \frac{\partial C_A}{\partial y}$	$N_A = - (D + E_D) \frac{\partial C_A}{\partial y}$	$N_A = - E_D \frac{\partial C_A}{\partial y}$

More detailed consideration will be given in Chapter 9 to the flow conditions and rates of transfer of heat and mass in the boundary layer, the region close to the surface over which the fluid is flowing.

QUANTITATIVE RELATIONS BETWEEN HEAT, MASS, AND MOMENTUM TRANSFER

We have seen in Chapter 6 that the general equation for heat transfer by forced convection without change in phase can be expressed in the form of dimensionless groups as,

$$Nu. = C . Re.^{w_1} . Pr.^{w_2} \quad (\text{from equations 6.28 and 6.30})$$

The Reynolds group determines, to a great extent, the nature of the flow pattern in the fluid, and in Chapter 3 it was shown that the drop in pressure along a pipe in which a fluid was flowing could be related to the Reynolds group by means of a friction chart. Since heat transfer and momentum transfer are thus both related to the Reynolds group, a number of proposals have been made to show a direct relation between the heat transfer and the distribution of momentum of the stream. The first of these was made in 1874 by REYNOLDS⁽¹⁰⁾, who considered the equilibrium set up when an element of fluid moved from a region at high temperature to a solid surface, at a lower temperature.

Suppose a mass M of fluid situated at a distance from the surface to be moving with a velocity u_s in the X -direction. If this element moves to the surface where the velocity is zero it will give up its momentum $M \cdot u_s$ in time t , say. If the temperature difference between the element and the surface is θ_s , and C_p is the specific heat of the fluid, the heat transferred to the surface will be $M \cdot C_p \cdot \theta_s$. Over a surface of area A we can then write that the rate of heat transfer given by

$$\frac{M \cdot C_p \cdot \theta_s}{t} = -q \cdot A \quad \dots(8.25)$$

where q is the heat transferred away from the surface per unit area per unit time.

If the shear stress at the surface is R , the shearing force over the area A will equal the rate of change in momentum, i.e.

$$\frac{M \cdot u_s}{t} = R \cdot A \quad \dots(8.26)$$

Thus
$$\frac{C_p \theta_s}{u_s} = -\frac{q}{R} \quad \dots(8.27)$$

and writing h as the heat transfer coefficient between the fluid and the surface,

$$-\frac{q}{\theta_s} = h = \frac{RC_p}{u_s}$$

or
$$\frac{h}{C_p \rho u_s} = \frac{R}{\rho u_s^2} \quad \dots(8.28)$$

The dimensionless group, $\frac{h}{C_p \rho u_s}$, is known as the Stanton Group St .

We have seen in Chapter 3 that for the flow through a tube the friction factor $R/\rho u^2$ is related to the Reynolds group by the Blasius equation, $R/\rho u^2 \propto Re^{-0.25}$, so that we obtain the relation between heat transfer and the Reynolds group as,

$$\frac{h}{C_p \rho u_s} \propto Re^{-0.25} \quad \dots(8.29)$$

since the mean velocity u is proportional to the velocity u_s remote from the surface.

In this analysis there have been a number of simplifying assumptions which should be noted. Thus the turbulent nature of the fluid was assumed to exist up to the surface and no allowance has been made for the variations in physical properties of the fluid. A more detailed consideration of the analogy between heat transfer and friction is given later (p. 289).

The j -Factor of Chilton and Colburn

For heat transfer by convection to a fluid flowing through a tube, CHILTON and COLBURN^{(23) (26)} have written equation 6.31 as

$$Nu. = 0.023 Re^{0.8} Pr^{0.33} \quad \dots(8.30)$$

where the viscosity is measured at the mean film temperature.

If both sides are divided by the product $Re.Pr.$, we get,

$$\frac{Nu.}{Re.Pr.} = St. = \frac{h}{C_p \rho u} = 0.023 \cdot Re.^{-0.2} \cdot Pr.^{-0.67} \quad \dots (8.31)$$

or
$$\left(\frac{h}{C_p \rho u} \right) \cdot \left(\frac{C_p \mu}{k} \right)^{0.67} = j_h = 0.023 \cdot Re.^{-0.2} \quad \dots (8.32)$$

This analysis, in which the j -factor for heat transfer j_h is introduced, is due to CHILTON and COLBURN, who found that a plot of j_h against $Re.$ gave approximately the same curve as the friction chart for flow through tubes. It differs from the simple Reynolds analogy given above in that the Prandtl group, which contains the important physical properties of the fluid, is introduced. Since $Pr.$ for a number of gases has the value of about 0.74, $Pr.^{0.67}$ is nearly unity and the j -factor method will give the same results as the original Reynolds analogy. It is sometimes useful to rewrite j_h for flow through a tube in terms of the change in temperature of the fluid $T_2 - T_1$, the mean temperature difference θ_m and the heat transfer surface A giving

$$j_h = \frac{h}{C_p \rho u} \left[\frac{C_p \cdot \mu}{k} \right]^{0.67} = \frac{(T_2 - T_1) \cdot S}{\theta_m \cdot A} \cdot \left[\frac{C_p \cdot \mu}{k} \right]^{0.67} \quad \dots (8.33)$$

where S is the cross-sectional area of flow.

Mass Transfer Factor j_d

It has been shown in Chapter 7 that the rate of transfer of material A to or from a surface can be expressed in the form of three-dimensionless groups, so that

$$\frac{h_D \cdot L}{D} = f[Re. Sc.] \quad \dots (7.26)$$

where L is some characteristic dimension of the system, and

D is the diffusion coefficient for the material.

By analogy with the derivation given above for heat transfer, CHILTON and COLBURN⁽²⁶⁾ have deduced a factor for mass transfer j_d which they have expressed in the form,

$$j_d = \frac{h_D \cdot P_{Bm}}{u \cdot P} \cdot \left[\frac{\mu}{\rho \cdot D} \right]^{0.67} \quad \dots (8.34)$$

The term P_{Bm}/P , the logarithmic mean of the partial pressure of the inert component B divided by the total pressure, is introduced because in transfer in one direction the concentration of B may alter substantially and $h_D P_{Bm}$ has been found to be more nearly constant than h_D . We have already seen that j_h can be related to the Reynolds group and the friction term and we shall now use the factor j_d obtained in experiments on mass transfer and thus find the experimental relation between j_d and j_h .

Mass Transfer Inside Vertical Tubes. Several workers have measured the rate of transfer from a liquid flowing down the inside wall of a tube to a gas

passing upwards. GILLILAND and SHERWOOD⁽²⁵⁾ have vaporised a number of liquids including water, toluene, aniline and propyl, amyl and butyl alcohols into an air stream flowing up the tube. They worked with a small tube 1 in. diameter (d) and 18 in. long, fitted with calming sections at the top and bottom, and varied the pressure from 0.14 to about 3 atmospheres. The data were plotted logarithmically as,

$$\frac{h_D \cdot d}{D} \cdot \frac{P_{Bm}}{P} \text{ against } Re.$$

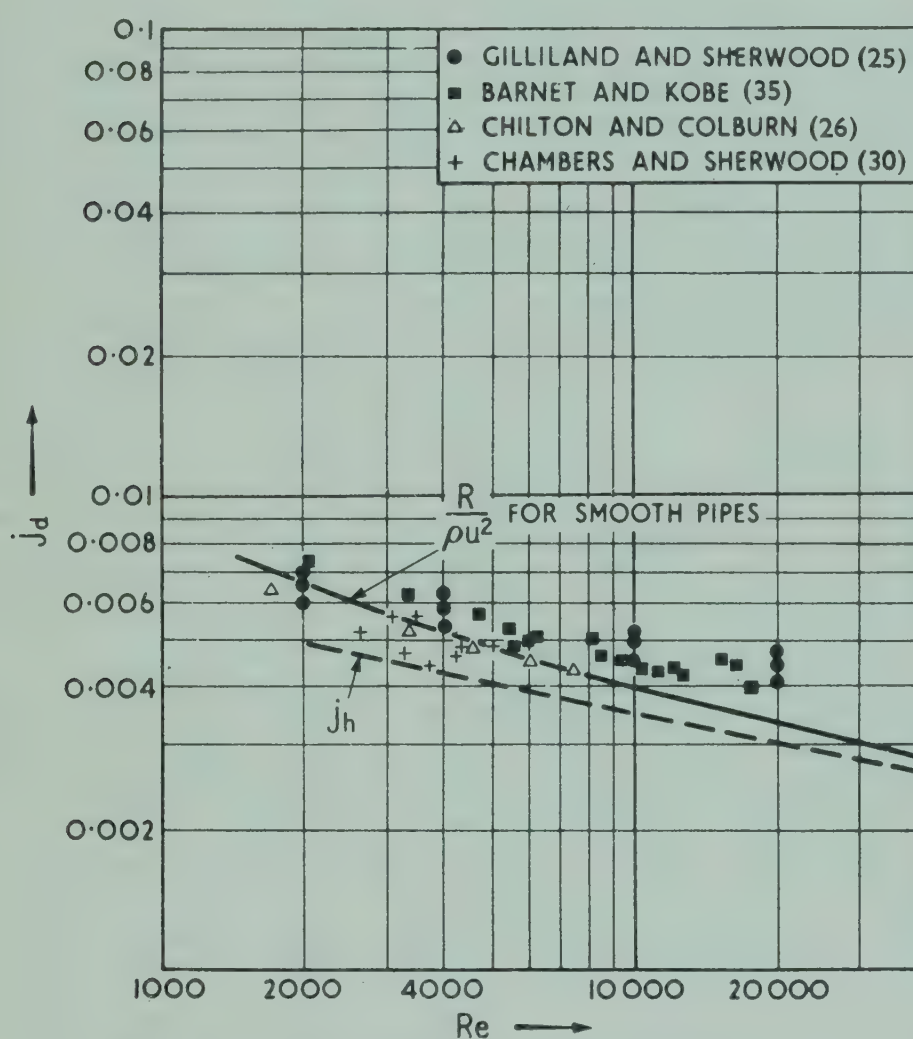


Fig. 8.2. Mass transfer in wetted wall columns

when straight lines are obtained. Introducing the Schmidt group, they were able to bring the points for all the liquids on to the same line, the equation of which was

$$\frac{h_D}{u} \cdot \frac{P_{Bm}}{P} \cdot \left(\frac{\mu}{\rho \cdot D} \right)^{0.56} = 0.023 Re^{-0.17} \quad \dots (8.35)$$

The index of the Schmidt group is much less than the index of 0.67 for the Prandtl group as obtained with heat transfer but the range of values of Sc . used was very small and insufficient to confirm this. SHERWOOD and PIGFORD⁽⁵⁾ have shown that if the data of GILLILAND and SHERWOOD and others are plotted with the Schmidt group raised to the power 0.67, as shown in Fig. 8.2, a reasonably good correlation is obtained. Although the points are rather more scattered than with heat transfer it is reasonable to assume that both j_d and j_h are approximately equal to $R/\rho u^2$.

Mass Transfer to Plane Surfaces. The rate of vaporisation of liquids into an air stream passing over the liquid surface under conditions of forced convection has been measured by several investigators. In addition, some experiments have been carried out in the absence of an airstream, under what have been termed still air conditions. Most of these experiments have been carried out in some form of wind tunnel where the rate of flow of the air could be controlled and its temperature and humidity measured. In such experiments it is important to keep the liquid level constant and to avoid any form of turbulence promoter on the leading edge of the pan.

HINCHLEY and HIMUS⁽¹⁷⁾ measured the rate of vaporisation of water from heated rectangular pans fitting flush with the floor of a wind tunnel 18 in. wide by 9 in. high. They were able to show that the rate of vaporisation was proportional to the pressure difference $P_s - P_w$, where P_s is the vapour pressure of the water and P_w the partial pressure of the water in the air. They presented their results in an empirical equation,

$$W = \text{constant} (P_s - P_w) \quad \dots (8.36)$$

where the constant varies with the geometry of the pan and the velocity.

The importance of this early work lies in the fact that the pressure difference $P_s - P_w$ was shown to be the driving force in the process. More systematic work in more elaborate equipment has been done by POWELL and GRIFFITHS⁽²⁸⁾, WADE⁽³⁶⁾, and by PASQUILL⁽³⁷⁾. WADE, working with a small pan of $3\frac{1}{2}$ in. side in a tunnel, vaporised a variety of organic liquids including acetone, benzene, and trichlorethylene, all at atmospheric pressure. POWELL and GRIFFITHS used a stretched canvas over rectangular pans in such a way that the canvas was always wet and gave the effect of a water surface. For all these liquids the relationships between the rate of vaporisation and both the partial pressure difference and the rate of flow of the air were the same. POWELL and GRIFFITHS found that the rate of vaporisation per unit area of the surface decreased with increase in the down wind length L of the wet surface and that for rectangular pans the rate of vaporisation was proportional to $L^{0.77}$. On this basis SHERWOOD⁽⁵⁾ has plotted the results in terms of the Reynolds group Re_w , with the length of the pan as the characteristic dimension. Fig. 8.3, taken from his work, shows j_d plotted against Re_w for a number of liquids. Although the individual points do not lie too well on a smooth curve it is reasonable to write $j_d \approx j_h$. It should be noted that the Schmidt group was varied only over a small range and that almost equally good results would be obtained by omitting it from the correlation. The inclusion of the term P_{Bm} is not satisfactory, since the results for a number of workers, including those of Wade on many liquids, show that the vaporisation is proportional to $P_s - P_w$ even when this term amounts to $\frac{1}{2}$ atmosphere.

The true influence of the Schmidt group was not established until LINTON and SHERWOOD⁽⁴⁶⁾ measured the rates of solution of benzoic acid, cinnamic acid, and β -naphthol in water. With these relatively insoluble materials the Schmidt

group had values from 1000 to 3000 and the results confirmed the index of $2/3$ for this group.

Since j_h and j_d are approximately equal, the values of mass transfer coefficients can be calculated from the corresponding value of the heat transfer coefficients.

$$\text{Now } \frac{h_D}{u} \cdot \frac{P_{Bm}}{P} Sc^{0.67} = \frac{h}{C_p \rho u} Pr^{0.67} \text{ from equations 8.32 and 8.34}$$

$$\text{so that } h_D = \frac{h}{C_p \rho} \cdot \frac{P}{P_{Bm}} \cdot \left(\frac{Pr}{Sc} \right)^{0.67} \dots (8.37)$$

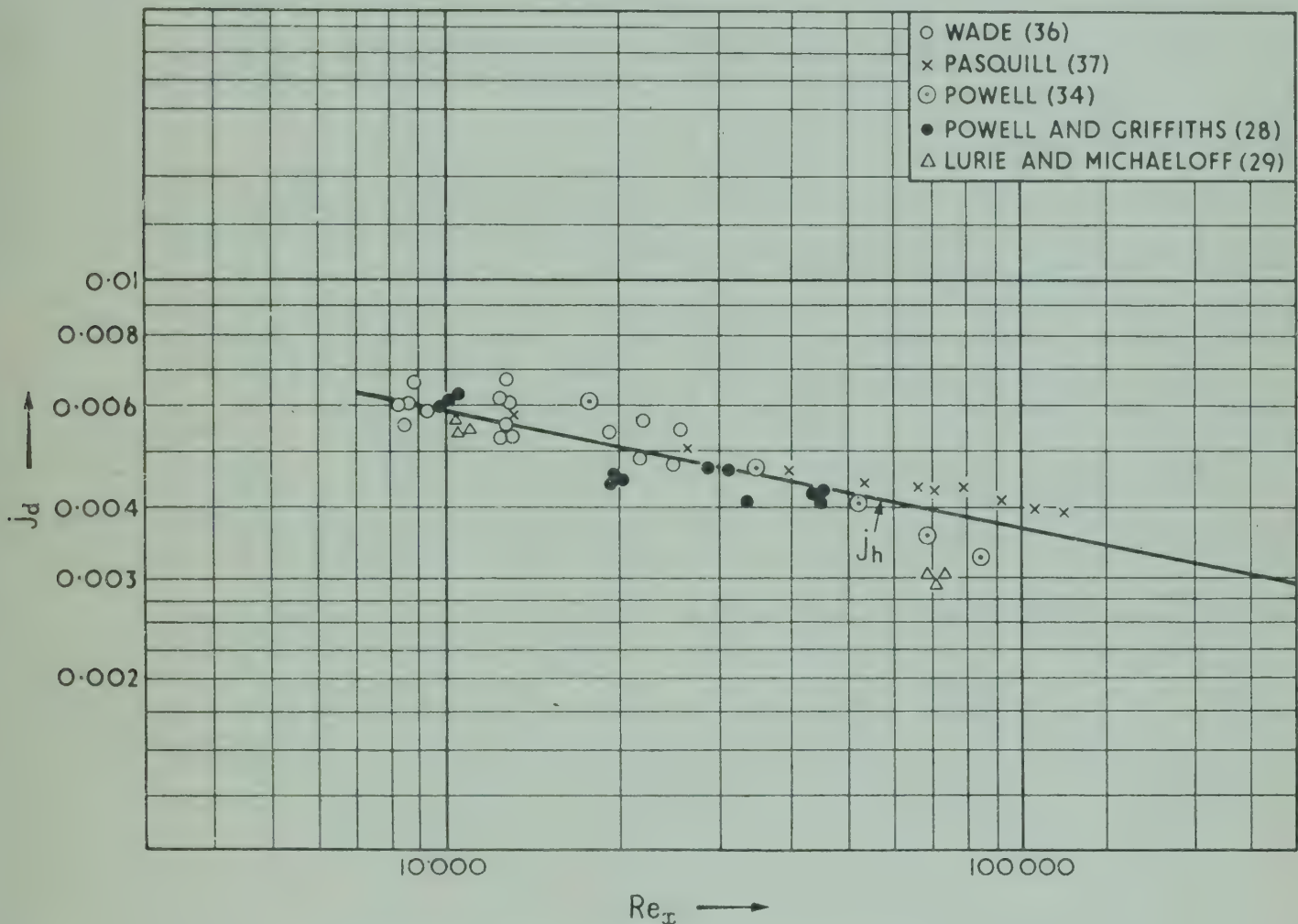


Fig. 8.3. Evaporation from plane surfaces

Difficulties arise in the evaluation of mass transfer coefficients since the composition of the material varies with distance from the surface, and the physical properties therefore are not constant and some mean value must be used. If the Prandtl and the Schmidt Numbers are nearly equal and P/P_{Bm} approaches unity, the above relation reduces to

$$h_D = \frac{h}{C_p \rho} \dots (8.38)$$

This is known as the Lewis relation⁽¹⁶⁾ and can be used to calculate mass transfer coefficients approximately even when the diffusivity of the mixture is not known. Because of the limitations already referred to, the Lewis relation is not generally accurate. However, it applies very closely for the water vapour-air system. The importance of this fact will be considered in detail in Chapter 10, where the problems involved in air humidification are discussed.

The results quoted above give reasonably good support to the treatment of heat, mass, and momentum transfer by the j -factor, but it is important to remember that in all the cases considered the drag is almost entirely in the form of skin friction (i.e. viscous drag at the surface). As soon as an attempt is made to apply the relation to cases where form drag (i.e. additional drag caused by the eddies set up as a result of the fluid impinging on an obstruction) is important, such as beds of granular solids or evaporation from cylinders or spheres, the j -factor and the friction term are found no longer to be equal. This problem will receive further consideration in Volume 2. SHERWOOD carried out experiments where the form drag was large compared with the skin friction which he calculated approximately by subtracting the form drag from the total drag force. He obtained reasonable agreement between the corresponding value of $R/\rho u^2$ and j_h and j_d .

GAMSON *et al.*⁽³⁸⁾ have successfully used the j -factor method to correlate their experimental results for heat and mass transfer between a bed of granular solids and a gas stream.

PRATT⁽⁴³⁾ has examined the effect of using artificially roughened surfaces and of introducing "turbulence promoters," which increase the amount of form drag. He found that the values of $R/\rho u^2$ and the heat and mass transfer coefficients were a minimum for smooth surfaces and all three quantities increased as the surface roughness was increased. $R/\rho u^2$ increased far more rapidly than either of the other two quantities however, and the heat and mass transfer coefficients were found to reach a maximum value whereas $R/\rho u^2$ could be increased almost indefinitely. PRATT has suggested that these maxima occur when the velocity gradient at the surface corresponds with that in the turbulent part of the fluid; i.e. at a condition where the buffer layer ceases to exist (see Chapter 9).

CHAPTER 9

The Boundary Layer

WHEN a fluid flows over a solid surface a velocity gradient is set up at right angles to the direction of flow, because of the viscous forces acting within the fluid. The fluid in contact with the surface must necessarily be brought to rest since, otherwise, the velocity gradient and the shear stress at the surface would be infinite. The drag force resulting from the retardation of the fluid at the surface is transmitted throughout the whole of the fluid and therefore the velocity gradient also extends through the whole of the fluid. At progressively greater distances from the surface, however, the effect of the drag becomes smaller and, for all practical purposes, can be regarded as being confined to a region close to the surface and known as the boundary layer. The whole of the velocity gradient is thus assumed to lie within the boundary layer and, outside it, the velocity is assumed to remain constant.

The thickness of the boundary layer will be a function of the distance from the leading edge of the surface (Fig. 9.1). Since the viscous drag of the fluid can be

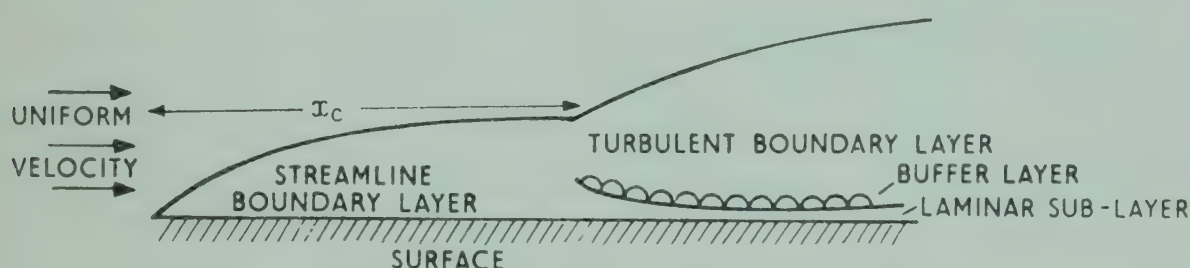


Fig. 9.1. Development of boundary layer

transmitted only at a finite rate, the boundary layer thickness will be zero at the leading edge and gradually increase as the distance from the leading edge increases. In any plane at right angles to the direction of flow, however, the velocity within the boundary layer will vary from zero at the surface to u_s , the velocity of the undisturbed stream, at its outer edge. Where the boundary layer thickness is small, the flow is streamline and the velocity at any distance from the surface is a simple function of that distance. At a certain critical thickness, however, the flow changes from streamline to turbulent, except within a very thin layer near the surface, where it remains streamline: this thin layer is known as the laminar sub-layer. Between the laminar sub-layer and the turbulent portion of the boundary layer is a region in which the flow is neither streamline nor fully turbulent; this is known as the buffer layer.

The change from streamline to turbulent conditions in the boundary layer occurs at a certain critical distance from the leading edge. This distance depends on the shape of the leading edge and the roughness of the surface, and also on the velocity and properties of the fluid: thus with a rough surface or a blunt edge

it is comparatively short. For a given surface, the transition takes place at some critical value of the Reynolds group with respect to distance x from the leading edge. This Reynolds group will be denoted by the symbol Re_x and its critical value at a distance x_c from the leading edge by the symbol Re_{x_c} ; Re_{x_c} is of the order of 10^5 .

A knowledge of conditions in the boundary layer is necessary in order to calculate the resultant drag force of the surface on the fluid and in the calculation of heat and mass transfer coefficients. Expressions will now be derived for the thickness and velocity of flow at any point in the boundary layer.

THE MOMENTUM EQUATION

A fluid of density ρ and viscosity μ flows over a plane surface and the velocity of flow outside the boundary layer is u_s . A boundary layer of thickness δ forms near the surface and at a height y above the surface the velocity of the fluid is reduced to a value u_y .

Consider the equilibrium of an element of fluid bounded by the planes 1-2 and 3-4 at distances x and $x + dx$ from the leading edge; the surface and the plane 2-4 parallel to the surface and at a distance l from it; and by two planes parallel to the plane of the paper and unit distance apart. The distance l is greater than the boundary layer thickness δ (Fig. 9.2).

Consider the velocities and forces in the X -direction.

At plane 1-2, mass rate of flow through a strip of thickness dy at distance y from the surface

$$= \rho \cdot u_y \cdot 1 \cdot dy$$

Total flow through plane 1-2

$$= \int_0^l \rho \cdot u_y \cdot dy$$

Rate of transfer of momentum through the elementary strip

$$= \rho \cdot u_y \cdot 1 \cdot dy \cdot u_y = \rho \cdot u_y^2 \cdot dy$$

Total rate of transfer of momentum through plane 1-2

$$= \int_0^l \rho \cdot u_y^2 \cdot dy$$

In passing from plane 1-2 to plane 3-4, the mass flow changes by

$$\frac{\partial}{\partial x} \left(\int_0^l \rho \cdot u_y \cdot dy \right) dx$$

and the rate of transfer of momentum changes by

$$\frac{\partial}{\partial x} \left(\int_0^l \rho \cdot u_y^2 \cdot dy \right) dx$$

A mass flow of fluid equal to the difference between the flows at planes 3–4 and 1–2 must therefore occur through plane 2–4.

Since plane 2–4 lies outside the boundary layer, this entering fluid must have a velocity u_s in the X -direction.

Thus the rate of transfer of momentum through plane 2–4 into the element

$$= u_s \cdot \frac{\partial}{\partial x} \left(\int_0^l \rho \cdot u_y \cdot dy \right) \cdot dx$$

The net rate of change of momentum in the X -direction on the element must be equal to the momentum added from outside together with the net force acting on it.

The forces in the X -direction acting on the element of fluid are:

(1) A shear force resulting from the shear stress R_0 acting at the surface. This is a retarding force and therefore R_0 is negative.

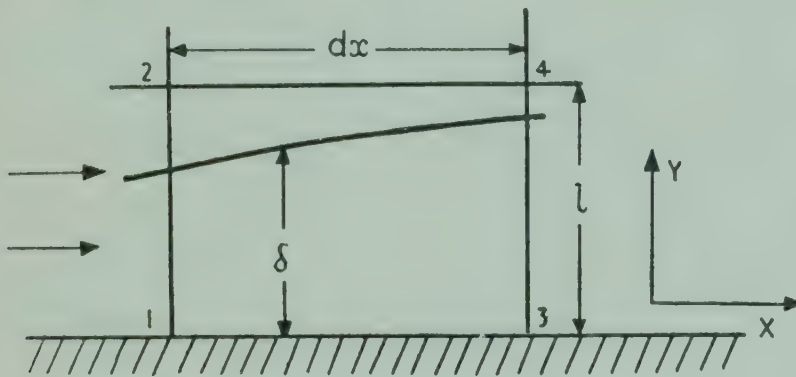


Fig. 9.2. Element of boundary layer

(2) The force produced as a result of a difference in pressure dP between the planes 3–4 and 1–2.

$$\text{Thus the net force acting} = R_0 dx - \frac{\partial P}{\partial x} dx \cdot l$$

Hence

$$\frac{\partial}{\partial x} \left(\int_0^l \rho \cdot u_y^2 \cdot dy \right) \cdot dx = u_s \frac{\partial}{\partial x} \left(\int_0^l \rho \cdot u_y \cdot dy \right) \cdot dx + R_0 \cdot dx - l \cdot \frac{\partial P}{\partial x} dx$$

$$\text{i.e.} \quad \frac{\partial}{\partial x} \int_0^l \rho (u_s - u_y) \cdot u_y \cdot dy = -R_0 + l \cdot \frac{\partial P}{\partial x} \quad \dots (9.1)$$

This expression, known as the Momentum Equation, can be integrated if the relation between u_y and y is known. The pressure drop in the X -direction will be negligible if the velocity of the main stream remains constant at u_s , and for an incompressible fluid or for a compressible fluid where the pressure changes are small compared with the total pressure, the expression can therefore be written

$$\rho \frac{\partial}{\partial x} \int_0^l (u_s - u_y) \cdot u_y \cdot dy = -R_0 \quad \dots (9.2)$$

It will be noted that no assumptions have been made concerning the nature of the flow within the boundary layer and therefore this relation is applicable to

both the streamline and the turbulent regions. The relation between u_y and y will now be derived for streamline and turbulent flow over a plane surface and equation 9.2 will be integrated.

THE STREAMLINE PORTIONS OF THE BOUNDARY LAYER

In the streamline boundary layer, the only forces acting within the fluid are pure viscous forces and no transfer of momentum takes place by eddy motion. Assume that the relation between u_y and y can be expressed approximately by

$$u_y = u_0 + a'y + b'y^2 + c'y^3 \quad \dots (9.3)$$

The coefficients a' , b' , c' , and u_0 can be evaluated because the boundary conditions which the relation must satisfy are known (Fig. 9.3).

At the surface, the velocity is reduced to zero and therefore $u_0 = 0$. Further, close to the surface, the velocity of flow and hence the momentum of the fluid is very low, so that the rate of change of momentum in the X -direction of a

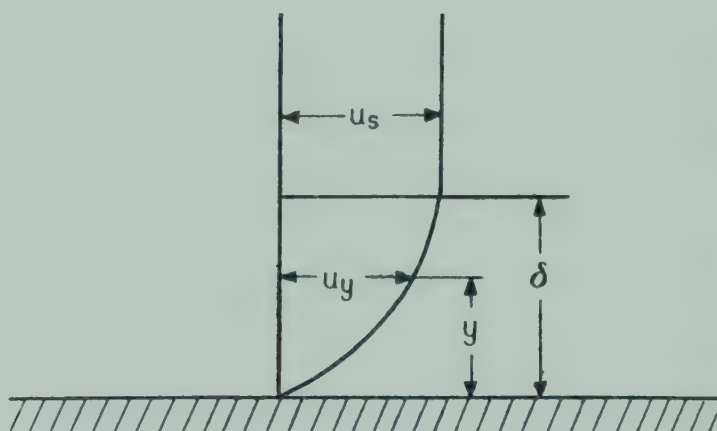


Fig. 9.3. Velocity distribution in streamline boundary layer

given layer of fluid is negligible and the frictional forces resulting from the viscous drag of the fluid must therefore be constant in the Y -direction near the surface since this fluid is not appreciably accelerated or retarded. The shear stress within the fluid at the surface

$$= R_0 = -\mu \left(\frac{\partial u_y}{\partial y} \right)_{y=0} = \text{constant}$$

Thus

$$\left(\frac{\partial^2 u_y}{\partial y^2} \right)_{y=0} = 0$$

At the distant edge of the boundary layer, it is assumed that the velocity is equal to the main stream velocity.

Thus, when $y = \delta$, $u_y = u_s$ and $\frac{\partial u_y}{\partial y} = 0$

and equation 9.3 becomes $u_y = a'y + b'y^2 + c'y^3$

$$\frac{\partial u_y}{\partial y} = a' + 2b'y + 3c'y^2$$

and

$$\frac{\partial^2 u_y}{\partial y^2} = 2b' + 6c'y$$

At $y = 0$,
$$\frac{\partial^2 u_y}{\partial y^2} = 0$$

Thus
$$b' = 0$$

At $y = \delta$,
$$u_y = a'\delta + c'\delta^3 = u_s$$

and
$$\frac{\partial u_y}{\partial y} = a' + 3c'\delta^2 = 0$$

Thus
$$a' = -3c'\delta^2$$

Hence
$$c' = -u_s/2\delta^3 \text{ and } a' = 3u_s/2\delta$$

The equation for the velocity profile is, therefore,

$$u_y = \frac{3u_s}{2} \frac{y}{\delta} - \frac{u_s}{2} \left(\frac{y}{\delta}\right)^3 \quad \dots(9.4)$$

i.e.
$$\frac{u_y}{u_s} = \frac{3}{2} \left(\frac{y}{\delta}\right) - \frac{1}{2} \left(\frac{y}{\delta}\right)^3 \quad \dots(9.4a)$$

This relation applies over the range $0 < y < \delta$.

When $y > \delta$,
$$u = u_s \quad \dots(9.5)$$

The integral in the momentum equation (9.2) can now be evaluated for the streamline boundary layer by considering the ranges $0 < y < \delta$ and $\delta < y < l$ separately.

Thus

$$\begin{aligned} \int_0^\delta (u_s - u_y) \cdot u_y \, dy &= \int_0^\delta u_s^2 \left(1 - \frac{3}{2} \frac{y}{\delta} + \frac{y^3}{2\delta^3}\right) \left(\frac{3}{2} \frac{y}{\delta} - \frac{y^3}{2\delta^3}\right) dy + \int_\delta^l (u_s - u_s) \cdot u_s \, dy \\ &= u_s^2 \int_0^\delta \left(\frac{3}{2} \frac{y}{\delta} - \frac{9}{4} \frac{y^2}{\delta^2} - \frac{1}{2} \frac{y^3}{\delta^3} + \frac{3}{2} \frac{y^4}{\delta^4} - \frac{1}{4} \frac{y^6}{\delta^6}\right) dy \\ &= u_s^2 \cdot \delta \left(\frac{3}{4} - \frac{3}{4} - \frac{1}{8} + \frac{3}{10} - \frac{1}{28}\right) \\ &= \frac{39}{280} u_s^2 \cdot \delta \quad \dots(9.6) \end{aligned}$$

Now
$$R_0 = -\mu \left(\frac{\partial u_y}{\partial y}\right)_{y=0} = \frac{-3}{2} \mu \frac{u_s}{\delta} \quad \dots(9.7)$$

Substitution from equations 9.6 and 9.7 in equation 9.2, gives,

$$\rho \frac{\partial}{\partial x} \left(\frac{39}{280} \delta u_s^2\right) = \frac{3}{2} \mu \cdot \frac{u_s}{\delta}$$

$$\therefore \delta \cdot d\delta = \frac{140}{13} \cdot \frac{\mu}{\rho} \cdot \frac{1}{u_s} \cdot dx$$

$$\therefore \frac{\delta^2}{2} = \frac{140}{13} \cdot \frac{\mu x}{\rho u_s} \text{ (since } \delta = 0 \text{ when } x = 0) \quad \dots(9.8)$$

Thus

$$\delta = 4.64 \sqrt{\frac{\mu x}{\rho u_s}} \quad \dots(9.8a)$$

and

$$\frac{\delta}{x} = 4.64 \sqrt{\frac{\mu}{x \rho u_s}} = 4.64 Re_x^{-\frac{1}{2}} \quad \dots(9.9)$$

This relation for the thickness of the boundary layer has been obtained on the assumption that the velocity profile can be described by a polynomial of the form of equation 9.3 and that the main stream velocity is reached at a distance δ from the surface, whereas, in fact, the stream velocity is approached asymptotically. Whereas equation 9.4 gives the velocity u_y accurately as a function of y , it does not provide a means of calculating accurately the distance from the surface at which u_y has a particular value, when u_y is near u_s . The thickness of the boundary layer as calculated is therefore a function of the particular approximate relation which is taken to represent the velocity profile. This difficulty can be overcome by introducing a new term, the displacement thickness δ^* .

When a viscous fluid flows over a surface, it is retarded and the overall flow rate is therefore reduced. A non-viscous fluid, however, would not be retarded and therefore a boundary layer would not form. The displacement thickness δ^* is defined as the distance the surface would have to be moved, in the Y -direction, in order to obtain the same rate of flow with this non-viscous fluid as would be obtained with the viscous fluid.

Mass rate of flow of a frictionless fluid between $y = \delta^*$ and $y = \infty$

$$= \rho \int_{\delta^*}^{\infty} u_s \cdot dy$$

Mass rate of flow of the real fluid between $y = 0$ and $y = \infty$

$$= \rho \int_0^{\infty} u_y \cdot dy$$

Then, by definition of the displacement thickness,

$$\rho \int_{\delta^*}^{\infty} u_s \cdot dy = \rho \int_0^{\infty} u_y \cdot dy$$

i.e.

$$\int_0^{\infty} u_s \cdot dy - \int_0^{\delta^*} u_s \cdot dy = \int_0^{\infty} u_y \cdot dy$$

giving

$$\delta^* = \int_0^{\infty} \left(1 - \frac{u_y}{u_s}\right) dy \quad \dots(9.10)$$

$$= \int_0^{\delta} \left(1 - \frac{3}{2} \frac{y}{\delta} + \frac{1}{2} \frac{y^3}{\delta^3}\right) dy$$

(since equation 9.4a applies only in the limits $0 < y < \delta$ and outside this region $u_y = u_s$ and the integral is zero).

i.e.

$$\delta^* = \delta \left(1 - \frac{3}{4} + \frac{1}{8}\right)$$

$$= 0.375\delta \quad \dots(9.11)$$

Shear Stress at the Surface

The shear stress in the fluid at the surface,

$$\begin{aligned}
 R_0 &= -\frac{3}{2} \mu \frac{u_s}{\delta} \quad (\text{from equation 9.7}) \\
 &= -\frac{3}{2} \mu \cdot u_s \cdot \frac{1}{x} \cdot \frac{1}{4.64} \sqrt{\frac{x \rho u_s}{\mu}} \\
 &= -0.323 \rho u_s^2 \sqrt{\frac{\mu}{x \rho u_s}} = -0.323 \rho u_s^2 \cdot Re_x^{-\frac{1}{2}}
 \end{aligned}$$

The shear stress R acting on the surface itself is equal to $-R_0$.

Thus
$$\frac{R}{\rho u_s^2} = 0.323 Re_x^{-\frac{1}{2}} \quad \dots (9.12)$$

This gives the point value of R and $\frac{R}{\rho u_s^2}$ at $x = x$. In order to calculate the total frictional force acting at the surface, it is necessary to multiply the average value of R , between $x = 0$ and $x = x$, by the area of the surface.

The average value of $\frac{R}{\rho u_s^2}$ denoted by the symbol $\left(\frac{R}{\rho u_s^2}\right)_m$ is given by,

$$\begin{aligned}
 \left(\frac{R}{\rho u_s^2}\right)_m \cdot x &= \int_0^x \frac{R}{\rho u_s^2} \cdot dx \\
 &= \int_0^x 0.323 \sqrt{\frac{\mu}{x \rho u_s}} \cdot dx \quad (\text{from equation 9.12}) \\
 &= 0.646 x \sqrt{\frac{\mu}{x \rho u_s}} \\
 \therefore \left(\frac{R}{\rho u_s^2}\right)_m &= 0.646 \sqrt{\frac{\mu}{x \rho u_s}} \\
 &= 0.646 Re_x^{-0.5} \approx 0.65 Re_x^{-0.5} \quad \dots (9.13)
 \end{aligned}$$

THE TURBULENT PORTION OF THE BOUNDARY LAYER

A useful simplified treatment of the flow conditions within the turbulent boundary layer is obtained by neglecting the existence of the buffer layer (Fig. 9.4). It is assumed that there is a region close to the surface, the laminar sub-layer, in which momentum transfer is by molecular motion alone; outside this region the viscous forces are assumed to be negligible so that transfer is effected entirely by eddy motion. The method is based on the assumption that the shear stress at a plane surface can be calculated from the simple power law developed by BLASIUS, already referred to in Chapter 3.

BLASIUS⁽¹³⁾ has given an approximate expression for the shear stress at a plane smooth surface over which a fluid is flowing with a velocity u_s , for conditions where $Re_x < 10^7$. His equation is as follows,

$$\frac{R}{\rho u_s^2} = 0.0228 \left(\frac{\mu}{u_s \delta \rho} \right)^{0.25} \quad \dots (9.14)$$

Thus the shear stress is expressed as a function of the boundary layer thickness δ and therefore implicitly assumes a certain velocity profile in the fluid. As a first assumption, assume that a simple power relation exists between the velocity and the distance from the surface in the boundary layer, i.e.

$$\left(\frac{u_y}{u_s} \right) = \left(\frac{y}{\delta} \right)^p \quad \dots (9.15)$$

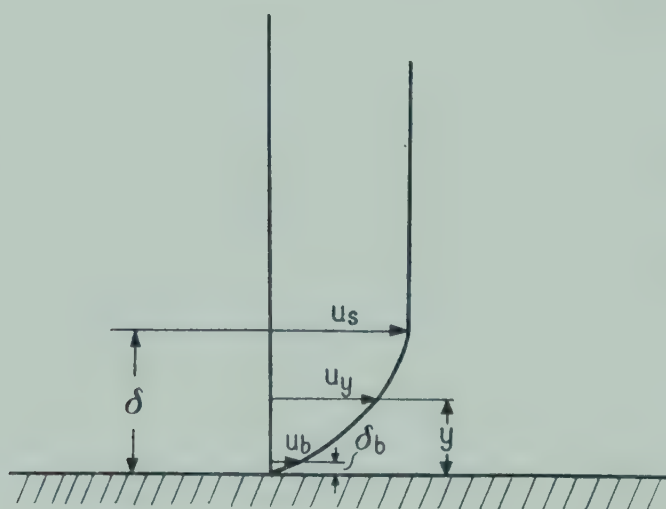


Fig. 9.4. Turbulent boundary layer

$$\begin{aligned} \text{Hence } R &= 0.0228 \rho u_s^2 \left(\frac{\mu}{u_s \delta \rho} \right)^{0.25} \\ &= 0.0228 \rho^{0.75} \mu^{0.25} \delta^{-0.25} u_s^{1.75} \\ &= 0.0228 \rho^{0.75} \mu^{0.25} \delta^{-0.25} u_y^{1.75} \left(\frac{\delta}{y} \right)^{1.75p} \quad (\text{from equation 9.15}) \\ &= 0.0228 \rho^{0.75} \mu^{0.25} u_y^{1.75} y^{-1.75p} \delta^{1.75p-0.25} \quad \dots (9.16) \end{aligned}$$

If the velocity profile is the same for all stream velocities, the shear stress must be defined by specifying the velocity u_y at any distance y from the surface. The boundary layer thickness (determined by the velocity profile) is no longer an independent variable so that the index of δ in equation 9.16 must be zero, i.e.

$$1.75p - 0.25 = 0$$

and

$$p = \frac{1}{7}$$

Thus

$$\frac{u_y}{u_s} = \left(\frac{y}{\delta} \right)^{1/7} \quad \dots (9.17)$$

This is sometimes known as the Prandtl seventh power law.

In the analysis, no assumption has been made concerning the shape of the surface and this relation therefore applies to cylindrical as well as plane surfaces.

Differentiating equation 9.17 with respect to y ,

$$\frac{\partial u_y}{\partial y} = \frac{1}{7} u_s \cdot \delta^{-1/7} y^{-6/7} \quad \dots (9.18)$$

This relation gives an infinite velocity gradient at the surface and a finite velocity gradient at the outer edge of the boundary layer. This is in contradiction to the conditions which we have seen will exist in the stream. However, little error will be introduced by using this relation for the whole of the boundary layer in the momentum equation, since the velocities and hence the momenta near the surface are very low and it gives the correct value of the velocity at the edge of the boundary layer. Accepting equation 9.17 for the limits $0 < y < \delta$, the integral in equation 9.2 becomes,

$$\begin{aligned} \int_0^l (u_s - u_y) u_y \cdot dy &= u_s^2 \left\{ \int_0^\delta \left[1 - \left(\frac{y}{\delta} \right)^{1/7} \right] \left(\frac{y}{\delta} \right)^{1/7} \cdot dy \right\} + \int_\delta^l (u_s - u_s) \cdot u_s \cdot dy \\ &= u_s^2 \int_0^\delta \left\{ \left(\frac{y}{\delta} \right)^{1/7} - \left(\frac{y}{\delta} \right)^{2/7} \right\} dy \\ &= u_s^2 \cdot \delta \left(\frac{7}{8} - \frac{7}{9} \right) \\ &= \frac{7}{72} u_s^2 \cdot \delta \quad \dots (9.19) \end{aligned}$$

Now, from the Blasius Equation,

$$-R_0 = R = 0.0228 \rho u_s^2 \left(\frac{\mu}{u_s \delta \rho} \right)^{1/4} \quad \dots (9.14)$$

Substituting from equations 9.14 and 9.19 in equation 9.2,

$$\rho \frac{\partial}{\partial x} \cdot \frac{7}{72} u_s^2 \cdot \delta = 0.0228 \rho u_s^2 \left(\frac{\mu}{u_s \delta \rho} \right)^{1/4}$$

$$\text{i.e.} \quad \delta^{5/4} dx = 0.235 \left(\frac{\mu}{u_s \cdot \rho} \right)^{1/4} \cdot dx$$

$$\text{i.e.} \quad \frac{4}{5} \delta^{5/4} = 0.235 x \left(\frac{\mu}{u_s \cdot \rho} \right)^{1/4}$$

This assumes that $\delta = 0$ when $x = 0$, i.e. that the turbulent boundary layer extends to the leading edge of the surface. An error is introduced by this assumption, but it is small, except where the surface is only slightly longer than the critical distance x_c .

$$\text{Thus} \quad \delta = 0.376 x^{0.8} \left(\frac{\mu}{u_s \cdot \rho} \right)^{0.2} \quad \dots (9.20)$$

$$= 0.376 x \left(\frac{\mu}{u_s \rho x} \right)^{0.2} \quad \dots (9.20a)$$

$$\text{i.e.} \quad \frac{\delta}{x} = 0.376 Re_x^{-0.2} \quad \dots (9.20b)$$

Now the displacement thickness δ^* is given by equation 9.10,

$$\delta^* = \int_0^\infty \left(1 - \frac{u_y}{u_s}\right) dy \quad \dots(9.10)$$

$$= \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/7}\right) dy \text{ (approximately)}$$

$$= \frac{1}{8}\delta \quad \dots(9.21)$$

As already explained δ^* is independent of the particular approximation used for the velocity profile.

It is seen from equations 9.9 and 9.20a that at any given value of x , the rate of increase of thickness of the boundary layer is greater in the turbulent region than in the streamline region. At the transition point between the two types of flow, there is a small critical region in which the flow is unstable.

The Laminar Sub-layer

Suppose that at $x = x$ the laminar sub-layer is of thickness δ_b and that the total thickness of the boundary layer is δ .

It has already been explained on page 262 that the shear stress and hence the velocity gradient are almost constant near the surface. Since the laminar sub-layer is very thin, the velocity gradient within it can therefore be taken as constant.

Thus the shear stress in the fluid at the surface R_0

$$= -\mu \left(\frac{\partial u_y}{\partial y}\right)_{y=0} = -\mu \frac{u_y}{y}, \text{ where } y < \delta_b$$

Equating this to the value obtained from equation 9.14,

$$0.0228\rho u_s^2 \left(\frac{\mu}{u_s\delta\rho}\right)^{\frac{1}{4}} = \mu \frac{u_y}{y}$$

so that

$$u_y = 0.0228\rho u_s^2 \cdot \frac{1}{\mu} \left(\frac{\mu}{u_s\delta\rho}\right)^{\frac{1}{4}} \cdot y$$

If the velocity at the edge of the laminar sub-layer is u_b , i.e. if $u = u_b$ when $y = \delta_b$,

$$u_b = 0.0228\rho u_s^2 \frac{1}{\mu} \left(\frac{\mu}{u_s\delta\rho}\right)^{\frac{1}{4}} \cdot \delta_b$$

$$= 0.0228 \frac{\rho u_s^2}{\mu} \frac{\mu}{u_s\delta\rho} \cdot \delta_b \left(\frac{\mu}{u_s\delta\rho}\right)^{-\frac{3}{4}}$$

Thus

$$\frac{\delta_b}{\delta} = \frac{1}{0.0228} \left(\frac{u_b}{u_s}\right) \left(\frac{\mu}{u_s\delta\rho}\right)^{\frac{3}{4}} \quad \dots(9.22)$$

Now the velocity at the inner edge of the turbulent region must also be given by the equation for the velocity distribution in the turbulent region.

Hence $\left(\frac{\delta_b}{\delta}\right)^{1/7} = \frac{u_b}{u_s}$ (from equation 9.17)

Thus $\left(\frac{u_b}{u_s}\right)^7 = \frac{1}{0.0228} \left(\frac{u_b}{u_s}\right) \left(\frac{\mu}{u_s \delta \rho}\right)^{3/4}$ (from equation 9.22)

i.e. $\frac{u_b}{u_s} = 1.87 \left(\frac{\mu}{u_s \delta \rho}\right)^{1/8}$
 $= 1.87 Re_{\delta}^{-1/8} \dots (9.23)$

Now since $\delta = 0.376 x^{0.8} \left(\frac{\mu}{u_s \rho}\right)^{0.2}$, (equation 9.20),

$$\begin{aligned} \frac{u_b}{u_s} &= 1.87 \left\{ \frac{u_s \rho}{\mu} \cdot 0.376 \cdot \frac{x^{0.8} \mu^{0.2}}{u_s^{0.2} \rho^{0.2}} \right\}^{-1/8} \\ &= \frac{1.87}{0.376^{1/8}} \cdot \left\{ \frac{u_s^{0.8} \cdot x^{0.8} \cdot \rho^{0.8}}{\mu^{0.8}} \right\}^{-1/8} \\ &= 2.11 Re_x^{-0.1} \approx 2.1 Re_x^{-0.1} \dots (9.24) \end{aligned}$$

The thickness of the laminar sub-layer is given by,

$$\frac{\delta_b}{\delta} = \left(\frac{u_b}{u_s}\right)^7 = \frac{190}{Re_x^{0.7}} \text{ (from equations 9.17 and 9.24)}$$

i.e. $\frac{\delta_b}{x} = \frac{190}{Re_x^{0.7}} \cdot \frac{0.376}{Re_x^{0.2}} \text{ (from equation 9.20b)}$
 $= 71.5 Re_x^{-0.9} \dots (9.25)$

Thus $\delta_b \propto x^{0.1}$, i.e. it increases very slowly as x increases. Further $\delta_b \propto u_s^{-0.9}$ and therefore decreases rapidly as the velocity is increased, and heat and mass transfer coefficients are therefore considerably influenced by the velocity.

The shear stress at the surface, at a distance x from the leading edge, is given by

$$R_0 = -\mu \frac{u_b}{\delta_b}$$

Then since $R_0 = -R$,

$$\begin{aligned} R &= \mu \cdot 2.11 \cdot u_s \cdot Re_x^{-0.1} \cdot \frac{1}{x} \cdot \frac{1}{71.5} \cdot Re_x^{0.9} \text{ (from equations 9.24 and 9.25)} \\ &= 0.0296 Re_x^{0.8} \cdot \frac{\mu u_s}{x} \\ &= 0.0296 \rho u_s^2 \cdot Re_x^{-0.2} \approx 0.03 \rho u_s^2 \cdot Re_x^{-0.2} \dots (9.26) \end{aligned}$$

i.e. $\frac{R}{\rho u_s^2} = \frac{0.0296}{Re_x^{0.2}} \dots (9.27)$

The mean value of $R/\rho u_s^2$ over the range $x = 0$ to $x = x$ is given by

$$\begin{aligned}\left(\frac{R}{\rho u_s^2}\right)_m \cdot x &= \int_0^x \left(\frac{R}{\rho u_s^2}\right) dx \\ &= \int_0^x 0.0296 \left(\frac{\mu}{u_s x \rho}\right)^{0.2} dx \\ &= 0.0296 \left(\frac{\mu}{u_s x \rho}\right)^{0.2} \frac{x}{0.8}\end{aligned}$$

i.e.
$$\left(\frac{R}{\rho u_s^2}\right)_m = 0.037 Re_{.x}^{-0.2} \quad \dots (9.28)$$

The total shear force acting on the surface is found by adding the forces acting in the streamline ($x < x_c$) and turbulent ($x > x_c$) regions. This can be done provided the critical value $Re_{.x_c}$ is known.

In the streamline region $R/\rho u_s^2 = 0.646 Re_{.x}^{-0.5} \quad \dots (9.13)$

In the turbulent region $R/\rho u_s^2 = 0.037 Re_{.x}^{-0.2} \quad \dots (9.28)$

In calculating the mean value of $R/\rho u_s^2$ in the turbulent region, it was assumed that the turbulent boundary layer extended to the leading edge. A more accurate value for the mean value of $R/\rho u_s^2$ over the whole surface can be obtained by using the expression for streamline conditions over the range from $x = 0$ to $x = x_c$ (where x_c is the critical distance from the leading edge) and the expression for turbulent conditions in the range $x = x_c$ to $x = x$.

Mean value of $R/\rho u_s^2$

$$\begin{aligned}&= \frac{1}{x} (0.646 Re_{.x_c}^{-0.5} \cdot x_c + 0.037 Re_{.x}^{-0.2} \cdot x - 0.037 Re_{.x_c}^{-0.2} x_c) \\ &= 0.646 Re_{.x_c}^{-0.5} \cdot \frac{Re_{.x_c}}{Re_{.x}} + 0.037 Re_{.x}^{-0.2} - 0.037 Re_{.x_c}^{-0.2} \cdot \frac{Re_{.x_c}}{Re_{.x}} \\ &= 0.037 Re_{.x}^{-0.2} + Re_{.x}^{-1} (0.646 Re_{.x_c}^{0.5} - 0.037 Re_{.x_c}^{0.8}) \quad \dots (9.29)\end{aligned}$$

Example. Water flows at a velocity of 3 f.p.s. over a plane surface, 2 ft wide and 3 ft long. Calculate the total drag force acting on the surface, if the transition from streamline to turbulent flow in the boundary layer occurs when the Reynolds Group, $Re_{.x}$, equals 10^5 .

Solution. Value of $Re_{.x}$ at far end of surface

$$\begin{aligned}&= \frac{3 \times 30.48 \times 3 \times 30.48 \times 1}{0.01} \\ &= 835,000\end{aligned}$$

Mean value of $R/\rho u_s^2$ (from equation 9.29)

$$\begin{aligned}&= \frac{0.037}{835,000^{0.2}} + \frac{1}{835,000} (0.646 \times 100,000^{0.5} - 0.037 \times 100,000^{0.8}) \\ &= 0.00222\end{aligned}$$

Total drag force

$$\begin{aligned}
 &= R/\rho u_s^2 \times \rho u_s^2 \times \text{area of surface} \\
 &= 0.00222 \times 62.3 \times 9 \times 6 \\
 &= 7.5 \text{ poundals} \\
 &= \underline{\underline{0.23 \text{ pound weight}}}
 \end{aligned}$$

Example. Calculate the thickness of the boundary layer at a distance of 6 in. from the leading edge of a surface over which oil, of viscosity 50 centipoises and density 62 lb/ft³, flows with a velocity of 1 f.p.s. What is the displacement thickness of the boundary layer?

Solution.

$$Re_x = \frac{6 \times 2.54 \times 30.48 \times 1}{0.5} = 930$$

$$\delta/x = 4.64 Re_x^{-0.5} \text{ for streamline boundary layer (from equation 9.9)}$$

$$\begin{aligned}
 \therefore \delta &= 4.64 \times 930^{-0.5} \times 6 \text{ in.} \\
 &= 0.91 \text{ in.}
 \end{aligned}$$

$$\text{Displacement thickness} = 0.375 \times 0.91 = \underline{\underline{0.34 \text{ in.}}} \text{ (from equation 9.11)}$$

FLOW OF FLUID IN A PIPE

Entry Conditions

When a fluid flowing with a uniform velocity enters a pipe, a boundary layer forms at the walls and gradually thickens as the distance from the entry point

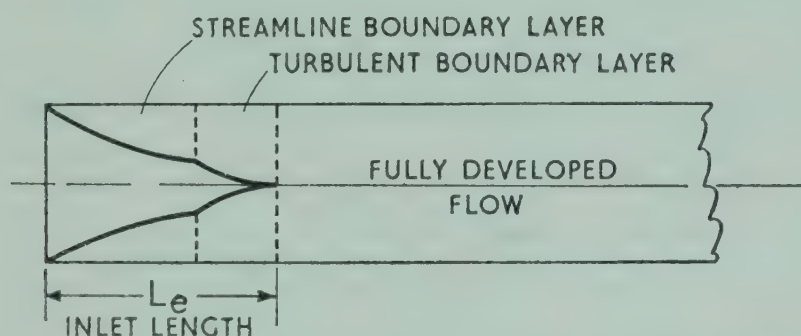


Fig. 9.5. Conditions at entry to pipe

increases. Since the fluid in the boundary layer is retarded and the total flow remains constant, the fluid in the central stream will be accelerated. At a certain distance from the inlet, the boundary layers, which have formed in contact with the walls, join at the axis of the pipe, and, from that point onwards, occupy the whole cross-section and consequently remain of constant thickness. Fully developed flow then exists. If the boundary layers are still streamline when fully developed flow commences, the flow in the pipe remains streamline. On the other hand, if the boundary layers are already turbulent, turbulent flow will persist (Fig. 9.5).

An approximate experimental expression for the inlet length L_e is

$$\frac{L_e}{d} = 0.0288 Re.$$

where d is the diameter of the pipe and Re is the Reynolds group with respect to pipe diameter, and based on the mean velocity of flow in the pipe. This expression is only approximate, and is inaccurate for Reynolds Numbers in the region of 2500 because the boundary layer thickness increases very rapidly in this region. An average value of L_e at a Reynolds number of 2500 is $50-100d$. The inlet length is somewhat arbitrary as steady conditions in the pipe are approached asymptotically—the boundary layer thickness being a function of the assumed velocity profile.

At the inlet to the pipe the velocity across the whole section will be constant. At some distance from the inlet, the velocity at the pipe axis will have increased and will reach a maximum value when the boundary layer joins. After this point the whole velocity distribution, and hence the velocity at the axis, will remain constant. Since the velocity of the fluid at the axis is increased, the

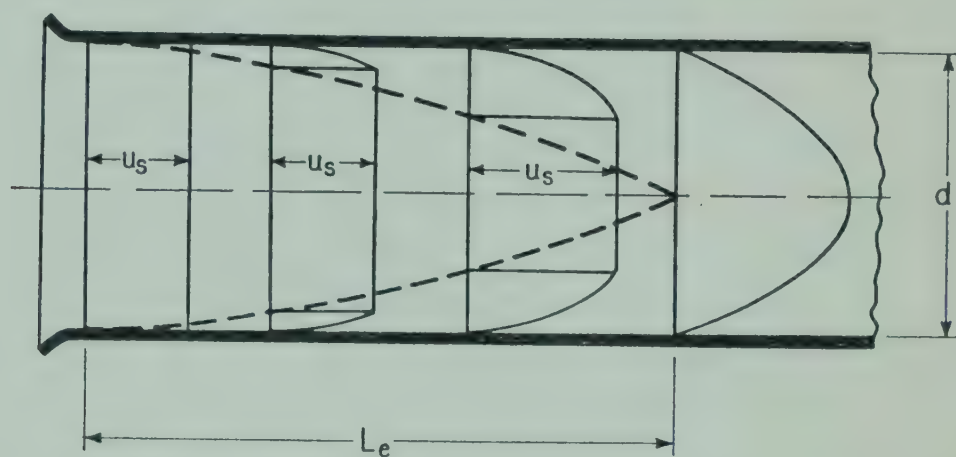


Fig. 9.6. Development of laminar velocity profile at entry to pipe

kinetic energy per unit mass increases, and therefore there must be a corresponding fall in the pressure energy.

Under streamline conditions, the velocity at the axis u_s will increase from a value u at the inlet to a value $2u$ where fully developed flow exists (Fig. 9.6).

Thus kinetic energy per unit mass of the fluid at the axis at inlet $= \frac{1}{2} u^2$.

Corresponding kinetic energy at end of inlet length $= \frac{1}{2} (2u)^2 = 2 \cdot u^2$.

Increase in kinetic energy per unit mass $= \frac{3}{2} u^2$.

Thus fall in pressure due to increase of velocity of fluid $= \frac{3}{2} \rho u^2$.

If the flow in the pipe is turbulent, the velocity at the axis increases from u to only about $\frac{u}{0.82}$.

Under these conditions, fall in pressure

$$\begin{aligned}
 &= \frac{1}{2} \rho u^2 \left(\frac{1}{0.82^2} - 1 \right) \\
 &= \frac{1}{4} \rho u^2
 \end{aligned}$$

The existence of a *vena contracta* near the entry to the pipe has been neglected here.

Application of Boundary Layer Theory

The velocity distribution and frictional resistance have already been calculated from purely theoretical considerations for the streamline flow of a fluid in a pipe. The boundary layer theory can now be applied in order to calculate, approximately, the conditions when the fluid is turbulent. For this purpose, it is assumed that the boundary layer expressions can be applied to flow over a cylindrical surface and that the flow conditions in the region of fully developed flow are the same as those when the boundary layers first join. The thickness of the boundary layer is thus taken to be equal to the radius of the pipe and the velocity at the outer edge of the boundary layer is assumed to be the velocity at the axis.

The velocity of the fluid can be assumed to obey the Prandtl seventh power law. If the boundary layer thickness δ is replaced by the pipe radius r this becomes

$$u_y = u_s \left(\frac{y}{r} \right)^{1/7} \quad \dots (9.17a)$$

The relation between the mean velocity and the velocity at the axis has already been derived using this expression in Chapter 3. The mean velocity u was found to be 0.82 times the velocity u_s at the axis but in this calculation the thickness of the laminar sub-layer was neglected and the Prandtl velocity distribution assumed to apply over the whole cross-section. The result therefore is strictly applicable only at very high Reynolds Numbers where the thickness of the laminar sub-layer is very small. At lower Reynolds Numbers, the mean velocity will be rather less than 0.82 times the velocity at the axis.

The expressions for the shear stress at the walls, the thickness of the laminar sub-layer, and the velocity at the outer edge of the laminar sub-layer can be applied to the turbulent flow of a fluid in a pipe. It is convenient to express these relations in terms of the mean velocity in the pipe, the pipe diameter, and the Reynolds group with respect to the mean velocity and diameter.

The shear stress at the walls is given by the Blasius Equation, (9.14),

$$\frac{R}{\rho u_s^2} = 0.0228 \left(\frac{\mu}{u_s r \rho} \right)^{1/4}$$

Writing $u = 0.82u_s$ and $d = 2r$,

$$\frac{R}{\rho u^2} = 0.0384 \left(\frac{\mu}{u d \rho} \right)^{1/4} = 0.0384 Re^{-1/4} \quad \dots (9.30)$$

This equation is more usually written

$$\frac{R}{\rho u^2} = 0.0396 Re^{-1/4} \quad \dots (9.31)$$

(corresponding to $u = 0.81u_s$).

Equation 9.31 is applicable for Reynolds Numbers up to 10^5 .

The velocity at the edge of the laminar sub-layer is given by

$$\frac{u_b}{u_s} = 1.87 \left(\frac{\mu}{u_s r \rho} \right)^{\frac{1}{8}} \quad \dots (9.23a)$$

which becomes

$$\begin{aligned} \frac{u_b}{u} &= 2.49 \left(\frac{\mu}{u d \rho} \right)^{\frac{1}{8}} \\ &= 2.49 Re^{-\frac{1}{8}} \quad \dots (9.32) \end{aligned}$$

The thickness of the laminar sub-layer is given by,

$$\begin{aligned} \frac{\delta_b}{r} &= \left(\frac{u_b}{u_s} \right)^7 \text{ (from 9.17a)} \\ &= 1.87^7 \cdot \left(\frac{\mu}{u_s r \rho} \right)^{\frac{7}{8}} \text{ (from 9.23a)} \end{aligned}$$

Thus

$$\begin{aligned} \frac{\delta_b}{d} &= 62 \left(\frac{\mu}{u d \rho} \right)^{\frac{7}{8}} \\ &= 62 Re^{-\frac{7}{8}} \quad \dots (9.33) \end{aligned}$$

Thus the thickness of the laminar sub-layer is approximately inversely proportional to the Reynolds Number and hence to the velocity.

Example. Calculate the thickness of the laminar sub-layer when benzene (sp. gr. 0.87, viscosity 0.7 centipoises) flows through a pipe, 2 in. in diameter, at 30 gal/min. What is the velocity of the benzene at the edge of the laminar sub-layer? Assume that fully developed flow exists within the pipe.

Solution.

$$30 \text{ gal/min} = \frac{1}{2} \times 8.7 \text{ lb/sec.}$$

$$= 1970 \text{ gm/sec.}$$

$$\text{Reynolds Number } Re. = \frac{4 \times 1970}{0.007 \times 5.08 \pi}$$

$$= 70,700$$

$$\delta_b/d = 62 Re^{-\frac{7}{8}} \text{ (equation 9.33)}$$

$$\begin{aligned} \therefore \delta_b &= 62 \times 70,700^{-\frac{7}{8}} \times 2 \text{ in.} \\ &= \underline{\underline{0.0072 \text{ in.}}} \end{aligned}$$

$$\text{Mean velocity of flow in pipe} = \frac{30 \times 10}{60 \times 62.5} \left(\frac{\pi}{4} \frac{1}{36} \right)^{-1}$$

$$= 3.67 \text{ f.p.s.}$$

$$u_b/u = 2.49 Re^{-\frac{1}{8}} \text{ (equation 9.32)}$$

$$\begin{aligned} \therefore u_b &= 2.49 \times 70,700^{-\frac{1}{8}} \times 3.67 \\ &= \underline{\underline{2.2 \text{ f.p.s.}}} \end{aligned}$$

UNIVERSAL VELOCITY PROFILE

PRANDTL⁽²⁴⁾, VON KARMAN⁽³¹⁾ and REICHARDT⁽³²⁾ have given alternative expressions for the velocity distribution in a circular pipe for turbulent flow of fluid. They assumed that the shear stress at any point in the fluid could be equated to that at the walls. The shear stress at any point is given by equation 8.22.

$$R_y = - \left(\frac{\mu}{\rho} + E \right) \frac{\partial(\rho u_y)}{\partial y}$$

Writing
$$E = \lambda_E^2 \left| \frac{\partial u_y}{\partial y} \right| \quad (\text{equation 8.16}),$$

$$R_y = - \mu \frac{\partial u_y}{\partial y} - \lambda_E^2 \rho \left| \frac{\partial u_y}{\partial y} \right| \frac{\partial u_y}{\partial y} \quad \dots (9.34)$$

Since the velocity increases with increase in distance from the surface, $\frac{\partial u_y}{\partial y}$ must be positive and therefore the modulus sign can be left out.

Thus
$$R_y = - \mu \frac{\partial u_y}{\partial y} - \lambda_E^2 \rho \left(\frac{\partial u_y}{\partial y} \right)^2 \quad \dots (9.35)$$

The first term will be small compared with the second, except close to the surface, giving

$$\sqrt{\frac{-R_y}{\rho}} = \lambda_E \frac{\partial u_y}{\partial y} \quad \dots (9.36)$$

PRANDTL assumed that the mixing length λ_E was directly proportional to the distance from the surface, i.e. $\lambda_E = I \cdot y$ (say). Taking the shear stress as constant and equal to the stress R_0 at the walls, we can write,

$$R = - R_0 = - R_y$$

Thus
$$\sqrt{\frac{R}{\rho}} = I \cdot y \cdot \frac{\partial u_y}{\partial y}$$

This expression will not apply near the axis where R_y approaches zero.

$\sqrt{\frac{R}{\rho}}$ is constant and has the dimensions of velocity. It is often called the "shearing stress velocity" and denoted by the symbol u^* .

Thus
$$u^* = I \cdot y \cdot \frac{\partial u_y}{\partial y} \quad \dots (9.37)$$

On integration, we obtain

$$u_y = u^* \left(\frac{1}{I} \ln y + \text{constant} \right)$$

Putting the constant equal to $I' - \frac{1}{I} \ln \frac{\mu}{\rho u^*}$, where I' is a second constant, we have

$$u_y = u^* \left(\frac{1}{I} \ln \frac{y \rho u^*}{\mu} + I' \right) \quad \dots (9.38)$$

This expression gives the velocity at the walls equal to $-\infty$ instead of 0 because the viscous forces have been neglected. It must not therefore be applied near the walls.

If $\frac{u_y}{u^*}$ is plotted against $\ln \frac{y\rho u^*}{\mu}$, for smooth round pipes, it is found that $I = 0.4$ and $I' = 5.5$, so that

$$u_y = u^* \left(2.5 \ln \frac{y\rho u^*}{\mu} + 5.5 \right)$$

For rough pipes, it is found experimentally that the value of I is unchanged at 0.4 but I' is dependent on the relative roughness of the surface e/d ; the higher the value of e/d , the lower is the value of I' . Now writing $u^+ = u_y/u^*$, which is a dimensionless derivative of u_y , and $y^+ = y\rho u^*/\mu$, which is a dimensionless derivative of y , for smooth pipes,

$$u^+ = 2.5 \ln y^+ + 5.5 \quad \dots(9.39)$$

and for rough pipes,

$$u^+ = 2.5 \ln y^+ + I' \quad \dots(9.40)$$

where $I' < 5.5$.

These relations apply to the fully turbulent region for which y^+ is greater than about 30.

In the laminar sub-layer, where the rate of transfer by eddy motion is negligible, the shear stress and the velocity gradient are approximately constant. From equation 9.35,

$$R = \mu \frac{u_y}{y}$$

$$\therefore u_y = \frac{R}{\mu} y$$

$$\therefore u^+ = \frac{u_y}{u^*} = \frac{\rho u^* y}{\mu} = y^+ \quad \dots(9.41)$$

This relation is found to apply for values of y^+ up to about 5, for both smooth and rough pipes.

In Fig. 9.7, u^+ is plotted against $\ln y^+$ for smooth pipes, and it is found that the following equation represents conditions in the buffer layer, where $5 < u^+ < 30$.

$$u^+ = -3.05 + 5.0 \ln y^+ \quad \dots(9.42)$$

This complete curve is known as the Universal Velocity Profile and enables the velocity distribution to be calculated for turbulent flow over a plane surface or in a pipe. It is clearly only an approximation, since it indicates discontinuities at $y^+ = 5$ and $y^+ = 30$ and also gives a finite velocity gradient at the axis of a pipe. The discontinuities are often obviated by the use of short connecting curves in these regions.

For rough pipes, I' in equation 9.40 is a function of e/d and a separate curve is obtained in both the buffer layer and the turbulent core for each particular value of e/d .

The velocity gradient at any point in the cross section can now be calculated.

(1) For y^+ less than 5 (the laminar sub-layer)

$$\frac{du_y}{dy} = \frac{u_y}{y} = \frac{R}{\mu} \quad \dots (9.43)$$

(2) From $y^+ = 5$ to $y^+ = 30$. For smooth pipes,

$$u^+ = -3.05 + 5.0 \ln y^+ = 5 \left(1 + \ln \frac{y^+}{5} \right) \quad \dots (9.42)$$

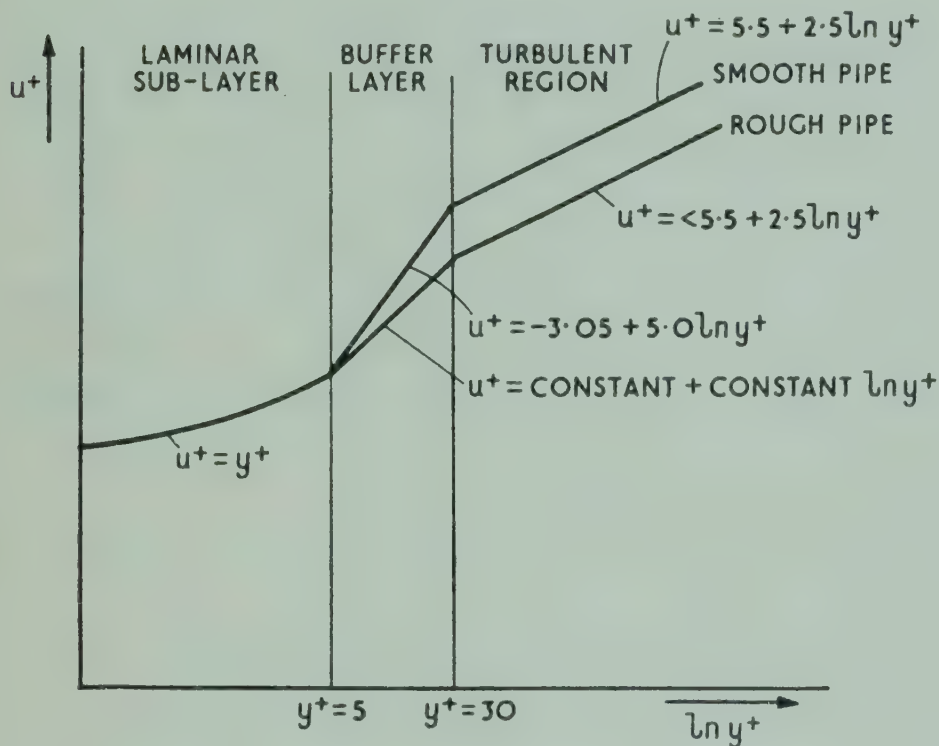


Fig. 9.7. The universal velocity profile

Thus

$$\frac{du^+}{dy^+} = \frac{5.0}{y^+}$$

so that

$$\begin{aligned} \frac{du_y}{dy} &= \sqrt{\frac{R}{\rho}} \cdot \frac{5.0}{y} \\ &= 5.0 \sqrt{\frac{R}{\rho u^2}} \cdot \frac{u}{y} \quad \dots (9.44) \end{aligned}$$

For rough pipes, the velocity gradient is a function of the relative roughness e/d .

(3) For $y^+ > 30$. For smooth pipes,

$$u^+ = 2.5 \ln y^+ + 5.5 \quad \dots (9.39)$$

and for rough pipes,

$$u^+ = 2.5 \ln y^+ + I' \quad \dots (9.40)$$

Thus, in both cases,

$$\frac{du^+}{dy^+} = \frac{2.5}{y^+}$$

and

$$\begin{aligned} \frac{du_y}{dy} &= \sqrt{\frac{R}{\rho}} \cdot \frac{2.5}{y} \\ &= 2.5 \sqrt{\frac{R}{\rho u^2}} \cdot \frac{u}{y} \end{aligned} \quad \dots (9.45)$$

EFFECT OF SURFACE ROUGHNESS ON SHEAR STRESS^{(21), (22), (43)}

Experiments have been carried out on artificially roughened surfaces in order to determine the effect of obstructions of various heights. Experimentally, it has been shown that the shear force is not affected by the presence of an obstruction of height e unless

$$\frac{u_e \cdot e \cdot \rho}{\mu} > 40 \quad \dots (9.46)$$

where u_e is the velocity of the fluid at a distance e above the surface.

If the obstruction lies entirely within the laminar sub-layer the velocity u_e is given by

$$\begin{aligned} R &= \mu \left(\frac{\partial u_y}{\partial y} \right)_{y=0} \\ &= \mu \cdot \frac{u_e}{e}, \text{ approximately} \end{aligned}$$

Now the shearing stress velocity,

$$u^* = \sqrt{\frac{R}{\rho}} = \sqrt{\frac{\mu u_e}{\rho e}}$$

so that

$$u_e = \frac{\rho \cdot e}{\mu} \cdot u^{*2}$$

Thus

$$\frac{u_e \cdot e \cdot \rho}{\mu} = \frac{e \cdot \rho}{\mu} \cdot u^{*2} \cdot \frac{e \cdot \rho}{\mu} = \left(\frac{e \cdot \rho \cdot u^*}{\mu} \right)^2 \quad \dots (9.47)$$

$\frac{e \cdot \rho \cdot u^*}{\mu}$ is known as the Roughness Reynolds Number, Re_r .

For the flow of a fluid in a pipe,

$$u^* = \sqrt{R/\rho} = u \cdot \sqrt{R/\rho u^2}$$

where u is the mean velocity over the whole cross section. Re_r will now be expressed in terms of the three dimensionless groups used in the friction chart.

Thus

$$\begin{aligned}
 Re_r &= \frac{e \cdot \rho \cdot u^*}{\mu} \\
 &= \frac{d \cdot \rho \cdot u}{\mu} \cdot \frac{e}{d} \cdot \sqrt{\frac{R}{\rho u^2}} \\
 &= Re \cdot \left(\frac{e}{d}\right) \cdot \sqrt{\frac{R}{\rho u^2}} \quad \dots (9.48)
 \end{aligned}$$

The shear stress should then be unaffected by the obstruction if $Re_r < \sqrt{40}$, i.e. if $Re_r < 6.5$ (from equations 9.46 and 9.47).

However, if the surface has a number of closely spaced obstructions, all of the same height e , the shear stress is affected when $Re_r > \text{about } 3$. If the obstructions are of varying heights, with e as the arithmetic mean, the shear stress is increased if $Re_r < \text{about } 0.3$, because the effect of one relatively large obstruction is greater than that of several small ones.

For hydrodynamically smooth pipes, through which fluid is flowing under turbulent conditions, the shear stress is given by the Blasius Equation,

$$\frac{R}{\rho u^2} \propto Re^{-\frac{1}{4}}$$

so that

$$R \propto u^{1.75}$$

and is independent of the roughness.

For smooth pipes, the frictional drag at the surface is known as skin friction. With rough pipes, however, an additional drag known as form drag results from the eddy currents caused by impact of the fluid on the obstructions and, when the surface is very rough, it becomes large compared with the skin friction. Since form drag involves dissipation of kinetic energy, the losses are proportional to the square of the velocity of the fluid, so that $R \propto u^2$. This applies when $Re_r > 50$.

Thus when $Re_r < 0.3$, $R \propto u^{1.75}$

when $Re_r > 50$, $R \propto u^2$

and when $0.3 < Re_r < 50$, $R \propto u^w$

where $1.75 < w < 2$.

When the thickness of the laminar sub-layer is large compared with the height of the obstructions the pipe behaves as a smooth pipe (when $e < \frac{1}{3}\delta_b$). Since the thickness of the laminar sub-layer decreases as the Reynolds Number is increased, a surface which is hydrodynamically smooth at low Reynolds Numbers may behave as a rough surface at higher values. This explains the shapes of the curves obtained for $R/\rho u^2$ plotted against Reynolds Number (Fig. 3.3). The curves, for all but the roughest of pipes, follow the curve for the smooth

pipe at low Reynolds Numbers and then diverge at higher values. The greater the roughness of the surface, the lower is the Reynolds Number at which the curve starts to diverge. At high Reynolds Numbers, the curves for rough pipes become parallel to the Reynolds Number axis, indicating that skin friction is negligible and $R \propto u^2$. Under these conditions, the shear stress can be calculated from equation 3.3.

THE BOUNDARY LAYER FOR HEAT TRANSFER

It has been shown that, when a fluid flows over a solid surface, a velocity gradient is set up in the boundary layer. If there is a difference in temperature between the surface and the fluid, heat transfer will take place and a temperature gradient will form in a similar manner to the velocity gradient. The whole of the temperature gradient can be considered to exist within a layer of fluid close to the surface, termed the thermal boundary layer. The idea of a boundary layer is only

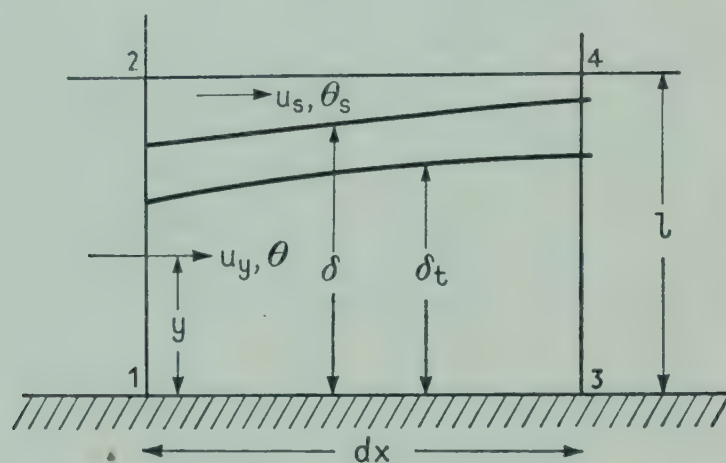


Fig. 9.8. The thermal boundary layer

hypothetical, since the temperature gradient extends to an indefinite depth in the fluid, but it provides a useful basis for heat transfer calculations. The thermal boundary layer does not necessarily correspond with the velocity boundary layer, which is not affected by the heat transfer, unless the physical properties of the fluid are appreciably altered.

When heat transfer is taking place between the surface and the element of fluid referred to on page 260, conditions are as shown in Fig. 9.8, in which the distance l is greater than either of the boundary layer thicknesses.

The heat transferred through an element of the plane 1-2, of thickness dy and at a distance y from the surface

$$= C_p \cdot \rho \cdot \theta \cdot u_y \cdot dy$$

where C_p is the specific heat of the fluid at constant pressure,

ρ is the density of the fluid,

θ and u_y are the temperature and velocity at a distance y from the surface.

The total rate of transfer of heat through the plane, 1-2,

$$= C_p \cdot \rho \cdot \int_0^l \theta \cdot u_y \cdot dy$$

if it is assumed that the physical properties of the fluid are independent of temperature.

In the distance dx this heat flow changes by an amount

$$C_p \cdot \rho \cdot \frac{\partial}{\partial x} \left\{ \int_0^l \theta \cdot u_y \cdot dy \right\} dx$$

It has already been shown (page 260) that there is a mass rate of flow of fluid through plane, 2-4, into the element equal to $\rho \cdot \frac{\partial}{\partial x} \left\{ \int_0^l u_y \cdot dy \right\} dx$ for an incompressible fluid.

Since the plane, 2-4, lies outside the boundary layers, the heat introduced into the element through the plane

$$= C_p \cdot \rho \cdot \theta_s \cdot \frac{\partial}{\partial x} \left\{ \int_0^l u_y \cdot dy \right\} dx$$

where θ_s is the temperature outside the thermal boundary layer.

The heat transferred by thermal conduction into the element through plane, 1-3,

$$= -k \cdot dx \left(\frac{\partial \theta}{\partial y} \right)_{y=0}$$

If the temperature θ_s of the main stream is unchanged, a heat balance on the element gives

$$C_p \cdot \rho \left\{ \frac{\partial}{\partial x} \int_0^l \theta \cdot u_y \cdot dy \right\} dx = C_p \cdot \rho \cdot \theta_s \cdot \left\{ \frac{\partial}{\partial x} \int_0^l u_y \cdot dy \right\} dx - k \cdot \left(\frac{\partial \theta}{\partial y} \right)_{y=0} dx$$

$$\text{i.e.} \quad \frac{\partial}{\partial x} \int_0^l u_y \cdot (\theta_s - \theta) dy = D_H \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \dots (9.49)$$

where $D_H = \frac{k}{C_p \cdot \rho}$, the thermal diffusivity of the fluid.

The relations between u_y and y have already been obtained for both streamline and turbulent flow. A relation between θ and y for streamline conditions in the boundary layer will now be derived, but it is not possible to define the conditions in the turbulent boundary layer sufficiently precisely to derive a similar expression for that case. The heat transfer through the turbulent boundary layer will therefore have to be considered later from a different point of view.

Heat Transfer for Streamline Flow over a Plane Surface

Consider the flow of fluid over a plane surface, which is heated at distances greater than x_0 from the leading edge. The velocity boundary layer will therefore start at the leading edge and the thermal boundary layer at a distance x_0 from it. Let the temperature of the heated portion of the plate remain constant and take

this as the datum temperature (Fig. 9.9). Assume that the temperature at a distance y from the surface can be represented by the expression,

$$\theta = a_0 y + b_0 y^2 + c_0 y^3 \quad \dots(9.50)$$

Since the fluid in contact with the surface is at rest, heat flow in the vicinity of the surface must be by pure thermal conduction. Thus heat transferred per unit area and unit time q

$$= -k \cdot \left(\frac{\partial \theta}{\partial y} \right)_{y=0}$$

$$\therefore \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \text{a constant}$$

and

$$\left(\frac{\partial^2 \theta}{\partial y^2} \right)_{y=0} = 0$$

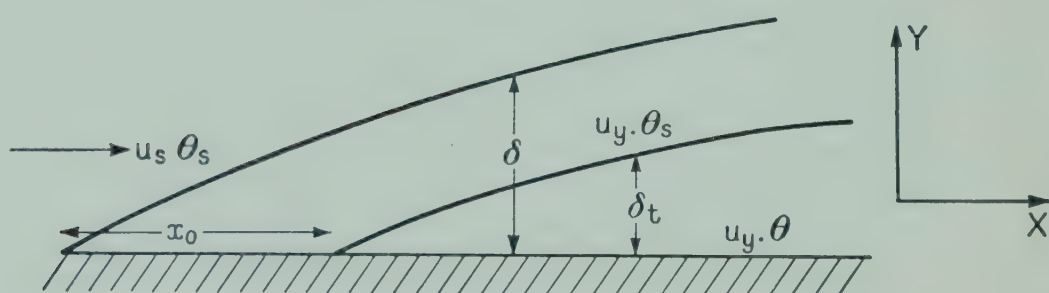


Fig. 9.9. Thermal boundary layer—streamline flow

At the outer edge of the thermal boundary layer, the temperature is θ_s , and the temperature gradient $\frac{\partial \theta}{\partial y} = 0$.

Thus the conditions for the thermal boundary layer, with respect to temperature, are the same as those for the velocity boundary layer with respect to velocity. Then if the thickness of the thermal boundary layer is δ_t , the temperature distribution is given by,

$$\frac{\theta}{\theta_s} = \frac{3}{2} \frac{y}{\delta_t} - \frac{1}{2} \left(\frac{y}{\delta_t} \right)^3 \quad \dots(9.51)$$

(cf. equation 9.4a)

and

$$\left(\frac{\partial \theta}{\partial y} \right)_{y=0} = \frac{3\theta_s}{2\delta_t} \quad \dots(9.52)$$

It will now be assumed that the velocity boundary layer is everywhere thicker than the thermal boundary layer, so that $\delta > \delta_t$ (Fig. 9.9). Thus the velocity distribution everywhere within the thermal boundary layer is given by equation 9.4a.

The integral in equation 9.49 clearly has a finite value within the thermal boundary layer, but is zero outside it. When the expression for the temperature distribution in the boundary layer is inserted therefore, the upper limit of integration must be altered from l to δ_t .

$$\begin{aligned}
\text{Thus } \int_0^l (\theta_s - \theta) \cdot u_y \cdot dy &= \theta_s \cdot u_s \int_0^{\delta_t} \left\{ 1 - \frac{3}{2} \frac{y}{\delta_t} + \frac{1}{2} \left(\frac{y}{\delta_t} \right)^3 \right\} \left\{ \frac{3y}{2\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right\} dy \\
&= \theta_s \cdot u_s \left\{ \frac{3}{4} \cdot \frac{\delta_t^2}{\delta} - \frac{3}{4} \cdot \frac{\delta_t^2}{\delta} - \frac{1}{8} \cdot \frac{\delta_t^4}{\delta^3} + \frac{3}{20} \left(\frac{\delta_t^2}{\delta} + \frac{\delta_t^4}{\delta^3} \right) - \frac{1}{28} \cdot \frac{\delta_t^4}{\delta^3} \right\} \\
&= \theta_s \cdot u_s \left\{ \frac{3}{20} \cdot \frac{\delta_t^2}{\delta} - \frac{3}{280} \cdot \frac{\delta_t^4}{\delta^3} \right\} \\
&= \theta_s \cdot u_s \cdot \delta \left(\frac{3}{20} \sigma^2 - \frac{3}{280} \sigma^4 \right) \dots (9.53)
\end{aligned}$$

where $\sigma = \delta_t/\delta$.

Since $\delta_t < \delta$, the second term is small compared with the first and

$$\int_0^l (\theta_s - \theta) u_y dy = \frac{3}{20} \theta_s \cdot u_s \cdot \delta \cdot \sigma^2 \dots (9.54)$$

Substituting from equations 9.52 and 9.54 in equation 9.49,

$$\frac{\partial}{\partial x} \left(\frac{3}{20} \theta_s \cdot u_s \cdot \delta \cdot \sigma^2 \right) = D_H \cdot \frac{3\theta_s}{2\delta_t} = D_H \cdot \frac{3\theta_s}{2\delta\sigma}$$

$$\therefore \frac{1}{10} u_s \cdot \delta\sigma \cdot \frac{\partial}{\partial x} \delta\sigma^2 = D_H$$

$$\therefore \frac{1}{10} u_s \left(\delta\sigma^3 \frac{\partial\delta}{\partial x} + 2\delta^2\sigma^2 \cdot \frac{\partial\sigma}{\partial x} \right) = D_H \dots (9.55)$$

It has already been shown that

$$\delta^2 = \frac{280\mu x}{13\rho u_s} = 21.5 \frac{\mu x}{\rho u_s} \text{ (equation 9.8)}$$

$$\text{Thus } \delta \cdot \frac{\partial\delta}{\partial x} = \frac{140}{13} \cdot \frac{\mu}{\rho u_s}$$

Substituting in equation 9.55,

$$\frac{u_s}{10} \cdot \frac{\mu}{\rho u_s} \left\{ \frac{140}{13} \sigma^3 + \frac{560}{13} x \cdot \sigma^2 \frac{\partial\sigma}{\partial x} \right\} = D_H$$

$$\therefore \frac{14}{13} \cdot \frac{\mu}{\rho D_H} \left(\sigma^3 + 4x\sigma^2 \cdot \frac{\partial\sigma}{\partial x} \right) = 1$$

$$\therefore \sigma^3 + \frac{4x}{3} \cdot \frac{\partial\sigma^3}{\partial x} = \frac{13}{14} Pr^{-1}$$

$$\left(\text{where the Prandtl number } Pr. = \frac{C_p \mu}{k} = \frac{\mu}{\rho D_H} \right)$$

$$\therefore \frac{3}{4} \cdot x^{-1} \cdot \sigma^3 + \frac{\partial \sigma^3}{\partial x} = \frac{13}{14} \cdot Pr.^{-1} \cdot \frac{3}{4} x^{-1}$$

$$\therefore \frac{3}{4} \cdot x^{-\frac{1}{2}} \cdot \sigma^3 + x^{\frac{3}{2}} \cdot \frac{\partial \sigma^3}{\partial x} = \frac{13}{14} \cdot Pr.^{-1} \cdot \frac{3}{4} x^{-\frac{1}{2}}$$

Integrating, $x^{\frac{3}{2}} \cdot \sigma^3 = \frac{13}{14} \cdot Pr.^{-1} \cdot x^{\frac{3}{2}} + \text{constant}$

i.e. $\sigma^3 = \frac{13}{14} \cdot Pr.^{-1} + \text{constant} \cdot x^{-\frac{3}{2}}$

Now when $x = x_0$, $\sigma = 0$

so that $\text{constant} = -\frac{13}{14} Pr.^{-1} \cdot x_0^{\frac{3}{2}}$

Hence $\sigma^3 = \frac{13}{14} Pr.^{-1} \cdot \left\{ 1 - \left(\frac{x_0}{x} \right)^{\frac{3}{2}} \right\}$

and $\sigma = 0.98 Pr.^{-\frac{1}{3}} \left\{ 1 - \left(\frac{x_0}{x} \right)^{\frac{3}{2}} \right\}^{\frac{1}{3}}$

$$\approx Pr.^{-\frac{1}{3}} \left\{ 1 - \left(\frac{x_0}{x} \right)^{\frac{3}{2}} \right\}^{\frac{1}{3}} \quad \dots (9.56)$$

If the whole length of the plate is heated, $x_0 = 0$ and

$$\sigma \approx Pr.^{-\frac{1}{3}} \quad \dots (9.57)$$

In the above derivation, it has been assumed that $\sigma < 1$.

For all liquids other than molten metals, $Pr. > 1$ and hence $\sigma < 1$ (from equation 9.57).

For gases $Pr. \leq 0.6$, so that $\sigma \geq 1.18$.

Thus only a small error is introduced when the above expression is applied to gases. The only serious deviations occur for molten metals, which have very low Prandtl Numbers.

If h is the heat transfer coefficient,

$$q = -h \cdot \theta_s$$

Thus $-h \cdot \theta_s = -k \left(\frac{\partial \theta}{\partial y} \right)_{y=0}$

i.e. $h = \frac{k}{\theta_s} \cdot \frac{3}{2} \cdot \frac{\theta_s}{\delta_t}$ (from equation 9.52)

$$= \frac{3k}{2\delta_t} = \frac{3k}{2\delta\sigma} \quad \dots (9.58)$$

Substituting for δ from equation 9.8a, and σ from equation 9.56,

$$h = \frac{3k}{2} \cdot \frac{1}{4.64} \sqrt{\frac{\rho u_s}{\mu x}} \cdot \frac{Pr^{\frac{1}{3}}}{\left\{1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right\}^{\frac{1}{3}}}$$

i.e.
$$\frac{hx}{k} = 0.323 \cdot Pr^{\frac{1}{3}} \cdot Re_x^{\frac{1}{2}} \cdot \frac{1}{\left\{1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right\}^{\frac{1}{3}}} \quad \dots (9.59)$$

If the surface is heated over its entire length, so that $x_0 = 0$,

$$Nu_x = \frac{h \cdot x}{k} = 0.323 \cdot Pr^{\frac{1}{3}} \cdot Re_x^{\frac{1}{2}} \quad \dots (9.60)$$

It is seen that the heat transfer coefficient has an infinite value at the leading edge, where the thickness of the thermal boundary layer is zero, and that

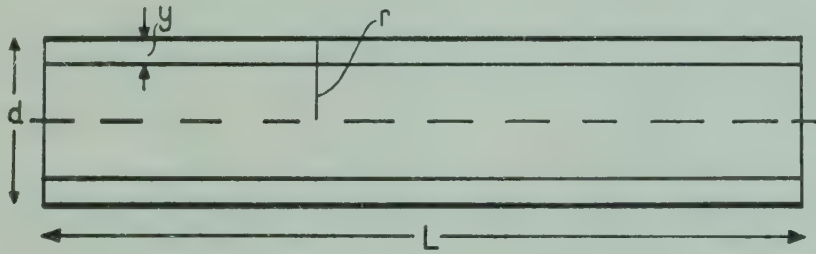


Fig. 9.10. Flow through heated tube

it decreases progressively as the boundary layer thickens. Equation 9.60 gives the point value of the heat transfer coefficient at a distance x from the leading edge. The mean value between $x = 0$ and $x = x$ is given by

$$\begin{aligned} h_m &= x^{-1} \int_0^x h \cdot dx \\ &= x^{-1} \int_0^x C' \cdot x^{-\frac{1}{2}} \cdot dx \end{aligned}$$

where C' is not a function of x

$$= x^{-1} \cdot \left[2 \cdot C' \cdot x^{\frac{1}{2}} \right]_0^x = 2h$$

Thus the mean value of the heat transfer coefficient between $x = 0$ and $x = x$ is equal to twice the point value at $x = x$. The mean value of the Nusselt Group is given by,

$$(Nu_x)_m = 0.65 Pr^{\frac{1}{3}} \cdot Re_x^{\frac{1}{2}} \quad \dots (9.61)$$

Streamline Flow of a Fluid through a Heated Tube

For the flow of a fluid through a tube as shown in Fig. 9.10, the rate of flow of heat Q through a cylindrical surface in the fluid at a distance y from the walls is given by,

$$Q = -k \cdot 2\pi L \cdot (r - y) \frac{\partial \theta}{\partial y}$$

Thus
$$\frac{\partial \theta}{\partial y} = -\frac{Q}{k2\pi L}(r-y)^{-1} \text{ and } \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \frac{Q}{2\pi k L r}$$

$$\frac{\partial^2 \theta}{\partial y^2} = -\frac{Q}{k2\pi L}(r-y)^{-2} \text{ and } \left(\frac{\partial^2 \theta}{\partial y^2}\right)_{y=0} = \frac{Q}{2\pi k L r^2}$$

Thus
$$\left(\frac{\partial^2 \theta}{\partial y^2}\right)_{y=0} = r^{-1} \left(\frac{\partial \theta}{\partial y}\right)_{y=0} \dots (9.62)$$

Assume that the temperature of the walls remains constant at the datum temperature and that the temperature at any distance y from the walls is given by,

$$\theta = a_0 y + b_0 y^2 + c_0 y^3 \dots (9.63)$$

thus
$$\frac{\partial \theta}{\partial y} = a_0 + 2b_0 y + 3c_0 y^2 \text{ and } \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = a_0$$

$$\frac{\partial^2 \theta}{\partial y^2} = 2b_0 + 6c_0 y \quad \text{and} \quad \left(\frac{\partial^2 \theta}{\partial y^2}\right)_{y=0} = 2b_0$$

Thus
$$2b_0 = \frac{a_0}{r} \text{ (from equation 9.62)}$$

i.e.
$$b_0 = \frac{a_0}{2r}$$

Now let the temperature of the fluid at the axis of the pipe be θ_s . The temperature gradient at the axis, from symmetry, must be zero.

Thus
$$0 = a_0 + 2r \cdot \frac{a_0}{2r} + 3c_0 r^2$$

$$\therefore c_0 = -\frac{2a_0}{3r^2}$$

and
$$\theta_s = a_0 r + r^2 \cdot \frac{a_0}{2r} + r^3 \cdot -\frac{2a_0}{3r^2}$$

$$= \frac{5a_0 r}{6}$$

$$\therefore a_0 = \frac{6\theta_s}{5r}$$

and
$$b_0 = \frac{3\theta_s}{5r^2}$$

and
$$c_0 = -\frac{4\theta_s}{5r^3}$$

Thus
$$\frac{\theta}{\theta_s} = \frac{6}{5} \frac{y}{r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3 \dots (9.64)$$

Thus the rate of heat transfer per unit area at the wall

$$q = -k \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= -\frac{6}{5} \cdot \frac{k\theta_s}{r} \dots (9.65)$$

In general, the temperature θ_s at the axis is not known and the heat transfer coefficient is related to the temperature difference between the walls and the bulk fluid. The bulk temperature of the fluid is defined as the ratio of the heat content to the heat capacity of the fluid flowing at any section. Thus the bulk temperature θ_B is given by

$$\begin{aligned}\theta_B &= \frac{\int_r^0 C_p \cdot \rho \cdot \theta \cdot u_y \cdot 2\pi(r-y) d(r-y)}{\int_r^0 C_p \cdot \rho \cdot u_y \cdot 2\pi(r-y) d(r-y)} \\ &= \frac{\int_r^0 \theta \cdot u_y \cdot (r-y) d(r-y)}{\int_r^0 u_y \cdot (r-y) d(r-y)} \quad \dots (9.66)\end{aligned}$$

Now from Poiseuille's Law (equation 3.13),

$$u_y = \frac{-\Delta P}{4\mu L} (r^2 - y^2) = \frac{-\Delta P}{4\mu L} (2ry - y^2)$$

Hence
$$u_s = \frac{-\Delta P}{4\mu L} r^2$$

where u_s is the velocity at the pipe axis, and

$$\frac{u_y}{u_s} = \frac{2y}{r} - \left(\frac{y}{r}\right)^2$$

$$\begin{aligned}\text{Thus } \int_r^0 u_y (r-y) d(r-y) &= r^2 u_s \int_0^1 \left\{ 2 \frac{y}{r} - \left(\frac{y}{r}\right)^2 \right\} \left\{ 1 - \frac{y}{r} \right\} d\left(\frac{y}{r}\right) \\ &= r^2 u_s \int_0^1 \left\{ 2 \left(\frac{y}{r}\right) - 3 \left(\frac{y}{r}\right)^2 + \left(\frac{y}{r}\right)^3 \right\} d\left(\frac{y}{r}\right) \\ &= \frac{1}{4} r^2 \cdot u_s \quad \dots (9.67)\end{aligned}$$

Since
$$\frac{\theta}{\theta_s} = \frac{6}{5} \frac{y}{r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3 \quad \dots (9.64)$$

$$\begin{aligned}\int_r^0 \theta \cdot u_y \cdot (r-y) d(r-y) &= r^2 u_s \theta_s \int_0^1 \left\{ \frac{6}{5} \frac{y}{r} + \frac{3}{5} \left(\frac{y}{r}\right)^2 - \frac{4}{5} \left(\frac{y}{r}\right)^3 \right\} \left\{ 2 \left(\frac{y}{r}\right) - 3 \left(\frac{y}{r}\right)^2 + \left(\frac{y}{r}\right)^3 \right\} d\left(\frac{y}{r}\right) \\ &= r^2 u_s \theta_s \int_0^1 \left\{ \frac{12}{5} \left(\frac{y}{r}\right)^2 - \frac{12}{5} \left(\frac{y}{r}\right)^3 - \frac{11}{5} \left(\frac{y}{r}\right)^4 - 3 \left(\frac{y}{r}\right)^5 - \frac{4}{5} \left(\frac{y}{r}\right)^6 \right\} d\left(\frac{y}{r}\right) \\ &= r^2 u_s \theta_s \left(\frac{4}{5} - \frac{3}{5} - \frac{11}{25} + \frac{1}{2} - \frac{4}{35} \right) \\ &= \frac{51}{350} r^2 u_s \theta_s \quad \dots (9.68)\end{aligned}$$

Substituting from equations 9.67 and 9.68 in equation 9.66,

$$\begin{aligned}\theta_B &= \frac{\frac{51}{350} r^2 u_s \theta_s}{\frac{1}{4} r^2 u_s} \\ &= \frac{102}{175} \theta_s = 0.583 \theta_s \quad \dots (9.69)\end{aligned}$$

Then, the heat transfer coefficient h is given by

$$h = - \frac{q}{\theta_B}$$

where q is the rate of heat transfer per unit area of tube.

$$\begin{aligned}\text{Thus} \quad h &= \frac{\frac{6}{5} \cdot k \frac{\theta_s}{r}}{0.583 \theta_s} \quad (\text{from equations 9.65 and 9.69}) \\ &= 2.06 \frac{k}{r} \\ &= 4.1 \frac{k}{d}\end{aligned}$$

$$\therefore Nu. = \frac{hd}{k} = 4.1 \quad \dots (9.70)$$

This expression is applicable only to the region of fully developed flow. The heat transfer coefficient for the inlet length can be calculated approximately, using the expressions that have been derived for the flow over a plane surface. It should be borne in mind that it has been assumed throughout that the physical properties of the fluid are not appreciably dependent on temperature and therefore the expressions will not be expected to hold accurately if the temperature differences are large and if the properties vary widely with temperature. An empirical equation (6.35) has already been given in Chapter 6 to cover the whole of the tube.

THE BOUNDARY LAYER FOR MASS TRANSFER

If mass transfer takes place as a result of a concentration gradient between the flowing fluid and the surface, the whole of the resistance to transfer can be considered as lying within a mass transfer boundary layer. If the concentration, and hence the rate of mass transfer, are small it can be shown⁽³⁾ that the velocity and thermal boundary layers are unaffected and that the basic equation for mass transfer is

$$\frac{\partial}{\partial x} \int_0^l (P_A - P_A) u_y \cdot dy = D \left(\frac{\partial P_A}{\partial y} \right)_{y=0} \quad \dots (9.71)$$

where P_A is the partial pressure of A at a distance y from the surface; P_A is the value outside the mass transfer boundary layer and l is a distance in the Y -direction, greater than the thickness of any of the three boundary layers. The similarity between equation 9.71 and equation 9.49 for heat transfer is apparent. It is not possible to obtain a simple relation connecting P_A and y however, so that integration is not possible in this case. In the next section, however, we shall show how the simple Reynolds analogy can be extended to give relations for heat and mass transfer between a turbulent fluid and a surface.

HEAT TRANSFER TO A FLUID IN TURBULENT MOTION

We have already indicated that it is not possible to apply equation 9.49 to heat transfer through the turbulent boundary layer because the temperature distribution near the surface is not known. A number of approximate methods have been developed for the calculation of the heat transfer coefficient and these are all refinements of the original Reynolds analogy given in Chapter 8. From equation 8.28, we have that

$$\frac{R}{\rho u_s^2} = - \frac{q}{C_p \theta_s \rho u_s} = \frac{h}{C_p \rho u_s} \quad \dots (8.28)$$

for the transfer of heat from a fluid at a temperature θ_s flowing over a surface with a velocity u_s . (The temperature of the surface is assumed to remain constant at the datum temperature.) In the derivation of this expression it was assumed that fluid was transferred from outside the boundary layer to the surface without mixing with the intervening fluid, that it was brought to rest at the surface and that thermal equilibrium was established. Various modifications have been made to this simple theory to take account of the existence of the laminar sub-layer and the buffer layer close to the surface.

TAYLOR⁽¹⁴⁾ and PRANDTL^{(12), (19)} allowed for the existence of the laminar sub-layer but ignored the existence of the buffer layer in their treatment and assumed that the simple Reynolds analogy was applicable to the transfer of heat and momentum from the main stream to the edge of the laminar sub-layer of thickness δ_b . Transfer through the laminar sub-layer was then presumed to be attributable solely to molecular motion.

Let au_s and $b\theta_s$ be the velocity and temperature, respectively, at the edge of the laminar sub-layer. Applying the Reynolds analogy (equation 8.27) for transfer across the turbulent region,

$$\frac{q}{R} = - \frac{C_p \cdot (\theta_s - b\theta_s)}{u_s - au_s} \quad \dots (9.72)$$

The rate of transfer of heat by conduction through the laminar sub-layer from a surface of area A is given by

$$qA = - \frac{kb\theta_s}{\delta_b}$$

The rate of transfer of momentum is equal to the shearing force and therefore

$$R_0 A = - \frac{\mu a u_s}{\delta_b} = - R A$$

Dividing,

$$\frac{q}{R} = - \frac{k b \theta_s}{\mu a u_s} \quad \dots (9.73)$$

Thus from equations 9.72 and 9.73,

$$\frac{k b \theta_s}{\mu a u_s} = \frac{C_p \cdot (1 - b) \cdot \theta_s}{(1 - a) \cdot u_s}$$

$$\therefore \frac{1 - b}{b} Pr. = \frac{1 - a}{a}$$

$$\therefore \frac{b}{a} = \frac{1}{a + (1 - a) Pr.^{-1}}$$

Substituting in equation 9.73,

$$\frac{q}{R} = - \frac{C_p \theta_s}{u_s} \cdot \frac{1}{1 + a(Pr. - 1)} \frac{h \theta_s}{R}$$

$$\text{i.e.} \quad St. = \frac{h}{C_p \rho u_s} = \frac{\frac{R}{\rho u_s^2}}{1 + a(Pr. - 1)} \quad \dots (9.74)$$

This general expression will now be applied to the flow over a plane surface and for fully developed flow in a pipe.

Heat Transfer for Turbulent Fluid over a Plane Surface

For the flow of a fluid over a plane surface,

$$R = 0.03 \rho u_s^2 \cdot Re._x^{-0.2} \quad \dots (9.26)$$

$$\text{and} \quad a = \frac{u_b}{u_s} = 2.1 Re._x^{-0.1} \quad \dots (9.24)$$

$$\text{Thus} \quad St. = Nu._x \cdot Re._x^{-1} \cdot Pr.^{-1} = \frac{0.03 Re._x^{-0.2}}{1 + 2.1 Re._x^{-0.1} (Pr. - 1)} \quad \dots (9.75)$$

This expression will give the point value of the Stanton Number, and hence of the heat transfer coefficient. The mean value over the whole surface is obtained by integration. No general expression for the mean coefficient can be obtained and a graphical or numerical integration must be carried out after the insertion of the appropriate values of the constants. If the denominator in equation 9.75 is put equal to unity, we have the simple Reynolds analogy and

$$St. = \frac{R}{\rho u_s^2} = 0.03 Re._x^{-0.2} \quad \dots (9.76)$$

Heat Transfer for Turbulent Flow in a Pipe

For the fully developed flow in a pipe, it is useful to obtain the heat transfer coefficient in terms of the Reynolds and Stanton groups with respect to pipe diameter, mean velocity of flow, and bulk temperature of the fluid.

If u is the mean velocity of the fluid and θ_B is the bulk temperature, we have for turbulent conditions that

$$\theta_B \approx \theta_s \text{ (because the fluid is well mixed)}$$

$$u = 0.82u_s$$

$$\frac{u_b}{u} = 2.43 Re^{-\frac{1}{4}} \quad \dots (9.32)$$

$$R = 0.0396 \rho u^2 \cdot Re^{-\frac{1}{4}} \quad \dots (9.31)$$

and

$$\frac{h}{C_p \rho u_s} = \frac{\frac{R}{\rho u_s^2}}{1 + a(Pr. - 1)} \quad \dots (9.74)$$

$$\therefore h = \frac{\frac{R}{\rho u^2} \cdot \left(\frac{u}{u_s}\right)^2 \cdot C_p \rho u_s}{1 + \left(\frac{u_b}{u}\right) \left(\frac{u}{u_s}\right) (Pr. - 1)} \cdot \frac{\theta_s}{\theta_B}$$

if h is based on the temperature difference between the bulk fluid and the surface.

$$= \frac{0.0396 \cdot Re^{-\frac{1}{4}} \cdot 0.82^2 C_p \cdot \rho \cdot u \cdot 0.82^{-1}}{1 + 2.43 Re^{-\frac{1}{4}} \cdot 0.82 (Pr. - 1)}$$

$$= \frac{0.032 Re^{-\frac{1}{4}}}{1 + 2.0 Re^{-\frac{1}{4}} (Pr. - 1)} C_p \cdot \rho \cdot u$$

and

$$St. = \frac{0.032 Re^{-\frac{1}{4}}}{1 + 2.0 Re^{-\frac{1}{4}} (Pr. - 1)} \quad \dots (9.77)$$

Whilst this expression is not entirely satisfactory, since it assumes that the properties of the fluid are not a function of temperature, it gives reasonable results for gases and liquids of low viscosities. For highly viscous liquids, it is necessary to use the more complex expression which is obtained when the temperature gradient in the buffer layer is taken into account. Again, putting the denominator in equation 9.77 equal to unity, we have the simple Reynolds analogy,

$$St. = \frac{h}{C_p \rho u} = 0.032 Re^{-\frac{1}{4}} \quad \dots (9.78)$$

It should be noted that, in two particular cases, equation 9.78 is applicable, i.e.:

(1) When the degree of turbulence is so high that no appreciable resistance to momentum or heat transfer exists in the laminar sub-layer, a approaches zero and equations 9.77 and 9.78 become identical.

(2) When the Prandtl Number is nearly unity, the two expressions are again identical. Thus for gases, the Prandtl Numbers of which are a little less than unity, quite accurate results are obtained from equation 9.78.

Equation 9.77 can be compared with the empirical equation given in Chapter 8, i.e.

$$St. = 0.023 Re^{-0.2} Pr^{0.67} \quad \dots (8.31)$$

It is seen that equations 8.31 and 9.77 differ mainly in the form in which the Prandtl group is introduced. When $Pr.$ is unity, both equations give a simple exponential relationship between $St.$ and $Re.$

The change in temperature of a fluid flowing under turbulent conditions in a tube can now be related to the pressure drop. For the conditions shown in

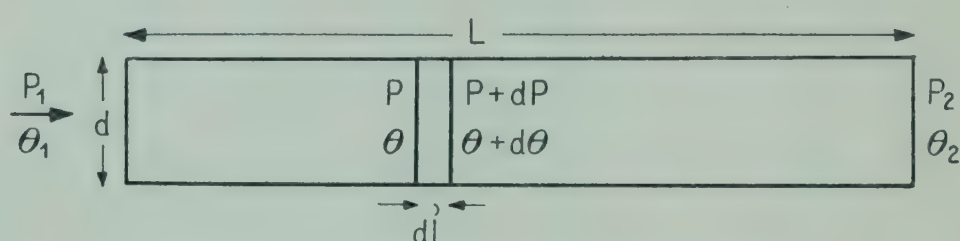


Fig. 9.11. Flow of heated fluid through tube

Fig. 9.11, the change in heat content of the fluid, per unit time, as it flows through a length dL of tube is

$$\frac{\pi}{4} d^2 \cdot \rho \cdot u \cdot C_p \cdot d\theta = -h \cdot \pi \cdot d \cdot dL \cdot \theta$$

Integrating,
$$\ln \frac{\theta_1}{\theta_2} = \frac{4h \cdot L}{d \cdot \rho \cdot u \cdot C_p}$$

$$\therefore \theta_1 - \theta_2 = 4 \cdot \frac{L}{d} \cdot \frac{h}{C_p \cdot \rho \cdot u} \cdot \theta_m$$

where θ_m is the logarithmic mean of θ_1 and θ_2 .

The change in pressure over the pipe is given by equation 3.7,

$$P_1 - P_2 = \frac{4R \cdot L}{d} \quad \dots (3.7)$$

Dividing,
$$\frac{\theta_1 - \theta_2}{P_1 - P_2} = St. \cdot \frac{\theta_m}{R}$$

$$= \frac{0.032 Re^{-\frac{1}{4}}}{1 + 2.0 Re^{-\frac{1}{8}} (Pr. - 1)} \cdot \frac{\theta_m}{\rho u^2} \cdot \frac{1}{0.0396 Re^{-\frac{1}{4}}}$$

(from equations 9.31 and 9.77)

$$= \frac{0.82}{1 + 2.0 Re^{-\frac{1}{8}} (Pr. - 1)} \cdot \frac{\theta_m}{\rho u^2} \quad \dots (9.79)$$

Use of Universal Velocity Profile⁽³⁹⁾

In the Taylor-Prandtl modification of the theory of heat transfer to a turbulent fluid, it was assumed that the heat passed directly from the turbulent fluid to the laminar sub-layer and the existence of the buffer layer was neglected. It was therefore possible to apply the simple theory for the boundary layer in order to calculate the heat transfer. In most cases, the results so obtained are sufficiently accurate but errors become significant when the relations are used to calculate heat transfer to liquids of high viscosities and liquids with very low Prandtl Numbers, such as molten metals. A more accurate expression can be obtained if the temperature difference across the buffer layer is taken into account. The exact conditions in the buffer layer are difficult to define and any mathematical treatment of the problem involves a number of assumptions. However, the conditions close to the surface over which fluid is flowing can be calculated approximately using the Universal Velocity Profile.

The method is based on a calculation of the temperature difference across the laminar sub-layer, the buffer layer, and the turbulent region in turn. This is carried out on the assumption that the laminar sub-layer extends from $y^+ = 0$ to $y^+ = 5$, the buffer layer from $y^+ = 5$ to $y^+ = 30$, and the turbulent region for y^+ greater than 30. The temperature difference across the turbulent region is obtained by direct application of the Reynolds analogy, with the velocity at the edge calculated by substituting $y^+ = 30$ in equation 9.39. For the buffer layer, it is shown that the eddy kinematic viscosity and the eddy thermal diffusivity are approximately equal, so that the temperature gradient can be expressed in terms of the eddy viscosity. The total temperature drop is then obtained by integration between the limits of 5 and 30 for y^+ . For the laminar sub-layer a constant temperature gradient is assumed and the thickness of the layer is obtained from the fact that y^+ extends from 0 to 5.

The total temperature difference across the three layers is then obtained as

$$\theta_s = -\frac{q}{C_p} \sqrt{\frac{1}{\rho R}} \left\{ u_s \sqrt{\frac{\rho}{R}} - 5(1 - Pr.) + 5 \ln \left(\frac{5Pr.}{6} + \frac{1}{6} \right) \right\} \dots (9.80)$$

For the flow over a plane surface,

$$\begin{aligned} St. &= Nu_x \cdot Re_x^{-1} \cdot Pr.^{-1} = -\frac{q}{C_p \cdot \rho \cdot u_s \cdot \theta_s} \\ &= \frac{1}{\rho u_s \sqrt{\frac{1}{\rho R}} \left\{ u_s \sqrt{\frac{\rho}{R}} + 5(Pr. - 1) + 5 \ln \left(\frac{5Pr.}{6} + \frac{1}{6} \right) \right\}} \\ &= \frac{R}{\rho u_s^2} \left\{ 1 + 5 \sqrt{\frac{R}{\rho u_s^2}} \left[(Pr. - 1) + \ln \left(\frac{5Pr. + 1}{6} \right) \right] \right\}^{-1} \\ &\dots (9.81) \end{aligned}$$

The point value of the Stanton Number at a distance x from the leading edge of the surface is obtained by writing

$$\frac{R}{\rho u_s^2} = 0.030 Re_x^{-0.2} \quad \dots (9.27)$$

The average value of the Stanton Number over the whole surface is then obtained by numerical or graphical integration.

The relations can also be applied for the fully developed flow of a fluid in a pipe, when the appropriate value for the shear stress at the walls is given by the Blasius Equation (9.31) or by equation 3.1. The mean velocity of the fluid is then used in the Stanton Number and the temperature difference is taken between the bulk fluid and the walls.

Thus for the flow of a fluid over a plane surface, using the Blasius Equation,

$$St. = \frac{h}{C_p \cdot \rho \cdot u_s} = \frac{\frac{R}{\rho u_s^2}}{1 + 5 \sqrt{\frac{R}{\rho u_s^2}} \left\{ (Pr. - 1) + \ln \left(\frac{5}{6} Pr. + \frac{1}{6} \right) \right\}} \quad \dots (9.81)$$

$$= \frac{0.030 Re_x^{-0.2}}{1 + 0.87 Re_x^{-0.1} \left[(Pr. - 1) + \ln \left(\frac{5}{6} Pr. + \frac{1}{6} \right) \right]} \quad \dots (9.82)$$

For fully developed flow in a round pipe,

$$St. = \frac{h}{C_p \cdot \rho \cdot u} = \frac{\frac{R}{\rho u^2} \cdot 0.82}{1 + 0.82 \sqrt{\frac{R}{\rho u^2}} 5 \left[(Pr. - 1) + \ln \left(\frac{5}{6} Pr. + \frac{1}{6} \right) \right]} \quad \dots (9.83)$$

$$= \frac{0.032 Re^{-\frac{1}{4}}}{1 + 0.82 Re^{-\frac{1}{4}} \left[(Pr. - 1) + \ln \left(\frac{5}{6} Pr. + \frac{1}{6} \right) \right]} \quad \dots (9.83)$$

Details of the derivation of the above equations are given by ECKERT⁽³⁾.

Example. Calculate the rise in temperature of water which is passed at 12 f.p.s. through a smooth 1 in. diameter pipe, 20 ft long. The water enters at 30°C and the temperature of the wall of the tube can be taken as approximately constant at 60°C. Use:

- The simple Reynolds analogy.
- The Taylor-Prandtl modification.
- The Buffer Layer equation (9.83).
- The equation $Nu. = 0.023 Re^{0.8} Pr^{0.33}$ (equation 8.30).

Solution. An approximate solution will be obtained, taking the fluid properties at 40°C and neglecting the effect of variation of properties with temperature. It will be assumed that fully developed flow exists.

$$Re. = \frac{(12 \times 30.48) \times 2.54 \times 1}{0.007} = 1.33 \times 10^5$$

$$Pr. = \frac{1 \times 0.007}{0.0041 \times 0.37} = 4.5$$

(a) Reynolds analogy.

$$\frac{h}{C_p \rho u} = 0.032 Re.^{-\frac{1}{4}} \quad \dots (9.78)$$

$$\begin{aligned} \therefore h &= 30.48 \times 12 \times 0.032 \times (1.33 \times 10^5)^{-\frac{1}{4}} \\ &= 0.61 \text{ g.-cal/cm}^2\text{-}^\circ\text{C-sec.} \end{aligned}$$

(b) Taylor-Prandtl equation,

$$\frac{h}{C_p \rho u} = 0.32 Re.^{-\frac{1}{4}} \{1 + 2.0 Re.^{-\frac{1}{4}} (Pr. - 1)\}^{-1} \quad \dots (9.77)$$

$$\begin{aligned} \therefore h &= 0.61 \left(1 + 2.0 \times 3.5 \times \frac{1}{4.37}\right)^{-1} \\ &= 0.23 \text{ g.-cal/cm}^2\text{-}^\circ\text{C-sec.} \end{aligned}$$

(c) Buffer layer equation.

$$\frac{h}{C_p \rho u} = 0.032 Re.^{-\frac{1}{4}} \left\{1 + 0.82 Re.^{-\frac{1}{4}} \left[(Pr. - 1) + \ln \left(5 \frac{Pr.}{6} + \frac{1}{6}\right)\right]\right\}^{-1} \quad \dots (9.83)$$

$$\begin{aligned} \therefore h &= 0.61 \left\{1 + \frac{0.82}{4.37} [3.5 + 2.30 \times 0.59]\right\}^{-1} \\ &= 0.61(1 + 0.91)^{-1} \\ &= 0.32 \text{ g.-cal/cm}^2\text{-}^\circ\text{C-sec.} \quad \dots (8.30) \end{aligned}$$

(d) $Nu. = 0.023 Re.^{0.8} . Pr.^{0.33}$

$$\begin{aligned} \therefore h &= \frac{0.00413 \times 0.37}{2.54} \times 0.023 \times (1.33 \times 10^5)^{0.8} \times 4.5^{0.33} \\ &= 0.000602 \times 0.023 \times 1.256 \times 10^4 \times 1.64 \\ &= 0.29 \text{ g.-cal/cm}^2\text{-}^\circ\text{C-sec.} \end{aligned}$$

Calculation of rise in temperature

Heat transferred per unit time in length dL of pipe $= h \cdot \pi \cdot 2.54 \cdot dL(60 - \theta)$ (where θ is the temperature at a distance L from the inlet).

Rate of increase of heat content of fluid $= \frac{1}{4}\pi \cdot 2.54^2 \times 12 \times 30.48 \times d\theta$.

Outlet temperature θ' is then given by

$$\int_{30}^{\theta'} \frac{d\theta}{60 - \theta} = 0.0043h \int_0^{20 \times 30.48} dL$$

$$\therefore \log_{10} (60 - \theta') = \log_{10} 30 - \frac{2.61}{2.303} \cdot h = 1.477 - 1.13h$$

The outlet temperature will be calculated in the following table:

	h	$1.13h$	$\log_{10} (60 - \theta')$	$60 - \theta'$	$\theta' (^{\circ}\text{C})$
(a)	0.61	0.69	0.787	6.1	53.9
(b)	0.23	0.26	1.217	16.5	43.5
(c)	0.32	0.36	1.117	13.1	46.9
(d)	0.29	0.33	1.147	14.0	46.0

The results confirm that the simple Reynolds analogy is far from accurate, when used to calculate the heat transfer to a liquid.

Example. The same tube is maintained at 80°C and air is passed through it at 12 f.p.s.; the initial temperature of the air is 20°C . Calculate the outlet temperature of the air for the four cases used in the above example.

Solution. Take the physical properties of the air at 40°C and assume that fully developed flow exists in the pipe.

$$Re. = \frac{(12 \times 30.48) \times 2.54 \times 29 \times 273}{0.00018 \times 22,400 \times 313} = 5800$$

$$Pr. = \frac{0.24 \times 0.00018}{0.014 \times 0.00413} = 0.75$$

The heat transfer coefficients and final temperatures are then calculated as in the first example. The results obtained are:

$$\begin{array}{ll} (a) \ h = 0.00038 \text{ g.-cal/cm}^2\text{-}^{\circ}\text{C-sec.} & \theta' = 78.1^{\circ}\text{C.} \\ (b) \ h = 0.00045 \text{ g.-cal/cm}^2\text{-}^{\circ}\text{C-sec.} & \theta' = 79.0^{\circ}\text{C.} \\ (c) \ h = 0.00044 \text{ g.-cal/cm}^2\text{-}^{\circ}\text{C-sec.} & \theta' = 78.9^{\circ}\text{C.} \\ (d) \ h = 0.00052 \text{ g.-cal/cm}^2\text{-}^{\circ}\text{C-sec.} & \theta' = 79.4^{\circ}\text{C.} \end{array}$$

In this case, the result obtained using the Reynolds analogy is far more nearly correct.

MASS TRANSFER TO A FLUID IN TURBULENT MOTION

If a fluid is flowing under turbulent conditions over a surface to which mass transfer is taking place, there will be a velocity and a partial pressure gradient near the surface. Transfer of momentum and of mass through the boundary layer can be considered in two parts: transfer through the turbulent portion of the boundary layer and transfer through the laminar sub-layer, if the existence of the buffer layer is neglected. The principle used in the derivation of the Reynolds analogy between heat transfer and fluid friction can be applied to the calculation of mass transfer rates. In the turbulent region, matter is considered as being transferred to the edge of the laminar sub-layer by means of eddy currents, and it is assumed that no mixing of the fluid takes place in the intervening space. The ratio of the mass transfer rate to the momentum transfer rate is then calculated. Another expression for this ratio is then obtained for the molecular transfer process which occurs in the laminar sub-layer and the two expressions are equated in order to eliminate an unknown condition at the interface between the two regions.

The heat transfer relations were not complicated by the existence of bulk flow in the direction of transfer. In mass transfer problems, it is necessary to

consider two important cases. One is the transfer process where no bulk flow takes place, i.e. equimolecular counter-diffusion; the other is the process in which there is no net motion of one of the constituents of the fluid mixture. Since the diffusing materials in a mixture will, in general, have different molecular weights, no simple relation can be obtained for the mass transfer coefficient, though COLBURN⁽²³⁾ gives an approximate expression. The problem is most satisfactorily handled by using the j -factor method already discussed in Chapter 8.

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LIST OF SYMBOLS USED IN CHAPTERS 7, 8 and 9

A	Area of surface	L^2
a	Ratio of u_b to u_s	—
a_0	Coefficient of y	$L^{-1}\theta$
a'	Coefficient of y	T^{-1}
b	Ratio of θ_b to θ_s	—

b_0	Coefficient of y^2	$L^{-2}\theta$
b'	Coefficient of y^2	$L^{-1}T^{-1}$
C	Coefficient in equation for heat transfer by convection	—
C_A, C_B	Molar concentration of A, B	ML^{-3}
C_p	Specific heat at constant pressure	—
C_T	Total molar concentration	ML^{-3}
C_v	Specific heat at constant volume	—
C_{Bm}	Logarithmic mean value of C_B	ML^{-3}
C'	Quantity independent of x	$ML^{-1.5}T^{-1}$
c	Mass concentration	ML^{-3}
c_m	Heat capacity of one molecule	M
c_o	Coefficient of y^3	$L^{-3}\theta$
c'	Coefficient of y^3	$L^{-2}T^{-1}$
D	Gas phase diffusivity	L^2T^{-1}
D_H	Thermal diffusivity	L^2T^{-1}
D_L	Liquid phase diffusivity	L^2T^{-1}
D_{AB}	Diffusivity of A in B	L^2T^{-1}
D_{BA}	Diffusivity of B in A	L^2T^{-1}
d	Pipe diameter	L
E	Eddy kinematic viscosity	L^2T^{-1}
E_D	Eddy diffusivity	L^2T^{-1}
E_H	Eddy thermal diffusivity	L^2T^{-1}
e	Surface roughness	L
F	Coefficient in MAXWELL'S Law of Diffusion	$M^{-1}L^3T^{-1}$
g	Acceleration due to gravity	LT^{-2}
h	Heat transfer coefficient	$ML^{-2}T^{-1}$
h_D	Mass transfer coefficient	LT^{-1}
h_m	Mean value of heat transfer coefficient	$ML^{-2}T^{-1}$
I	Ratio of mixing length to distance from surface	—
I'	Integration constant	—
i_1, i_2, i_3	Fractions of root mean square velocity of molecules	—
j	Fraction of mean free path of molecules	—
j_a	" j -factor" for mass transfer	—
j_h	" j -factor" for heat transfer	—
k_G	Transfer coefficient for transfer through stationary gas	$L^{-1}T$
k_G'	Transfer coefficient for equimolecular counterdiffusion	$L^{-1}T$
L	Length of pipe or surface, or characteristic dimension	L
L_e	Inlet length of pipe	L
l	Thickness of element of fluid in Y -direction	L
M	Molecular weight	—
M	Mass of fluid	M
m	Mass of gas molecule	M
N	Molar rate of diffusion per unit area	$ML^{-2}T^{-1}$
N'	Total molar rate of transfer per unit area	$ML^{-2}T^{-1}$
N	Numerical concentration of molecules at $y = y$	L^{-3}
N'	Numerical concentration of molecules at $y = y + j\lambda$	L^{-3}
n	Number of moles of gas	M
P	Total pressure	$ML^{-1}T^{-2}$
P_A, P_B	Partial pressure of A, B	$ML^{-1}T^{-2}$
P_s	Vapour pressure of water	$ML^{-1}T^{-2}$
P_w	Partial pressure of water in gas stream	$ML^{-1}T^{-2}$
P_{As}	Partial pressure of A outside boundary layer	$ML^{-1}T^{-2}$
P_{Bm}	Logarithmic mean value of P_B	$ML^{-1}T^{-2}$
p	An index	—
Q	Rate of transfer of heat	$MT^{-1}\theta$

q	Rate of transfer of heat per unit area at walls	$ML^{-2}T^{-1}\theta$
q_y	Rate of transfer of heat per unit area at $y = y$	$ML^{-2}T^{-1}\theta$
R	Shear stress acting on surface	$ML^{-1}T^{-2}$
R_0	Shear stress acting on fluid at surface	$ML^{-1}T^{-2}$
R_y	Shear stress in fluid at $y = y$	$ML^{-1}T^{-2}$
R	Universal gas constant	$L^2T^{-2}\theta^{-1}$
r	Radius of pipe	L
S	Cross-sectional area of flow	L^2
T	Absolute temperature	θ
t	Time	T
u	Mean velocity	LT^{-1}
u_A, u_B	Mean molecular velocity in direction of transfer	LT^{-1}
u_b	Velocity at edge of laminar sub-layer	LT^{-1}
u_E	Mean velocity in eddy	LT^{-1}
u_e	Velocity at distance e from surface	LT^{-1}
u_F	Velocity due to bulk flow	LT^{-1}
u_m	Root mean square velocity of molecules	LT^{-1}
u_0	Velocity of fluid at surface	LT^{-1}
u_s	Velocity of fluid outside boundary layer, or at pipe axis	LT^{-1}
u_y	Velocity in X -direction at $y = y$	LT^{-1}
u_y'	Velocity in X -direction at $y = y + j\lambda$	LT^{-1}
u^+	Ratio of u_y to u^*	—
u^*	Shearing stress velocity, $\sqrt{R/\rho}$	LT^{-1}
V	Volume	L^3
V	Molecular volume	$M^{-1}L^3$
V_0	Correction term in equation 7.25	$M^{-1}L^3$
W	Mass rate of evaporation	MT^{-1}
w_1, w_2, w_3	Indices	—
X	Direction of flow of fluid	—
x	Distance from leading edge of surface in X -direction	L
x_e	Value of x at which flow becomes turbulent	L
x_0	Unheated length of surface	L
Y	Direction perpendicular to surface	—
y	Distance from surface	L
y_L	Equivalent thickness of liquid film	L
y_r	Reference distance from surface	L
y^+	Ratio of y to $\mu/\rho u^*$	—
Z	Third principal direction	—
z	Distance in Z -direction	L
Δ	Finite difference in a quantity	—
β	Coefficient of thermal expansion	θ^{-1}
γ	Ratio of C_p to C_v	—
δ	Thickness of boundary layer	L
δ_b	Thickness of laminar sub-layer	L
δ_t	Thickness of thermal boundary layer	L
δ^*	Displacement thickness of boundary layer	L
λ	Mean free path of molecules	L
λ_E	Mixing length	L
μ	Viscosity of fluid	$ML^{-1}T^{-1}$
ρ	Density of fluid	ML^{-3}
ρ_r	Density of fluid at $y = y_r$	ML^{-3}
ρ_w	Density of fluid at surface	ML^{-3}
σ	Ratio of δ_t to δ	—
θ	Temperature at $y = y$	θ
θ_B	Bulk temperature of fluid	θ

θ_m	Logarithmic mean temperature	θ
θ_r	Temperature at $y = y_r$	θ
θ_s	Temperature outside boundary layer, or at pipe axis	θ
θ'	Temperature at $y = y + j\lambda$	θ
$Nu.$	Nusselt Number hd/k	—
Nu_x	Nusselt Number hx/k	—
$Re.$	Reynolds Number $ud\rho/\mu$	—
Re_r	Roughness Reynolds Number $u_s e \rho/\mu$	—
Re_x	Reynolds Number $u_s x \rho/\mu$	—
Re_{x_0}	Reynolds Number $u_s x_c \rho/\mu$	—
$Pr.$	Prandtl Number $C_p \mu/k$	—
$Sc.$	Schmidt Number $\mu/\rho D$	—
$St.$	Stanton Number $h/C_p \rho u$ or $h/C_p \rho u_s$	—

Section D

Humidification and Water Cooling

CHAPTER 10

Humidification and Water Cooling

It is frequently necessary to alter the amount of vapour present in a gas stream. If the moisture content is to be reduced, partial condensation must be effected and the condensed vapour removed. The moisture content can be increased by passing the gas over water, which then evaporates into the gas stream. Increasing the moisture content is referred to as humidification and decreasing as dehumidification. In humidification moisture is transferred to the gas in contact with the water by evaporation and then to the main stream by diffusion, and simultaneous transfer of heat and mass takes place according to the relations developed in previous chapters. The mass of vapour present in unit mass of dry gas is known as the humidity.

Humidification and dehumidification are widely used in air conditioning and in gas drying. Thus water vapour is removed from wet chlorine which is very corrosive so that the gas can be handled in steel equipment, and the gases used in sulphuric acid manufacture are dehumidified or dried before they enter the convertors. The moisture may be removed by passing the gas through some dehydrating agent, such as sulphuric acid, when the process is one of absorption, or alternatively, the gas may be dehumidified by one of the methods described later.

The cooling of water by bringing it into contact with an air stream is a widely used process for treating hot condenser water, so that it can be returned to the plant. In the usual form of water cooling tower the hot water is run in at the top and flows downwards over some form of packing against the air current which enters at the bottom. The design of these towers is discussed later in the chapter and, although the air-water system is much the most important and is in some ways unique, the same basic principles can be applied to other liquids and gases.

Definition of Terms. A number of terms are used in relation to humidification and they are defined below:

Humidity \mathcal{H} . . . Mass of vapour associated with unit mass of dry gas.

Humidity of saturated gas \mathcal{H}_0 . . . The humidity of the gas when saturated with vapour at the given temperature.

Percentage humidity . $\frac{\text{Humidity of the gas} \times 100}{\text{Humidity of saturated gas}}$

Humid heat	. The heat required to raise unit mass of dry gas and its associated vapour through unit temperature difference at constant pressure (approx. $0.24 + 0.45 \mathcal{H}$ for air and water vapour).
Humid volume	. The volume occupied by unit mass of dry gas and its associated vapour.
Saturated volume	. The humid volume of saturated gas.
Dew point	. The temperature at which the gas is saturated with vapour. It is the temperature at which condensation will first occur when the gas is cooled.
Percentage relative humidity	. $\frac{\text{Partial pressure of vapour in the gas} \times 100}{\text{Partial pressure of vapour in saturated gas}}$

The humidity of a gas is related to the partial pressure of the vapour in the following manner. In unit volume of gas, the mass of vapour is $\frac{P_w}{RT} \cdot M_w$ and the mass of non-condensable gas is $\frac{P - P_w}{RT} \cdot M_A$ and therefore the humidity \mathcal{H} is given by

$$\mathcal{H} = \frac{P_w}{P - P_w} \cdot \frac{M_w}{M_A} \quad \dots(10.1)$$

Similarly, the humidity of saturated gas is given by

$$\mathcal{H}_0 = \frac{P_{w0}}{P - P_{w0}} \cdot \frac{M_w}{M_A} \quad \dots(10.1a)$$

For the air-water system, P_w is frequently small compared with the total pressure P , and we can write

$$\mathcal{H} = \frac{18}{29} \frac{P_w}{P}$$

Then, at atmospheric pressure $\mathcal{H} = \frac{1.8}{2.9} P_w$ where P_w is in atmospheres.

The percentage humidity of a gas can now be expressed in terms of the percentage relative humidity as follows.

The percentage humidity

$$\begin{aligned}
 &= \frac{\mathcal{H}}{\mathcal{H}_0} \times 100 \text{ (by definition)} \\
 &= \frac{\frac{P_w}{P - P_w} \cdot \frac{M_w}{M_A}}{\frac{P_{w0}}{P - P_{w0}} \cdot \frac{M_w}{M_A}} = \frac{P - P_{w0}}{P - P_w} \times \frac{P_w}{P_{w0}} \times 100 \\
 &= \frac{P - P_{w0}}{P - P_w} \times \text{percentage relative humidity} \quad \dots(10.2)
 \end{aligned}$$

In the above expressions,

P_w is the partial pressure of the vapour in the gas,

P_{w0} is the partial pressure of vapour in saturated gas at the same temperature,

M_A is the mean molecular weight of the gas,

M_w is the molecular weight of the vapour,

P is the total pressure,

R is the gas constant, and

T is the absolute temperature.

Thus the percentage relative humidity and the percentage humidity are the same only when $\frac{P - P_{w0}}{P - P_w}$ is equal to unity. This condition is approached in nearly saturated gas (where $P_w \rightarrow P_{w0}$) and when the partial pressure of the vapour is only a small proportion of the total pressure.

WET BULB TEMPERATURE

If a stream of unsaturated gas is passed over the surface of a liquid, evaporation takes place and the humidity of the gas is increased. The temperature of the liquid falls below that of the gas, so that heat then flows from the gas to the liquid and, at equilibrium, the heat supplied by the gas is just sufficient to vaporise the liquid. The temperature finally attained by the liquid is known as the wet bulb temperature and the rate at which this temperature is reached depends on the initial temperatures and the rate of flow of the gas past the surface. If the gas rate is high and the area of contact between the gas and liquid is small, there will be no significant change in either the temperature or the humidity of the gas stream.

The rate of transfer of heat from the gas to the liquid can then be written as

$$Q = h \cdot A \cdot (\theta - \theta_w) \quad \dots(10.3)$$

where Q is the heat flow per unit time,

h is the heat transfer coefficient,

A is the area for transfer, and

θ and θ_w the temperatures of the gas and liquid phases.

The material evaporating into the gas is transferred by diffusion from the interface to the gas stream as a result of the concentration difference $c_0 - c$, where c_0 is the concentration of vapour (mass per unit volume) at the surface and c is the concentration in the gas stream. The rate of evaporation W is then given by

$$W = h_D A (c_0 - c) = \frac{h_D A M_w}{RT} (P_{w0} - P_w) \quad \dots(10.4)$$

where h_D is the mass transfer coefficient. (See equation 7.15.)

The partial pressures P_w and P_{w0} of the vapour can be expressed in terms of the corresponding humidities \mathcal{H} and \mathcal{H}_w by equations 10.1 and 10.1a.

If P_w and P_{w0} are small compared with P , $(P - P_w)$ and $(P - P_{w0})$ can be replaced by the mean partial pressure of the gas P_A , so that

$$\begin{aligned} W &= \frac{h_D A (\mathcal{H}_w - \mathcal{H}) M_w}{RT} \cdot P_A \cdot \frac{M_A}{M_w} \\ &= h_D A \rho_A (\mathcal{H}_w - \mathcal{H}) \end{aligned} \quad \dots(10.5)$$

where ρ_A is the density of the gas at the partial pressure P_A .

The heat transfer required to maintain this rate of evaporation

$$Q = h_D \rho_A A (\mathcal{H}_w - \mathcal{H}) \cdot \lambda \quad \dots(10.6)$$

where λ is the latent heat of vaporisation per unit mass of liquid.

Thus equating the values of Q given by equations 10.3 and 10.6

$$\mathcal{H} - \mathcal{H}_w = - \frac{h}{h_D \rho_A \lambda} (\theta - \theta_w) \quad \dots(10.7)$$

Both h and h_D are dependent on the equivalent gas film thickness and thus any decrease in the thickness, as effected for instance by increasing the velocity of the gas, increases both h and h_D . At normal temperatures the ratio h/h_D has been found to be almost independent of the gas velocity provided this is greater than about 15 ft/sec. Under these conditions the heat transfer by convection from the gas stream is large compared with that by radiation and conduction from the surroundings.

Equation 10.7 shows that the difference in the humidity between the liquid surface and the gas is proportional to the difference in temperature between the gas and liquid. The wet bulb temperature θ_w depends only on the temperature and humidity of the gas. The ratio $h/h_D \rho_A$ has a value of about 0.26 for air and water, but varies from 0.4 to 0.52 for organic liquids. The values of θ_w which are normally quoted are determined under conditions where the gas velocity is comparatively high, so that the condition of the gas does not change appreciably as a result of contact with the liquid and the ratio h/h_D has reached a constant value.

ADIABATIC SATURATION TEMPERATURE

In the system we have just considered, neither the humidity nor the temperature of the gas is appreciably changed. If the gas is passed over the liquid at such a rate that the time of contact is sufficient for equilibrium to be established, the gas will become saturated and both phases will be brought to the same temperature. In a thermally insulated system, the total sensible heat falls by an amount equal to the latent heat of the liquid evaporated. As a result of continued passage of the gas the temperature of the liquid gradually approaches an equilibrium value which is known as the adiabatic saturation temperature.

These conditions are achieved in an infinitely tall thermally insulated humidification column, through which gas of a given initial temperature and humidity flows counter-currently to the liquid, under conditions where the gas is completely saturated at the top of the column. If the liquid is continuously circulated round the column, and if any fresh liquid which is added is at the same temperature as the circulating liquid, the temperature of the liquid at the top and bottom of the column, and of the gas at the top, approach the adiabatic saturation temperature. Temperature and humidity differences are a maximum at the bottom and zero at the top, and therefore the rates of transfer of heat and mass decrease progressively from the bottom to the top of the tower.

If we now take a heat balance over the column, we see that the whole of the heat of vaporisation of the liquid must come from the sensible heat in the gas. The temperature of the gas falls from θ to the adiabatic saturation temperature, θ_s , and its humidity increases from \mathcal{H} to \mathcal{H}_s (the saturation value at θ_s). Then working to a basis of unit mass of dry gas, we have,

$$(\theta - \theta_s) \cdot s = (\mathcal{H}_s - \mathcal{H}) \cdot \lambda$$

$$\text{i.e.} \quad \mathcal{H} - \mathcal{H}_s = -\frac{s}{\lambda}(\theta - \theta_s) \quad \dots(10.8)$$

where s is the humid heat of the gas (for small changes in \mathcal{H} , s will be almost constant) and λ is the latent heat of vaporisation at a temperature θ_s .

Equation 10.8 shows that there is an approximately linear relation between humidity and temperature for all mixtures of gas and vapour having the same adiabatic saturation temperature θ_s . A curve of humidity versus temperature for gases with a given adiabatic saturation temperature is known as an adiabatic cooling line. For a range of adiabatic saturation temperatures a family of curves, approximating to straight lines with slopes of $-\frac{s}{\lambda}$, is obtained. Because λ and s vary slightly the lines will not be exactly straight and parallel.

If equations 10.7 and 10.8 are compared, it is seen that the adiabatic saturation and wet bulb temperatures are identical if s is equal to $\frac{h}{h_{D\rho_A}}$. This is approximately true generally for water vapour systems, and accurately so when $\mathcal{H} = 0.047$.

The identity of the two temperatures for water vapour has long been known but little experimental work appears to have been carried out with other vapours until that of SHERWOOD and COMINGS⁽³⁾ in 1932. They worked with water, ethanol, n-propanol, n-butanol, benzene, toluene, carbon tetrachloride, and n-propyl acetate, and found that the wet bulb temperature was always higher than the adiabatic saturation temperature, except in the case of water.

In Chapter 8, it was shown that, when the Schmidt and Prandtl Numbers

for a mixture of gas and vapour are approximately equal to unity, the Lewis relation applies, i.e.

$$\frac{h}{h_D} = C_p \cdot \rho \quad \dots (8.38)$$

where C_p and ρ are the mean specific heat and density of the vapour phase.

$$\therefore \frac{h}{h_D \rho_A} = C_p \frac{\rho}{\rho_A} \quad \dots (10.9)$$

Unless the humidity is very high, the specific heat and the humid heat are of the same order, and the density of the gas-vapour mixture approximates to that of the gas alone. Under these conditions, therefore,

$$s \approx \frac{h}{h_D \rho_A} \quad \dots (10.10)$$

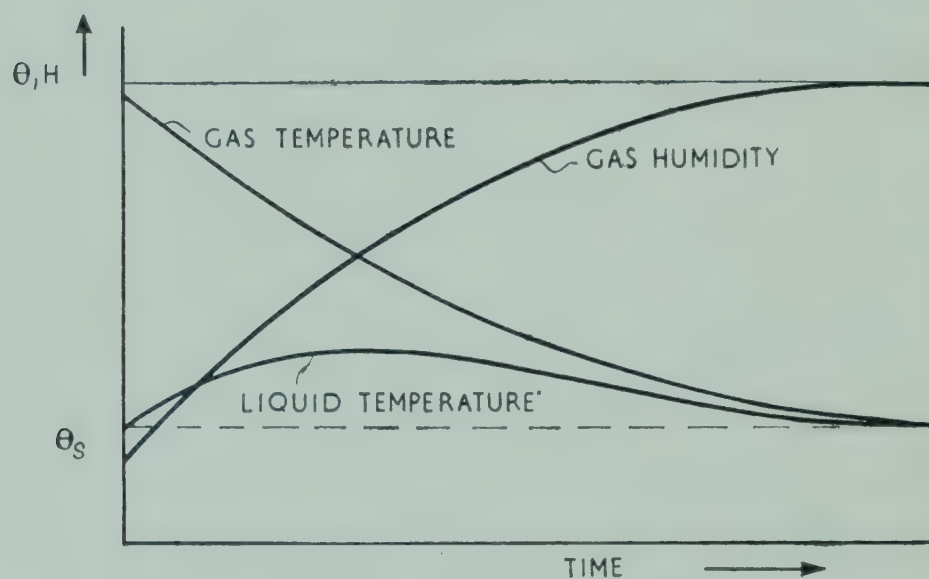


Fig. 10.1. Saturation of gas with liquid other than water at adiabatic saturation temperature

For all systems containing vapour other than water vapour, the humid heat s is only approximately equal to the ratio $\frac{h}{h_D \rho_A}$ and the difference between the two quantities may be as much as 50%.

If an unsaturated gas is brought into contact with a liquid which is at the adiabatic saturation temperature of the gas, a simultaneous transfer of heat and mass takes place. The temperature of the gas falls and its humidity increases (see Fig. 10.1). The temperature of the liquid at any instant tends to change and approach the wet bulb temperature corresponding to the particular condition of the gas at that moment. For a liquid other than water the adiabatic saturation temperature is less than the wet bulb temperature and therefore in the initial stages the temperature of the liquid rises. As the gas becomes humidified, however, its wet bulb temperature falls and consequently the temperature to which the liquid is tending decreases as evaporation takes place. In due course, therefore, a point is reached where the liquid actually reaches the wet bulb temperature of the gas in contact with it. It does not remain at this temperature, however, because the gas is not then completely saturated and further humidification is accompanied by a continued lowering of the wet bulb temperature. The temperature of the liquid therefore starts to fall and continues to fall until the gas is completely saturated. The liquid and gas are then both at the adiabatic saturation temperature.

The air-water system is unique, however, in that the Lewis relation holds quite accurately, so that the adiabatic saturation temperature is the same as the wet bulb temperature. If, therefore, an unsaturated gas is brought into contact with water at the adiabatic saturation temperature of the gas, there is no tendency for the temperature of the water to change and it remains in a condition of dynamic equilibrium throughout the whole of the humidification process (see Fig. 10.2). In this case, the adiabatic cooling line represents the conditions of gases of constant wet bulb temperatures as well as constant adiabatic saturation temperatures. The change in the condition of a gas as it is humidified with water vapour is therefore represented by the adiabatic cooling line and the intermediate conditions of the gas during the process are readily obtained. This is particularly useful because only partial humidification is normally obtained in practice.

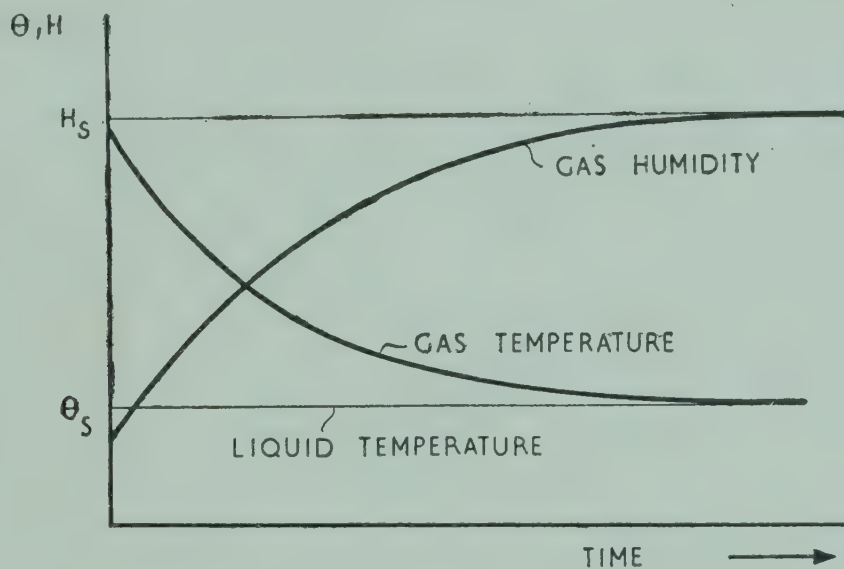


Fig. 10.2. Saturation of air with water at adiabatic saturation temperature

AIR-WATER SYSTEM

To facilitate calculations, various properties of the air-water system are plotted on the Humidity Chart. These charts are based on either the temperature or the enthalpy of the gas. The temperature-humidity chart is the more commonly used but the enthalpy-humidity chart is particularly useful for determining the effect of mixing two gases or of mixing a gas and a liquid. Each chart refers to a particular total pressure of the system. A humidity-temperature chart for the air-water system at one atmosphere total pressure is given in Fig. 10.3* and the corresponding humidity-enthalpy chart is given in Fig. 10.4.

Temperature-humidity Chart

It will be seen that the following quantities are plotted against temperature in Fig. 10.3.*

(1) *The Humidity \mathcal{H}* , for various values of the percentage relative humidity. For saturated gas

$$\mathcal{H}_0 = \frac{P_{w0}}{P - P_{w0}} \times \frac{M_w}{M_A} \quad \dots (10.1a)$$

* See folded chart facing page 370.

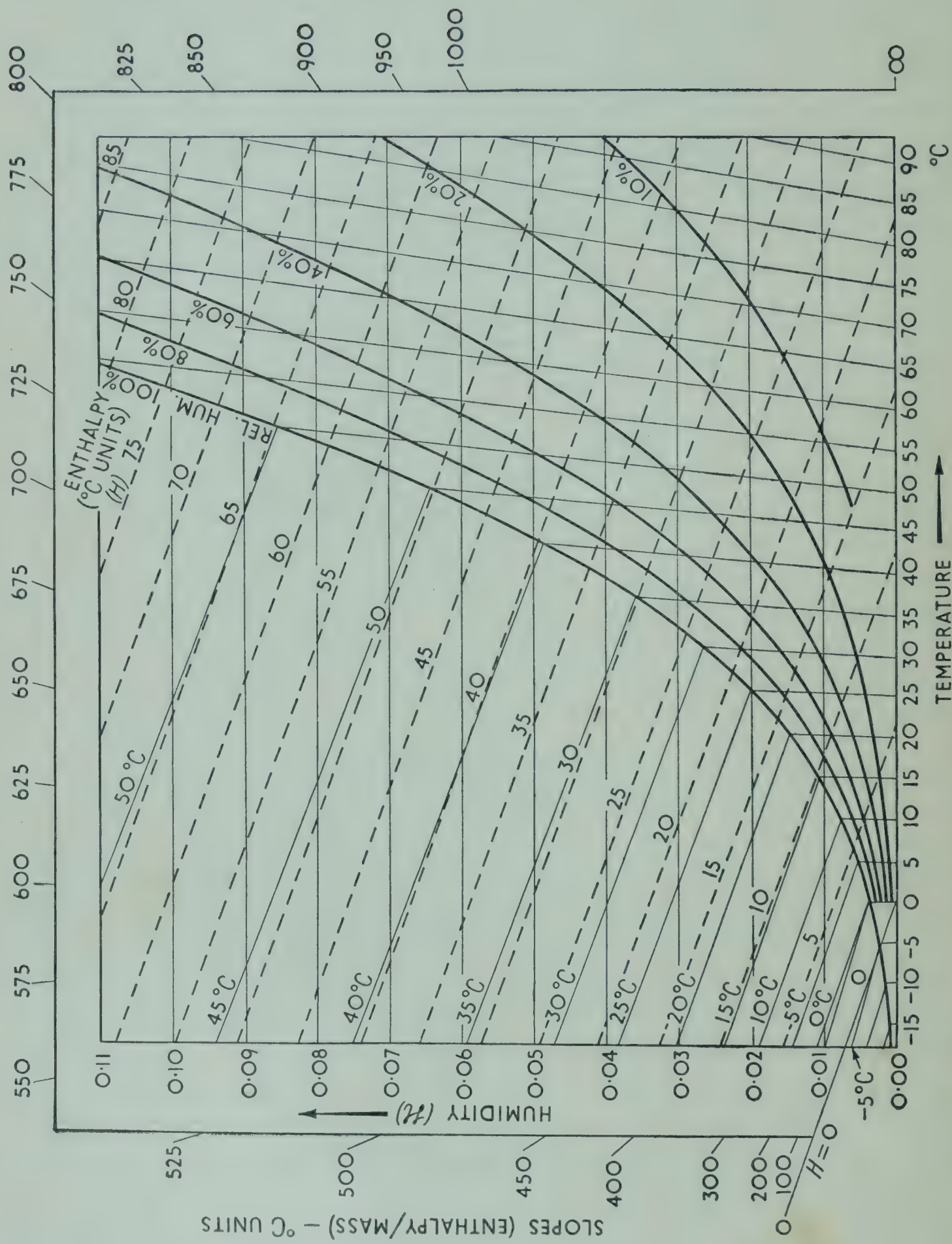


Fig. 10.4. Humidity-enthalpy diagram for air-water vapour system at atmospheric pressure

For a gas of relative humidity $Z\%$, we have from equations 10.1 and 10.2,

$$\mathcal{H} = \mathcal{H}_0 \cdot \frac{Z}{100} \cdot \frac{P - P_{w0}}{P - \frac{Z}{100} P_{w0}} \quad \dots (10.11)$$

(2) *The specific volume of dry gas*, which increases linearly with temperature.

(3) *The saturated volume*, which increases more rapidly because both the quantity and the specific volume of vapour increase with temperature. At a given temperature the humid volume varies linearly with humidity and therefore the humid volume of unsaturated gas can be found by interpolation.

(4) *The latent heat of vaporisation*.

In addition the *humid heat* is plotted as abscissa, against *humidity* as ordinate.

A number of adiabatic cooling lines are also plotted in Fig. 10.3. It has already been pointed out that these lines have a slope of $-\frac{s}{\lambda}$; they are slightly curved because s is a function of \mathcal{H} . They are drawn exactly straight on the diagram and the inclination of the axis is correspondingly adjusted. Each adiabatic cooling line represents the composition of all gases whose adiabatic saturation temperature is given by its point of intersection with the 100% relative humidity curve. For the air-water system, the adiabatic cooling lines represent conditions of constant wet bulb temperature as well and, as previously mentioned, enable the change in composition of a gas to be followed as it is humidified by contact with water at the adiabatic saturation temperature of the gas.

Enthalpy-humidity Chart

In the calculation of enthalpies, it is necessary to define some standard reference state at which the enthalpy is taken as zero. It is most convenient to take the melting point of material constituting the vapour as the reference temperature, and the liquid state of the material as its standard state.

Then if H is the enthalpy of the humid gas per unit mass of dry gas,

H_a is the enthalpy of the dry gas per unit mass,

H_w is the enthalpy of the vapour per unit mass,

C_a is the specific heat of the gas at constant pressure,

C_w is the specific heat of the vapour at constant pressure,

θ is the temperature of the humid gas,

θ_0 is the reference temperature,

λ is the latent heat of vaporisation of the liquid at θ_0 , and

\mathcal{H} is the humidity of the gas,

for unsaturated gas

$$H_a = C_a(\theta - \theta_0) \quad \dots (10.12)$$

$$H_w = C_w(\theta - \theta_0) + \lambda \quad \dots (10.13)$$

and

$$H = H_a + H_w \cdot \mathcal{H}$$

$$= C_a(\theta - \theta_0) + \mathcal{H}[C_w(\theta - \theta_0) + \lambda] \quad \dots(10.14)$$

If the gas contains more liquid or vapour than is required to saturate it at the temperature in question, either the gas will be supersaturated or the excess material will be present in the form of liquid or solid, according to whether the temperature θ is greater or less than the reference temperature, θ_0 . The supersaturated condition is unstable and therefore will not be considered further.

If the temperature θ is greater than θ_0 and if the humidity \mathcal{H} is greater than the humidity \mathcal{H}_0 of saturated gas, the enthalpy H per unit mass of dry gas is given by,

$$H = C_a(\theta - \theta_0) + \mathcal{H}_0[C_w(\theta - \theta_0) + \lambda] + C_L(\mathcal{H} - \mathcal{H}_0)(\theta - \theta_0) \quad \dots(10.15)$$

where C_L is the specific heat of the liquid.

If the temperature is less than θ_0 , the corresponding enthalpy H is given by,

$$H = C_a(\theta - \theta_0) + \mathcal{H}_0[C_w(\theta - \theta_0) + \lambda] + (\mathcal{H} - \mathcal{H}_0)[C_s(\theta - \theta_0) + \lambda_f] \quad \dots(10.16)$$

where C_s is the specific heat of the solid, and

λ_f is the latent heat of freezing of the liquid, i.e. a negative quantity.

The above three equations give the relation between enthalpy, temperature, and humidity for humid gas. For a given percentage relative humidity and temperature, the humidity can be found from the humidity-temperature diagram. Thus, for any given relative humidity, the enthalpy of a gas can be calculated from equations 10.14, 10.15, and 10.16 and plotted against its humidity; the enthalpy is usually plotted as the abscissa. Curves for 100% relative humidity, and for some lower relative humidity ($Z\%$, say), are shown in Fig. 10.5 for the air-water system. The nature of the isothermals will now be considered for the above three conditions.

At a constant temperature θ , the relation between enthalpy and humidity for an unsaturated gas is

$$H = \text{constant} + [C_w(\theta - \theta_0) + \lambda]\mathcal{H} \quad \dots(10.14a)$$

Thus the isothermal is a straight line of slope $[C_w(\theta - \theta_0) + \lambda]$ with respect to the humidity axis. At the reference temperature θ_0 , the slope is λ ; at higher temperatures the slope is greater than λ , and at lower temperatures it is less than λ . Because the latent heat is normally large compared with the sensible heat, the slope of the isothermals remains positive down to very low temperatures. Since the humidity is plotted as the ordinate, the slope of the isothermal relative to the X -axis decreases with increase in temperature.

At temperatures greater than θ_0 and humidities greater than the saturation humidity \mathcal{H}_0 , the vapour phase consists of saturated gas together with liquid

droplets in suspension, and the relation between enthalpy and humidity at constant temperature θ , is

$$H = \text{constant} + C_L \cdot (\theta - \theta_0) \cdot \mathcal{H} \quad \dots (10.15a)$$

The isothermal is therefore a straight line of slope $C_L \cdot (\theta - \theta_0)$. At the reference temperature θ_0 , the slope is zero and the isothermal is parallel to the humidity axis. At higher temperatures the slope has a small positive value.

When the humidity is greater than \mathcal{H}_0 and the temperature is less than θ_0 , solid particles are formed and the equation of the isothermal is

$$H = \text{constant} + [C_s \cdot (\theta - \theta_0) + \lambda_f] \cdot \mathcal{H} \quad \dots (10.16a)$$

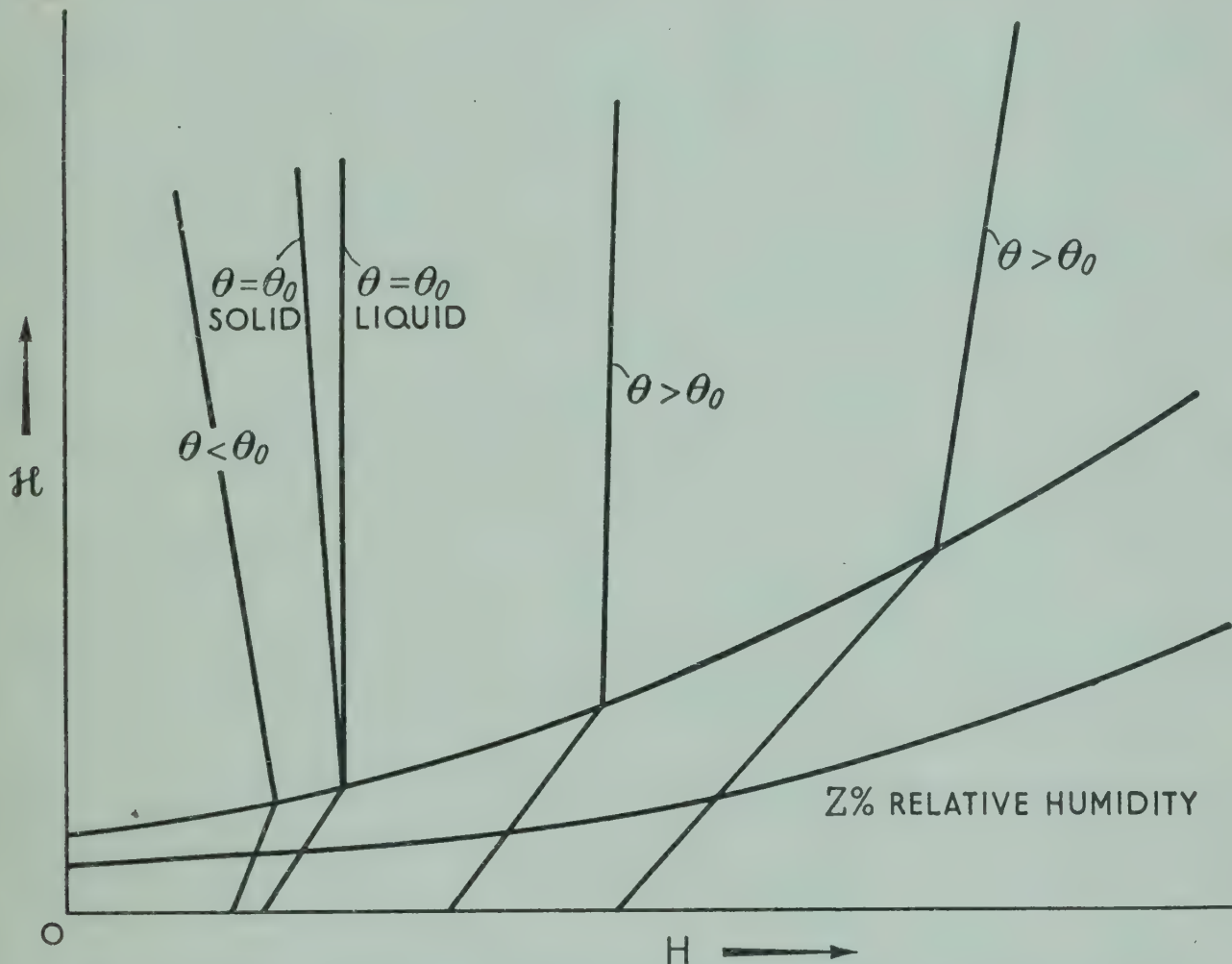


Fig. 10.5. Humidity-enthalpy diagram for air-water system—rectangular axes

This represents a straight line of slope, $[C_s(\theta - \theta_0) + \lambda_f]$. Both $C_s(\theta - \theta_0)$ and λ_f are negative and therefore the slopes of all these isothermals are negative. When $\theta = \theta_0$, the slope is λ_f . In the supersaturated region therefore there are two distinct isothermals at temperature θ_0 ; one corresponds to the condition where the excess of vapour is present in the form of liquid droplets and the other to the condition where it is present as solid particles. The region between these isothermals represents conditions where a mixture of liquid and solid is present in the saturated gas at the temperature θ_0 .

The diagram is unsatisfactory in this form for the water vapour-air system, because the shape of the humidity-enthalpy line for 100% relative humidity is such that the unsaturated region, which is the one most commonly referred to, occupies only a small fraction of the total area of the diagram, and accurate readings are difficult to make. The diagram is therefore usually drawn with

oblique axes, with the humidity axis vertical as before but with the lines of constant enthalpy inclined at such an angle that the isothermal in the unsaturated region for the temperature θ_0 is parallel to the humidity axis (Fig. 10.6). The isothermals remain straight lines but their apparent slopes are altered.

An illustration of a process which may be followed on the chart is the cooling of a gas at a constant total humidity. If a gas of dew-point greater than θ_0 and initially unsaturated is progressively cooled, the change in its composition will be indicated by a straight line at right angles to the humidity axis. This line will cut the 100% relative humidity line when the temperature

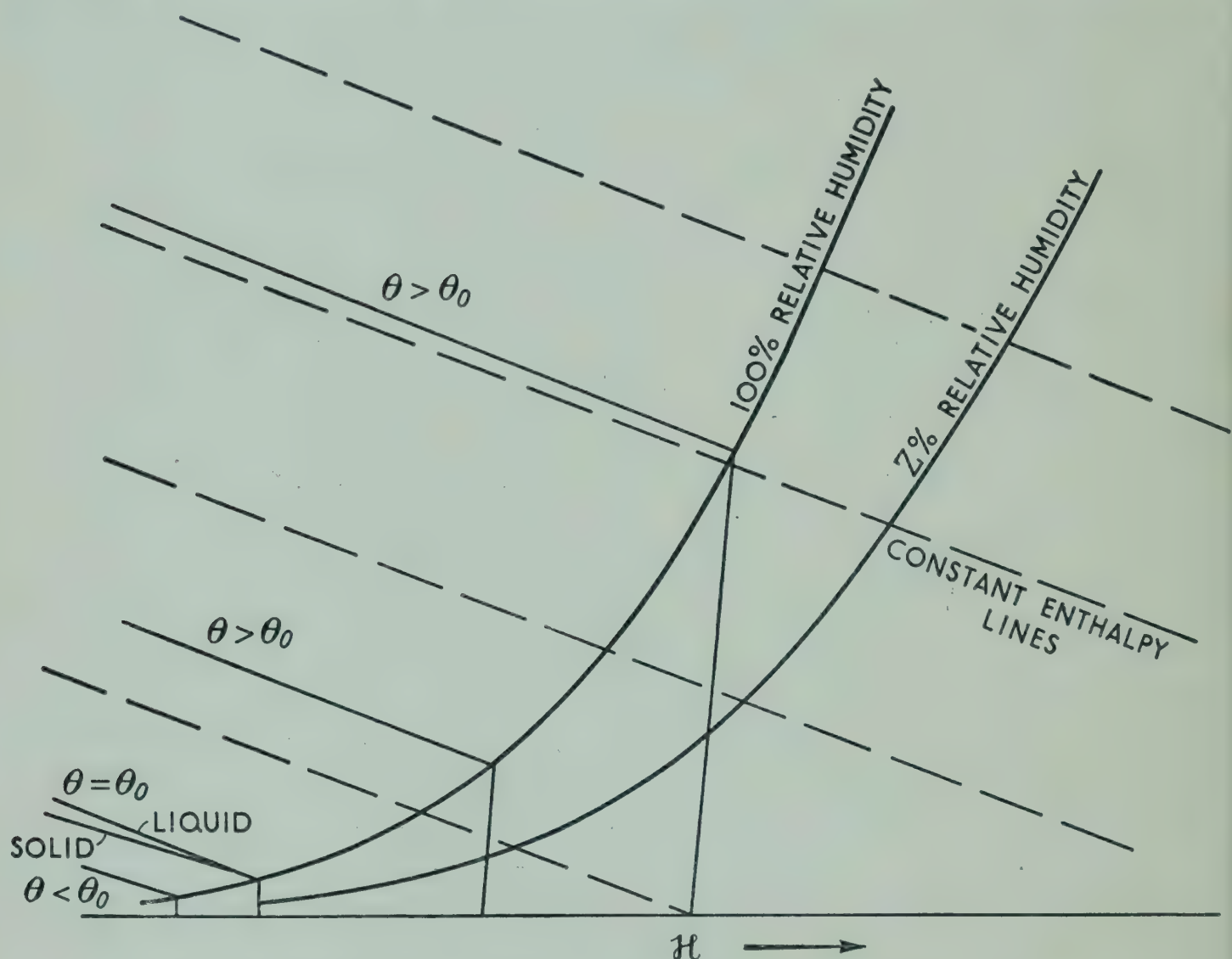


Fig. 10.6. Humidity-enthalpy diagram for air-water system—oblique axes

of the gas has been reduced to its dew-point. Further cooling will then result in the condensation of moisture and the condition of the mist-laden gas will be represented by a point to the left of the 100% relative humidity line. The composition of the gas phase, however, will correspond to the point where the isothermal through this point cuts the 100% relative humidity curve. The difference in the ordinates at the two points then represents the quantity of liquid which has been condensed. After further cooling, the temperature will fall to the freezing point of the liquid and further abstraction of heat will simply cause the liquid droplets to freeze. This process continues until all the liquid is frozen at the temperature θ_0 . Further cooling then results in a reduction of temperature and condensation of vapour as solid. If the dew-point of the gas had been below the freezing point θ_0 , liquid droplets would not have been formed at any stage in the cooling process. The gas would have fallen in temperature

without condensation until the dew-point was reached and further cooling would then have resulted in condensation of solid.

It should be noted that the curves of humidity plotted against either temperature or enthalpy have a discontinuity at the point corresponding to the freezing point of the humidifying material. Above the temperature θ_0 the lines are determined by the vapour-liquid equilibrium and below it by the vapour-solid equilibrium.

Mixing of Two Streams of Humid Gas. The condition of the gas resulting from the mixing of two streams of humid gas can be readily determined with the aid of the humidity-enthalpy diagram.

Consider the mixing of two gases of humidities \mathcal{H}_1 and \mathcal{H}_2 , temperatures θ_1 and θ_2 , and enthalpies H_1 and H_2 to give a mixed gas of temperature θ , enthalpy H , and humidity \mathcal{H} . Let the masses of dry gas concerned be m_1 , m_2 , and m respectively.

Then taking a balance on the dry gas, vapour, and enthalpy, we have,

$$m_1 + m_2 = m \quad \dots(10.17)$$

$$m_1\mathcal{H}_1 + m_2\mathcal{H}_2 = m\mathcal{H} \quad \dots(10.18)$$

and
$$m_1H_1 + m_2H_2 = mH \quad \dots(10.19)$$

Elimination of m gives,

$$m_1(\mathcal{H} - \mathcal{H}_1) = m_2(\mathcal{H}_2 - \mathcal{H}) \quad \dots(10.20)$$

and
$$m_1(H - H_1) = m_2(H_2 - H)$$

Dividing these two equations

$$\frac{\mathcal{H} - \mathcal{H}_1}{H - H_1} = \frac{\mathcal{H}_2 - \mathcal{H}}{H_2 - H} \quad \dots(10.21)$$

The condition of the resultant gas is therefore represented by a point on the straight line joining (\mathcal{H}_1, H_1) and (\mathcal{H}_2, H_2) . The humidity \mathcal{H} is given, from equation 10.20, by

$$\frac{\mathcal{H} - \mathcal{H}_1}{\mathcal{H}_2 - \mathcal{H}} = \frac{m_2}{m_1} \quad \dots(10.20a)$$

The gas formed by mixing two unsaturated gases may be either unsaturated, saturated, or supersaturated. The possibility of producing supersaturated gas arises because the 100% relative humidity line on the humidity-enthalpy diagram is concave towards the humidity axis.

Addition of Liquid or Vapour to a Gas. Let a mass m_3 of liquid or vapour of enthalpy H_3 be added to a gas of humidity \mathcal{H}_1 and enthalpy H_1 and containing a mass m_1 of dry gas. Then,

$$m_1(\mathcal{H} - \mathcal{H}_1) = m_3 \quad \dots(10.22)$$

$$m_1(H - H_1) = m_3H_3 \quad \dots(10.23)$$

Thus
$$\frac{H - H_1}{\mathcal{H} - \mathcal{H}_1} = H_3 \quad \dots (10.24)$$

where \mathcal{H} and H are the humidity and enthalpy of the gas produced on mixing.

The composition of the final gas is therefore represented by some point on the straight line, through (H_1, \mathcal{H}_1) , of slope, H_3 , relative to the humidity axis. The edge of the humidity chart is marked with points which, when joined to the origin, give straight lines of specified slopes. Thus a line of the required slope is drawn through the origin and a second line is then drawn parallel to it through the point (H_1, \mathcal{H}_1) . The position representing the final composition of the gas is given by the relation, $\mathcal{H} - \mathcal{H}_1 = m_3/m_1$ (for equation 10.22).

It can be seen, by reference to Fig. 10.4, that for the air-water system a straight line, of slope equal to the enthalpy of dry saturated steam (640 centigrade units), is almost parallel to the isothermals, so that the addition of live steam has only a small effect on the temperature of the gas. The addition of water spray, even if the water is considerably above the temperature of the gas, results in a lowering of the temperature provided that the whole of the water is evaporated. This arises because the latent heat of vaporisation of the liquid constitutes the major part of the enthalpy of the vapour. Thus when steam is added it gives up a small amount of sensible heat to the gas, whereas when hot liquid is added a small amount of sensible heat is given up and a very much larger amount of latent heat is absorbed from the gas.

Determination of Humidity

The most important methods for determining humidity are as follows:

(1) *Chemical Methods.* A known volume of the gas is passed over a suitable absorbent, the increase in weight of which is measured. The efficiency of the process can be checked by arranging a number of vessels containing absorbent in series and ascertaining that the increase in weight in the last of these is negligible. The method is very accurate but is laborious. Satisfactory absorbents for water vapour are phosphorus pentoxide dispersed on pumice, and concentrated sulphuric acid.

(2) *Determination of the Wet Bulb Temperature.* A relation has already been obtained for the humidity of a gas in terms of its temperature, its wet bulb temperature, and various physical properties of the gas and vapour. The wet bulb temperature is normally determined as the temperature attained by the bulb of a thermometer which is covered with a piece of material which is maintained saturated with the liquid. The gas should be passed over the surface of the wet bulb at a high enough velocity (> 15 f.p.s.) (a) for the condition of the gas stream not to be affected appreciably by the evaporation of liquid, (b) for the heat transfer by convection to be large compared with that by radiation and conduction from the surroundings and (c) for the ratio of the coefficients of heat and mass transfer to have reached a constant value. The gas should be passed long enough for equilibrium to be attained and, for accurate work, the

liquid should be cooled nearly to the wet bulb temperature, before it is applied to the material.

The stream of gas over the liquid surface may be produced by a small fan or other similar means (Fig. 10.7). The crude forms of wet bulb thermometer which make no provision for the rapid passage of gas cannot be used for accurate determinations of humidity.

(3) *Determination of the Dew-point.* The dew-point is determined by cooling a highly polished surface (Fig. 10.8) in the gas and observing the highest temperature at which condensation takes place. The humidity of the gas is equal to the humidity of saturated gas at the dew-point.

(4) *Measurement of the Torsion in a Hair or Fibre.* The torsional force exerted by a hair or fibre may be influenced by the humidity of the surrounding atmosphere (Fig. 10.9). Many forms of apparatus for automatic recording of humidity depend on this property. The method has the disadvantage that the apparatus needs frequent calibration because the zero tends to shift. This difficulty is most serious when the instrument is used over a wide range of humidities.

Some of the more important methods of determining humidity have been outlined above but reference should be made to standard works on psychrometry for details of the apparatus and experimental techniques.

Methods of Increasing Humidity

The following methods may be used for increasing the humidity of a gas:

(1) Live steam may be added directly in the required quantity. It has been shown that this produces only a slight increase in the temperature but the method is not generally favoured because any impurities that are present in the steam are added at the same time.

(2) Water may be sprayed into the gas at such a rate that, on complete vaporisation, it gives the required humidity. In this case, the temperature of the gas will fall as the latent heat of vaporisation must be supplied from the sensible heat of the gas and liquid.

(3) The gas may be mixed with a stream of gas of higher humidity. This method is frequently used in laboratory work when the humidity of a gas supplied to an apparatus is controlled by varying the proportions in which two gas streams are mixed.

(4) The gas may be brought into contact with water in such a way that only part of the liquid is evaporated. This is perhaps the most common method and will now be considered in more detail.

In order to obtain a high rate of humidification, the area of contact between the air and the water is made as large as possible by supplying the water in the form of a fine spray; alternatively the interfacial area is increased by using a packed column. Evaporation occurs if the humidity at the surface is greater

than that in the bulk of the air; that is, if the temperature of the water is above the dew-point of the air.

When humidification is carried out in a packed column, the water which is not evaporated can be recirculated so as to reduce the requirements of fresh water. As a result of continued recirculation, the temperature of the water will approach the adiabatic saturation temperature of the air, and the air leaving the column will be cooled—in some cases to within 1°C of the temperature of the water. If the temperature of the air is to be maintained constant or raised, the water must be heated.

Two methods of changing the humidity and temperature of a gas from A (θ_1, \mathcal{H}_1) to B (θ_2, \mathcal{H}_2), will be traced on the humidity chart (Fig. 10.10). The first method consists of saturating the air by water artificially maintained at the dew-point of air of humidity \mathcal{H}_2 (line AC) and then heating at constant humidity

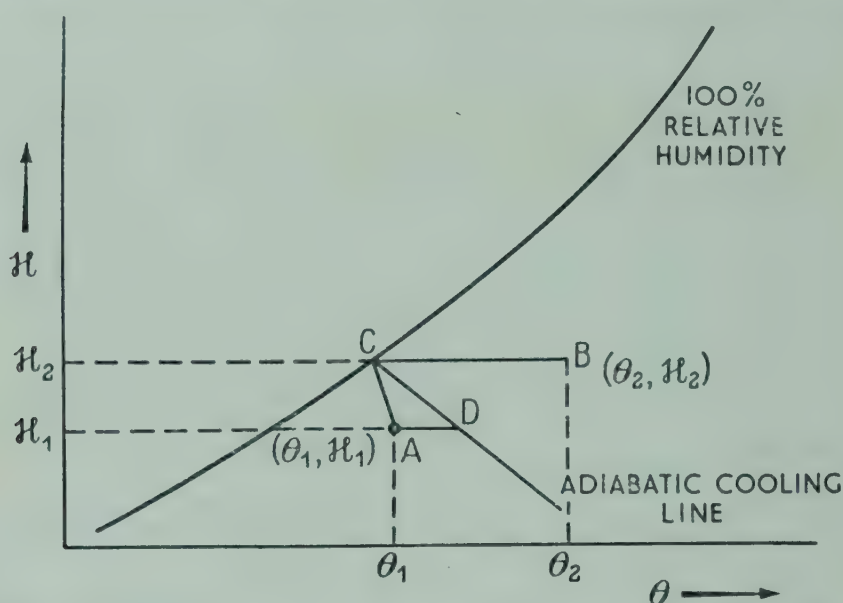


Fig. 10.10. Two methods of changing condition of gas from $(\theta_1, \mathcal{H}_1)$ to $(\theta_2, \mathcal{H}_2)$

to θ_2 (line CB). In the second method, the air is heated (line AD) so that its adiabatic saturation temperature corresponds with the dew-point of air of humidity \mathcal{H}_2 . It is then saturated by water at the adiabatic saturation temperature (line DC) and heated at constant humidity to θ_2 (line CB). In this second method, an additional operation—the preliminary heating—is carried out on the air but the water temperature automatically adjusts itself to the required value.

Since complete humidification is not always obtained, an allowance must be made when designing air humidification cycles. For example, if only 95% saturation is obtained the adiabatic cooling line should be followed only to the point corresponding to that degree of saturation, and therefore the gas must be heated to a slightly higher temperature before adiabatic cooling is commenced.

Dehumidification

Dehumidification of air can be effected by bringing it into contact with a cold surface, either liquid or solid. If the temperature of the surface is lower than the dew point of the gas, condensation takes place and the temperature of the gas falls. The temperature of the surface tends to rise because of the transfer of latent

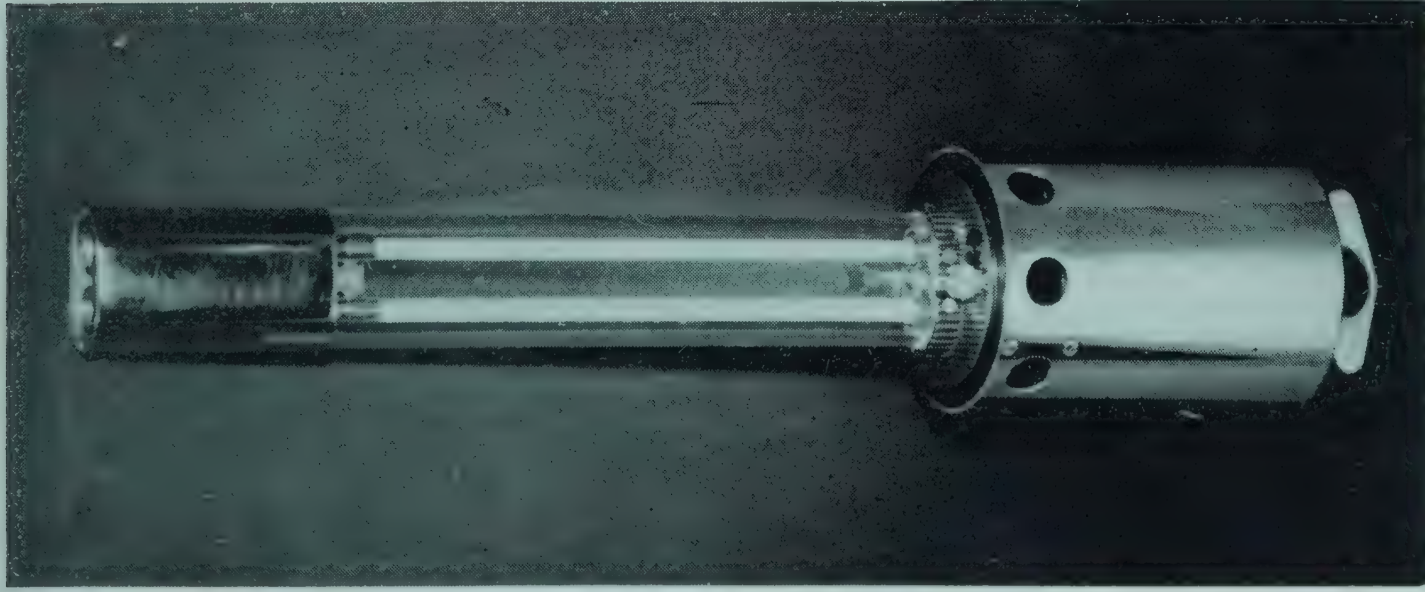


Fig. 10.7. Wet bulb thermometer



Fig. 10.8. Dew point apparatus
(Courtesy of C. F. Casella & Co. Ltd.)

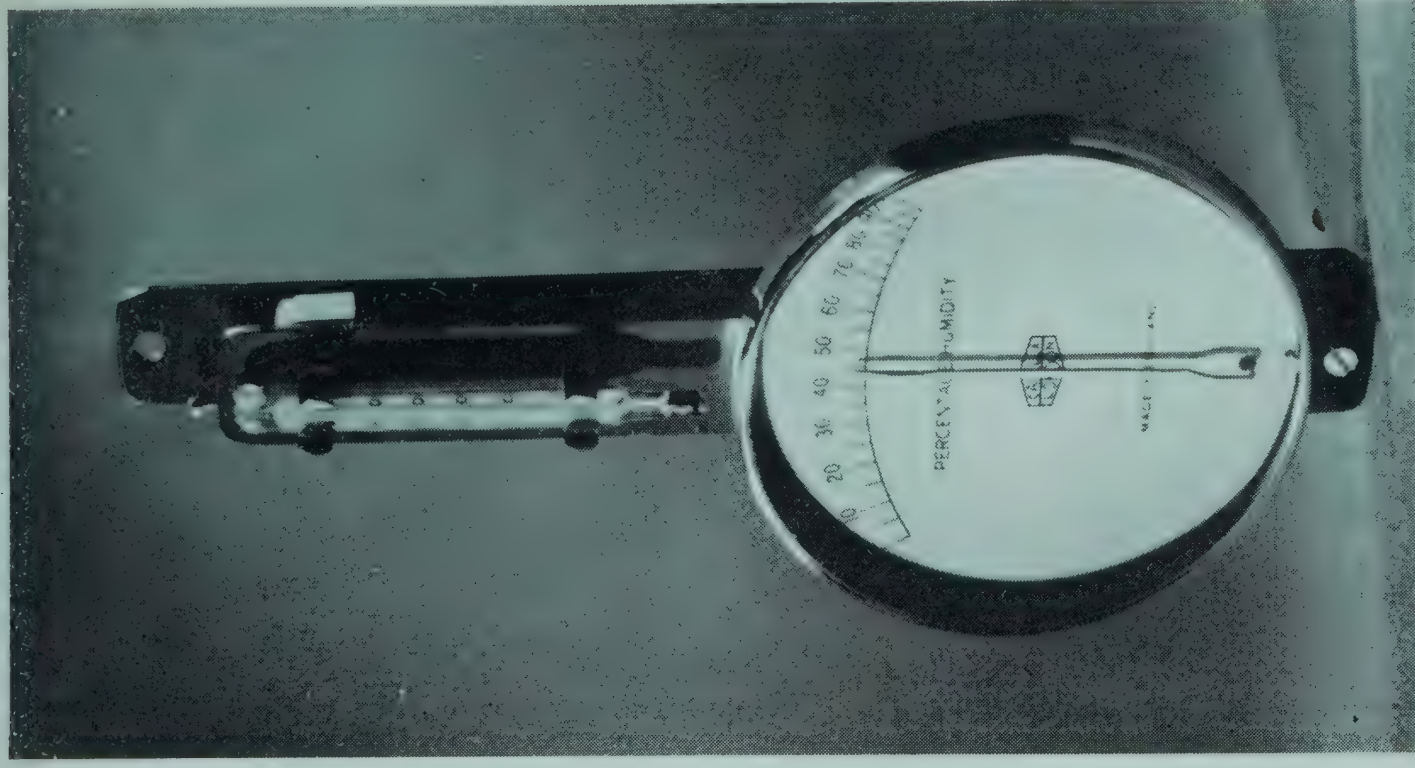
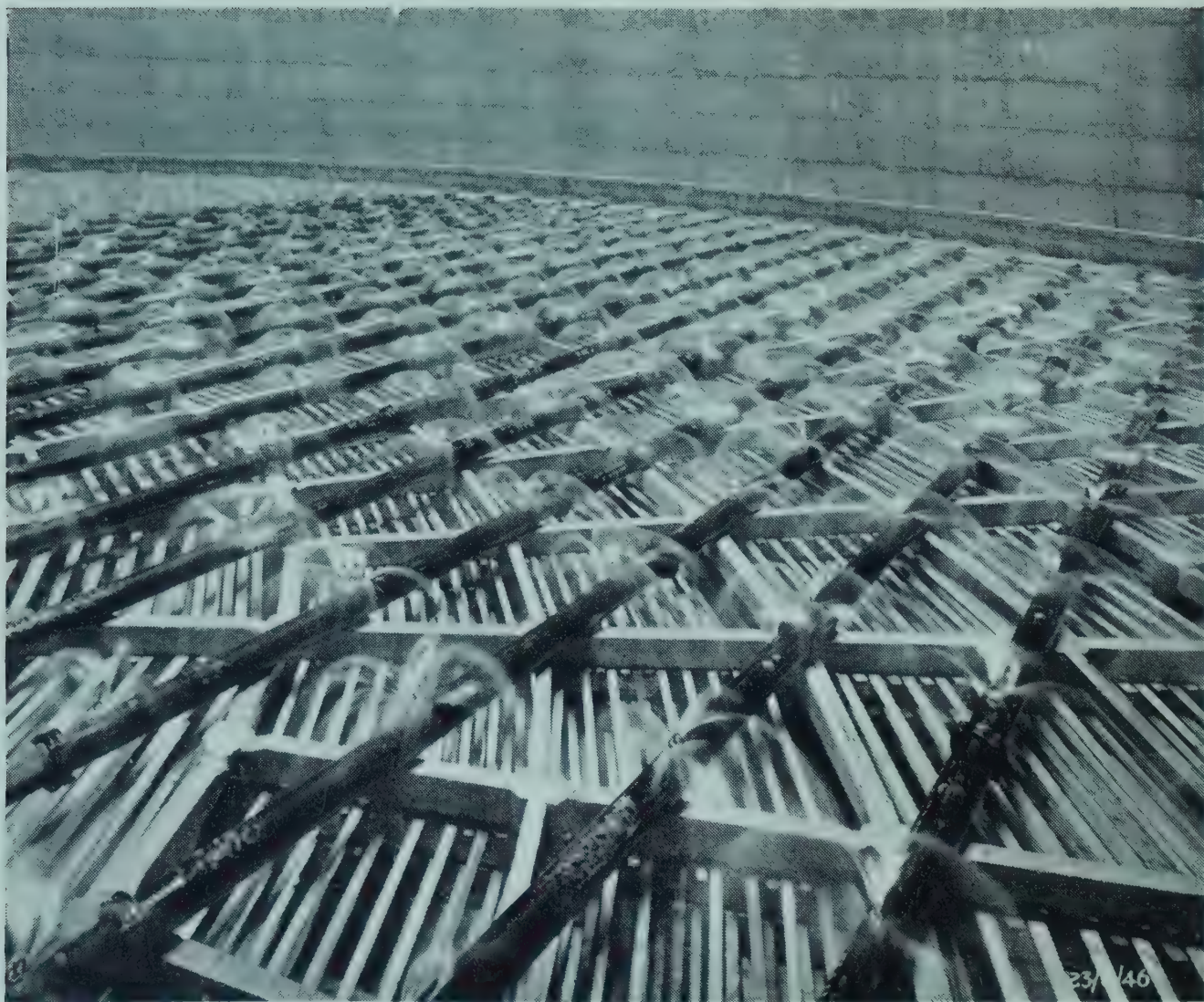


Fig. 10.9. Hair hygrometer

(a)



(b)

Fig. 10.11. Water cooling tower

(a) Outside view. (b) View of spray distributors.

(Reproduced by permission from *Cooling Towers*, by J. JACKSON;
Butterworths, 1952—in association with I.C.I. Ltd.)

and sensible heat to it. It would be expected that the air would cool at constant humidity, until the dew-point was reached, and that subsequent cooling would be accompanied by condensation. It is found, in practice, that this occurs only when the air is well mixed. Normally the temperature and humidity are reduced simultaneously throughout the whole of the process. The air in contact with the surface is cooled below its dew-point, and condensation of vapour therefore occurs before the more distant air has time to cool. Where the gas stream is cooled by cold water, counter-current flow should be employed because the temperatures of the water and air are changing in opposite directions.

The humidity can be reduced by compressing air, allowing it to cool again to its original temperature, and draining off the water which has condensed. During compression, the partial pressure of the vapour is increased and condensation takes place as soon as it reaches the saturation value. Thus, if air is compressed to a high pressure, it becomes saturated with vapour but the partial pressure is a small proportion of the total pressure. Compressed air from a cylinder therefore has a low humidity. Coal gas is frequently compressed before it is circulated, so as to prevent condensation in the mains.

A further method of reducing the humidity consists in removing the water vapour either by absorption or by adsorption.

Many large air conditioning plants incorporate automatic control of the humidity and temperature of the issuing air. Temperature control is effected with the aid of a thermocouple or resistance thermometer, and humidity control by means of a thermocouple recording the difference between the wet and dry bulb temperatures.

Example. Air containing 0.005 lb of water vapour per pound of dry air is heated to 52°C in a dryer and passed to the lower shelves. It leaves these shelves at 60% relative humidity and is reheated to 52°C and passed over another set of shelves, again leaving at 60% relative humidity. This is again repeated for the third and fourth sets of shelves, after which the air leaves the dryer. On the assumption that the material on each shelf has reached the wet bulb temperature and that heat losses from the dryer can be neglected, determine:

- (a) The temperature of the material on each tray.
- (b) The amount of water removed, in lb/hr, if 10,000 ft³/min of moist air leaves the dryer.
- (c) The temperature to which the inlet air would have to be raised to carry out the drying in a single stage.

Solution. For each of the four sets of shelves, the condition of the air is changed to 60% relative humidity along an adiabatic cooling line.

Air is initially at condition	$\theta = 52, \mathcal{H} = 0.005.$
On humidifying to 60% relative humidity,	$\theta = 28, \mathcal{H} = 0.015, \text{ and } \theta_w = 23.$
At end of second pass,	$\theta = 35, \mathcal{H} = 0.022, \text{ and } \theta_w = 28.$
At end of third pass,	$\theta = 39, \mathcal{H} = 0.027, \text{ and } \theta_w = 32.$
At end of fourth pass,	$\theta = 42, \mathcal{H} = 0.032, \text{ and } \theta_w = 34.$

Thus the temperatures of the material on each of the trays are:

23°C, 28°C, 32°C, and 34°C

The total increase in humidity = $0.032 - 0.005 = 0.027$.

Air leaving the system has a temperature of 42°C and relative humidity of 60%.

From humidity chart, specific volume of dry air = 14.3 cu ft/lb.

saturated volume = 15.5 cu ft/lb.

By interpolation, humid volume of air of 60% relative humidity = 15.0 cu ft/lb.

Mass of dry air passing = $\frac{10,000}{15.0} \times 60 = 40,000$ lb/hr.

Mass of water evaporated = $40,000 \times 0.027 = \underline{\underline{1080}}$ lb/hr.

If the material is to be dried by a single passage of air, the air must be heated initially until its wet bulb temperature is 34°C .

For air of humidity 0.005, this corresponds to a dry bulb temperature θ of $\underline{\underline{107^{\circ}\text{C}}}$.

WATER COOLING

On a small scale, water is cooled by allowing it to stand in open ponds, or by spraying and collecting in ponds; the latter method is known as the spray pond technique. Water cooling is carried out on a very large scale by allowing the water to come into intimate contact with a stream of air. Cooling takes place by transfer of sensible heat from the water to the air, and by virtue of the latent heat required to vaporise a part of the liquid. With a cooling tower, the water is introduced into the top of the tower and flows either over a series of baffles, or over packing, commonly in the form of wooden slats (Fig. 10.11) and the interfacial area is increased. The air current is produced either by natural convection caused by the lower density of the gas within the tower, or by means of a forced draught produced by a fan or blower. In most water cooling towers, the air current is in an upward direction and counter-current flow of the air and water is obtained. Sometimes, however, the air current is produced by the wind blowing horizontally into the tower. Counter-current flow is not then obtained but fresh air is introduced throughout the length of the tower. Higher gas velocities are obtained in forced draught systems and therefore the transfer coefficients are greater.

In the cooling tower, the temperature of the liquid falls and the temperature and humidity of the air rise and its action is thus similar to that of an air humidifier. The limiting temperature to which the water can be cooled is the wet bulb temperature corresponding to the condition of the air at inlet. The enthalpy of the air stream does not remain constant since the temperature of the liquid changes rapidly in the upper portion of the tower. Towards the bottom, however, the temperature of the liquid changes less rapidly because the temperature differences are smaller. At the top of the tower, the temperature falls from the bulk of the liquid to the interface and then again from the interface to the bulk of the gas. Thus the liquid is cooled by transfer of sensible heat, and by evaporation at the surface. At the bottom of a tall tower, however, the temperature gradient in the liquid is in the same direction, though smaller but the temperature gradient in the gas is in the opposite direction.

Transfer of sensible heat to the interface therefore takes place from the bulk of the liquid and from the bulk of the gas, and all the cooling is caused by the evaporation at the interface.

The height of a water cooling tower will be determined by setting up a material balance on the water and an enthalpy balance, and rate equations for the transfer of heat in the liquid and gas phases, and for mass transfer in the gas phase. There is no concentration gradient in the liquid and therefore there is no resistance to mass transfer in the liquid phase. The height of the tower must also be sufficient to prevent the humidified gas from recycling through the system, due to convection currents outside the tower. In many cases the tower is made considerably taller than the calculated height and the water is introduced some distance from the top. The upper portion of the tower then serves merely to give an increased draught.

Calculation of Height of Water Cooling Tower⁽¹⁰⁾

Consider the counter-current flow of water and air in a tower of height z (Fig. 10.12). The air rate G is constant throughout the whole height of the tower and, if only a small proportion of the total supply of water is evaporated, the liquid rate L can be taken as constant. The temperature, enthalpy, and humidity will be denoted by the symbols θ , H , and \mathcal{H} respectively, suffixes G , L , 1, 2, and f being used to denote conditions in the gas and liquid, at the bottom and top of the column, and of the air in contact with the water.

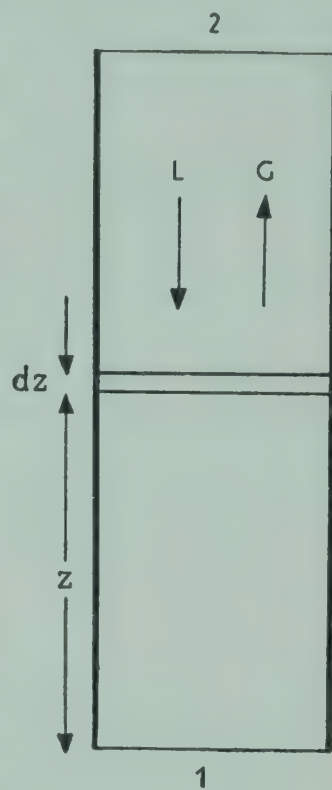


Fig. 10.12. Flow in water cooling tower

The five basic equations for a height dz of column are as follows:

(1) Water balance

$$dL = G \cdot d\mathcal{H} \quad \dots(10.25)$$

(2) Enthalpy balance

$$G \cdot dH_G = L \cdot dH_L \quad \dots(10.26)$$

since only a small proportion of the liquid is evaporated.

$$\text{Now} \quad H_G = s \cdot (\theta_G - \theta_0) + \lambda \cdot \mathcal{H} \quad \dots(10.27)$$

$$\text{and} \quad H_L = C_L \cdot (\theta_L - \theta_0) \quad \dots(10.28)$$

$$\text{Thus} \quad G \cdot dH_G = L \cdot C_L \cdot d\theta_L \quad \dots(10.29)$$

Integration of this expression over the whole length of the column, on the assumption that the physical properties of the materials do not change appreciably, gives

$$G \cdot (H_{G2} - H_{G1}) = L \cdot C_L \cdot (\theta_{L2} - \theta_{L1}) \quad \dots(10.30)$$

(3) Heat transfer from the body of the liquid to the interface

$$h_L \cdot a \cdot dz \cdot (\theta_L - \theta_f) = L \cdot C_L \cdot d\theta_L \quad \dots(10.31)$$

where h_L is the heat transfer coefficient in the liquid phase and a is the interfacial area per unit height of column.

$$\text{i.e.} \quad \frac{d\theta_L}{\theta_L - \theta_f} = \frac{h_L \cdot a}{L \cdot C_L} dz \quad \dots(10.31a)$$

(4) Heat transfer from the interface to the bulk of the gas

$$h_G \cdot a \cdot dz \cdot (\theta_f - \theta_G) = G \cdot s \cdot d\theta_G \quad \dots(10.32)$$

where h_G is the heat transfer coefficient in the gas phase.

$$\text{i.e.} \quad \frac{d\theta_G}{\theta_f - \theta_G} = \frac{h_G \cdot a}{G \cdot s} dz \quad \dots(10.32a)$$

(5) Mass transfer from the interface to the gas

$$h_D \cdot \rho \cdot a \cdot dz \cdot (\mathcal{H}_f - \mathcal{H}) = G \cdot d\mathcal{H} \quad \dots(10.33)$$

where h_D is the mass transfer coefficient for the gas and ρ is the mean density of the air (see equation 10.5).

$$\text{i.e.} \quad \frac{d\mathcal{H}}{\mathcal{H}_f - \mathcal{H}} = \frac{h_D \cdot \rho \cdot a}{G} dz \quad \dots(10.33a)$$

The above equations cannot be integrated directly since the conditions at the interface are not necessarily constant; nor can they be expressed directly in terms of the corresponding property in the bulk of the gas or liquid.

If the Lewis relation (equation 10.10) is applied, it is possible to obtain workable equations in terms of enthalpy instead of temperature and humidity. Thus writing h_G as $h_D \cdot \rho \cdot s$, from equation 10.32,

$$G \cdot s \cdot d\theta_G = h_D \cdot \rho \cdot a \cdot dz \cdot (s\theta_f - s\theta_G) \quad \dots(10.32b)$$

and from equation 10.33

$$G \cdot \lambda \cdot d\mathcal{H} = h_D \cdot \rho \cdot a \cdot dz \cdot (\lambda\mathcal{H}_f - \lambda\mathcal{H})$$

Adding these two equations gives

$$G(s d\theta_G + \lambda d\mathcal{H}) = h_D \cdot \rho \cdot a \cdot dz \cdot \{(s\theta_f + \lambda\mathcal{H}_f) - (s\theta_G + \lambda\mathcal{H})\}$$

$$\text{i.e.} \quad G \cdot dH_G = h_D \cdot \rho \cdot a \cdot dz \cdot (H_f - H_G) \quad \text{(from equation 10.27)} \quad \dots(10.34)$$

$$\text{i.e.} \quad \frac{dH_G}{H_f - H_G} = \frac{h_D \cdot \rho \cdot a}{G} \cdot dz \quad \dots(10.34a)$$

Combination of equations 10.29, 10.31, and 10.34 gives

$$\frac{H_G - H_f}{\theta_L - \theta_f} = - \frac{h_L}{h_D \cdot \rho} \quad \dots(10.35)$$

From equations 10.32b and 10.34,

$$\frac{H_G - H_f}{\theta_G - \theta_f} = \frac{dH_G}{d\theta_G} \quad \dots(10.36)$$

and from equations 10.32*b* and 10.33,

$$\frac{\mathcal{H}_f - \mathcal{H}}{\theta_f - \theta_G} = \frac{d\mathcal{H}}{d\theta_G} \quad \dots (10.37)$$

These equations will now be employed in the determination of the required height of a cooling tower for a given duty. The method consists in the graphical evaluation of the relation between the enthalpy of the body of gas and the enthalpy of the gas at the interface with the liquid. The required height of the tower is then obtained by integration of equation 10.34*a*.

Suppose water is to be cooled at a mass rate L from a temperature θ_{L2} to θ_{L1} . The air will be assumed to have a temperature θ_{G1} , a humidity \mathcal{H}_1 , and an

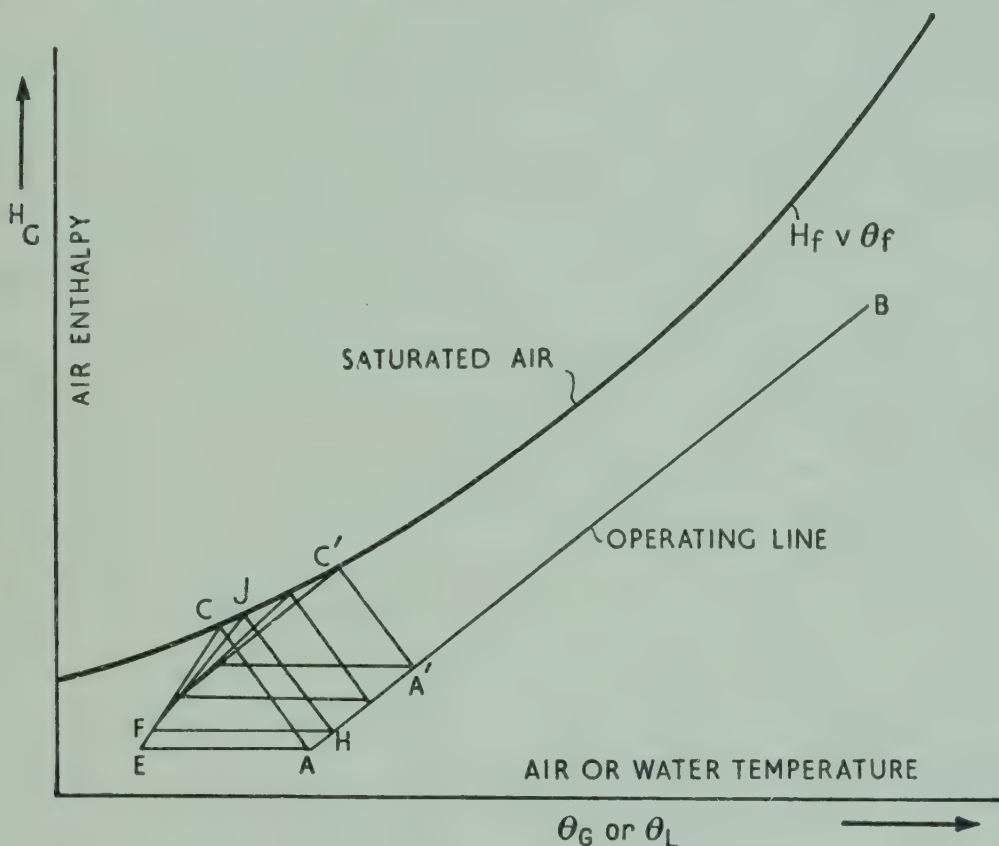


Fig. 10.13. Construction for height of water cooling tower

enthalpy H_{G1} (which can be calculated from the temperature and humidity), at the inlet point at the bottom of the tower, and its mass flow will be taken as G . The change in the condition of the liquid and gas phases will now be followed on an enthalpy-temperature diagram (Fig. 10.13). The enthalpy-temperature curve for saturated air is plotted either using calculated data or from the humidity chart (Fig. 10.3). The region below this line relates to unsaturated air and the region above it to supersaturated air. If it is assumed that the air in contact with the liquid surface is saturated with water vapour, this curve represents the relation between air enthalpy H_f and temperature θ_f at the interface.

The curve connecting air enthalpy and water temperature is now drawn using equation 10.30. This is known as the operating line and is a straight line of slope $\frac{L \cdot C_L}{G}$, passing through the points A (θ_{L1} , H_{G1}), and B (θ_{L2} , H_{G2}). Since θ_{L1} and H_{G1} are specified, the procedure is to draw a line through (θ_{L1} , H_{G1})

of slope $\frac{L \cdot C_L}{G}$, and to produce it to a point whose abscissa is equal to θ_{L2} . This point B then corresponds to conditions at the top of the tower and the ordinate gives the enthalpy of the air leaving the column.

Equation 10.35 gives the relation between liquid temperature, air enthalpy, and conditions at the interface, for any position in the tower and is represented by a family of straight lines of slope $-\frac{h_L}{h_D \cdot \rho}$. The line for the bottom of the column passes through the point A (θ_{L1} , H_{G1}) and cuts the enthalpy-temperature curve for saturated air at the point C , representing conditions at the interface. The difference in ordinates of points A and C is the difference in the enthalpy of the air at the interface, and that of the bulk air at the bottom of the column.

Similarly, line $A'C'$, parallel to AC , enables the difference in the enthalpies of the bulk air and the air at the interface to be determined at some other point in the column. The procedure can be repeated for a number of points and the value of $H_f - H_G$ obtained as a function of H_G for the whole tower.

The height of tower required to cool water from a temperature θ_{L2} to a temperature θ_{L1} is then obtained from equation 10.34a,

$$\frac{dH_G}{H_f - H_G} = \frac{h_D \cdot a \cdot \rho}{G} \cdot dz \quad \dots(10.34a)$$

On integration,

$$z = \int_1^2 dz = \frac{G}{h_D \cdot \rho \cdot a} \int_1^2 \frac{dH_G}{H_f - H_G} \quad \dots(10.38)$$

assuming h_D to remain approximately constant.

Since $H_f - H_G$ is now known as a function of H_G , $(H_f - H_G)^{-1}$ can be plotted against H_G , and the integral evaluated between the required limits. The height of the tower is thus determined.

Change in Condition of Air

The change in the humidity and temperature of the air is obtained as follows. The enthalpy and temperature of the air are known only at the bottom of the tower, where fresh air is admitted. Here the condition of the air can be represented by a point E with co-ordinates (H_{G1} , θ_{G1}). Thus the line AE is parallel to the temperature axis.

$$\text{Since} \quad \frac{H_G - H_f}{\theta_G - \theta_f} = \frac{dH_G}{d\theta_G} \quad \dots(10.36)$$

the slope of the line EC is $\frac{dH_G}{d\theta_G}$ and represents the rate of change of air enthalpy with air temperature at the bottom of the column. If the gradient $\frac{dH_G}{d\theta_G}$ is taken as constant over a small section, the point F , on EC , will represent the condition of the gas at a small distance from the bottom. The corresponding

liquid temperature is found by drawing through F a line parallel to the temperature axis. This cuts the operating line at some point H , which indicates the liquid temperature. The corresponding value of the temperature and enthalpy of the gas at the interface is then obtained by drawing a line through H , parallel to AC . This line then cuts the curve for saturated air at a point J , which represents the conditions of the gas at the interface. The rate of change of enthalpy with temperature for the gas is then given by the slope of the line FJ . Again, this slope can be considered to remain constant over a small height of the column and the condition of the gas is thus determined for the next point in the tower. The procedure is then repeated until the curve representing the condition of the gas has been extended to a point whose ordinate is equal to the enthalpy of the gas at the top of the column. This point is obtained by drawing a straight line through B , parallel to the temperature axis. The final point on the line then represents the condition of the air which leaves the top of the water cooling tower.

The size of the individual increments of height which are considered must be decided for the particular problem under consideration and will depend, primarily, on the rate of change of the gradient $\frac{dH_G}{d\theta_G}$. It should be noted that, for the gas to remain unsaturated throughout the whole of the tower, the line representing the condition of the gas must be below the curve for saturated gas. If, at any point, the two curves cross, the gas has become supersaturated and the relations which have been derived will not be applicable because of the condensation of water vapour to give liquid droplets.

Evaluation of Heat and Mass Transfer Coefficients

In general, the transfer coefficients for heat and mass in the gas phase and the heat transfer coefficient for the liquid phase will not be known. These coefficients are best determined by carrying out experiments with a laboratory or pilot plant tower using the same packing. This is done for the air-water system in the following manner, by means of a single experiment for any given air and water rate. The small column is operated at a steady liquid and gas rate and the temperature of the liquid at the top and bottom and the initial and final temperatures and humidities of the air stream are measured. The operating line for the system can then be drawn. Assuming a value of the ratio $-\frac{h_L}{h_{D\rho}}$ for the slope of the tie-lines AC , etc., the graphical construction is carried out, starting with the conditions at the bottom of the tower. The condition of the gas at the top of the tower is thus calculated and compared with the measured value. If the difference is significant, another value of $-\frac{h_L}{h_{D\rho}}$ is taken and the procedure repeated. Now that the slope of the tie line is known, the value of the integral of $\frac{dH_G}{H_f - H_G}$ over the whole column can be calculated. Since the height of the column is known, the product $h_D a$ is found by solution of equation 10.34a. $h_G a$ can then be calculated using the Lewis relation.

The values of the three transfer coefficients are therefore obtained at any given flow rates from a single experimental run. The effect of liquid and gas rate can be found by means of a series of similar experiments.

Approximate Solutions

The heat transfer coefficient for the liquid is often large compared with that for the gas phase. As a first approximation, therefore, it can be assumed that the whole of the resistance to heat transfer lies within the gas phase and that the temperature at the water-air interface is equal to the temperature of the bulk of the liquid. Thus, everywhere in the tower, $\theta_f = \theta_L$. This simplifies the calculations, because the lines AC , HJ , etc., have a slope of $-\infty$, i.e. they become parallel to the enthalpy axis.

Some workers have attempted to base the design of humidifiers on the overall heat transfer coefficient between the liquid and gas phases. This treatment is not satisfactory, since the quantities of heat transferred through the liquid and through the gas are not the same, as some of the heat is utilised in effecting evaporation at the interface. In fact, at the bottom of a tall tower, the transfer of heat in both the liquid and the gas phases is towards the interface, as already mentioned. A further objection to the use of overall coefficients is that the Lewis relation can be applied only to the heat and mass transfer coefficients in the gas phase.

Example. Water is to be cooled in a small tower from 55°C to 20°C by means of air flowing counter-currently. The liquid flows at a rate of $20\text{ gal/ft}^2\text{-hr}$ and the air at $8000\text{ ft}^3/\text{ft}^2\text{-hr}$. The entering air has a temperature of 20°C and a relative humidity of 20%. Calculate the required height of tower and the condition of the air leaving at the top.

The whole of the resistance to heat and mass transfer can be considered as being within the gas phase and the product of the mass transfer coefficient and the transfer surface per unit volume of column can be taken as 0.2 sec.^{-1} .

Solution. Liquid flow rate $= 20\text{ gal/ft}^2\text{-hr} = 200\text{ lb/ft}^2\text{-hr}$.

Gas flow rate $= 8000\text{ ft}^3/\text{ft}^2\text{-hr}$ of moist gas at inlet.

Inlet gas has 20% relative humidity at 20°C .

Thus the humidity \mathcal{H} is found from the humidity chart to be 0.003.

Latent heat of vaporisation of water at $0^\circ\text{C} = 597\text{ lb-calories/lb}$.

Specific heat of air $= 0.24$.

Specific heat of water vapour $= 0.48$.

Enthalpy of entering air, per pound of dry air, H_{G1} ,

$$\begin{aligned} &= 0.24 \times 20 + 0.003(0.48 \times 20 + 597) \\ &= 6.8\text{ lb-calories.} \end{aligned}$$

1 mole of air at inlet is associated with $0.003 \times \frac{2.9}{1.8} = 0.005$ moles water vapour.

$$\begin{aligned} \text{Flow rate of dry air} &= 8000 \times \frac{2.73}{35.9} \times \frac{2.9}{35.9} \times 0.995 \\ &= 604\text{ lb/ft}^2\text{-hr.} \end{aligned}$$

$$\text{Slope of operating line} = \frac{LC_L}{G} = 200/604 = 0.332.$$

Bottom point of operating line has co-ordinates $\theta_{L1} = 20$, $H_{G1} = 6.8$.

Top point of operating line has abscissa $\theta_{L2} = 55$.

Operating line is now drawn in.

H_{G2} is found to be 18.3.

The curve representing the enthalpy of saturated air as a function of temperature is now drawn in (Fig. 10.14). This gives the relation between enthalpy and temperature at the interface, ($H_f \text{ v } \theta_f$).

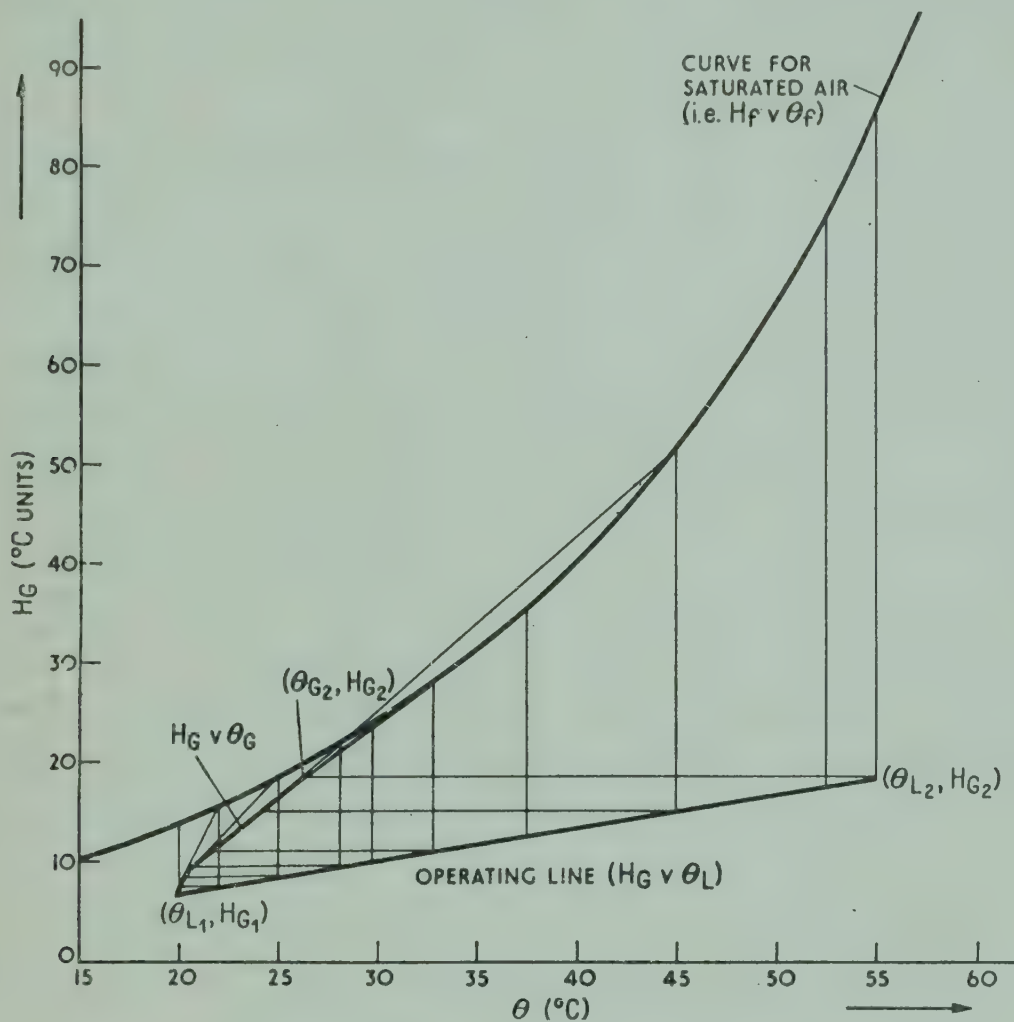


Fig. 10.14. Calculation of height of water cooling tower

Values of H_f are now read off from the graph for various values of H_G and a tabulated numerical solution is made of $\int \frac{dH_G}{H_f - H_G}$ between the limits of H_G of 6.8 and 18.3.

H_G	H_f	$H_f - H_G$	$\frac{1}{H_f - H_G}$	Mean value in interval	Interval	Value of integral over interval
6.8	13.8	7.0	0.143			
7.5	15.5	8.0	0.125	0.134	0.7	0.0938
10.0	23.5	13.5	0.074	0.100	2.5	0.2500
12.5	35.8	23.3	0.043	0.059	2.5	0.1475
15.0	51.5	36.5	0.027	0.035	2.5	0.0875
17.5	74.5	57.0	0.018	0.023	2.5	0.0575
18.3	85.0	66.7	0.015	0.017	0.8	0.0136

Value of integral = 0.6499

Height of tower = $\int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G} \times \frac{G}{h_d a \rho}$ (equation 10.38)

$= 0.65 \times \frac{604}{3600} \times \frac{293 \times 359}{273 \times 29} \times \frac{1}{0.2}$

$= 7.25 \text{ ft}$

The condition of the air leaving the tower can now be found using the graphical construction. Assuming that the whole of the resistance to transfer lies within the gas phase, the lines connecting θ_L with θ_f will be parallel to the enthalpy axis.

It is seen from the graph that, when $H_G = H_{G2} = 18.3$, the temperature of the gas, $\theta_{G2} = 27^\circ\text{C}$.

Gas at a temperature of 27°C and enthalpy 18.3°C units has a humidity \mathcal{H} of 0.019; this corresponds to a relative humidity of about 83%.

DESIGN OF A HUMIDIFYING TOWER

If the main function of the tower is to produce a stream of humidified air, the final temperature of the liquid will not be specified but the humidity of the gas leaving the top of the tower will be given instead. It is not therefore possible to fix any point on the operating line, though its slope can be calculated from the liquid and gas rates. In designing a humidifier, therefore, it is necessary to calculate the temperature and enthalpy, and hence the humidity, of the gas leaving the tower for a number of assumed water outlet temperatures and thereby determine the outlet water temperature resulting in the air leaving the tower with the required humidity. The operating line for this water outlet temperature is then used in the calculation of the height of the tower required to effect this degree of humidification. The calculation of the dimensions of a humidifier is therefore rather more tedious than that for the water cooling tower.

Temperature-humidity Relations—Water Temperature Constant

In a humidifier in which the make-up liquid is only a small proportion of the total liquid circulating, its temperature approaches the adiabatic saturation temperature θ_s , and remains constant, so that there is no temperature gradient in the liquid. The gas in contact with the liquid surface is approximately saturated and has a humidity \mathcal{H}_s .

$$\text{Thus} \quad d\theta_L = 0$$

$$\text{and} \quad \theta_{L1} = \theta_{L2} = \theta_L = \theta_f = \theta_s$$

Hence,

$$-G \cdot s \cdot d\theta_G = h_G \cdot a \cdot dz \cdot (\theta_G - \theta_s) \quad (\text{from equation 10.32})$$

$$\text{and} \quad -G \cdot d\mathcal{H} = h_D \cdot \rho \cdot a \cdot dz \cdot (\mathcal{H} - \mathcal{H}_s) \quad (\text{from equation 10.33})$$

Integration of these equations then gives,

$$\ln \frac{\theta_{G1} - \theta_s}{\theta_{G2} - \theta_s} = \frac{h_G \cdot a \cdot z}{G \cdot s} \quad \dots (10.39)$$

$$\text{and} \quad \ln \frac{\mathcal{H}_s - \mathcal{H}_1}{\mathcal{H}_s - \mathcal{H}_2} = \frac{h_D \cdot \rho \cdot a \cdot z}{G} \quad \dots (10.40)$$

assuming h_G , h_D , and s to remain approximately constant.

From these equations the temperature θ_{G2} and the humidity \mathcal{H}_2 of the gas leaving the humidifier can be calculated in terms of the height of the tower. Re-arrangement of equation 10.40 gives,

$$\ln \left\{ 1 + \frac{\mathcal{H}_1 - \mathcal{H}_2}{\mathcal{H}_s - \mathcal{H}_1} \right\} = - \frac{h_D \cdot \rho \cdot a \cdot z}{G}$$

i.e.
$$\frac{\mathcal{H}_2 - \mathcal{H}_1}{\mathcal{H}_s - \mathcal{H}_1} = 1 - e^{-h_D \rho a z / G} \quad \dots (10.41)$$

Thus the ratio of the actual increase in humidity produced in the saturator, to the maximum possible increase in humidity (i.e. the production of saturated gas) is equal to $1 - e^{-h_D \rho a z / G}$, and complete saturation of the gas is reached exponentially. A similar relation exists for the change in the temperature of the gas stream, namely,

$$\frac{\theta_{G1} - \theta_{G2}}{\theta_{G1} - \theta_s} = 1 - e^{-h_G a z / G s} \quad \dots (10.42)$$

Further, the relation between the temperature and the humidity of the gas at any stage in the adiabatic humidifier is given by,

$$\frac{d\mathcal{H}}{d\theta_G} = \frac{(\mathcal{H} - \mathcal{H}_s)}{\theta_G - \theta_s} \quad (\text{from equation 10.37})$$

On integration,

$$\ln \frac{\mathcal{H}_s - \mathcal{H}_2}{\mathcal{H}_s - \mathcal{H}_1} = \ln \frac{\theta_{G2} - \theta_s}{\theta_{G1} - \theta_s}$$

i.e.
$$\frac{\mathcal{H}_s - \mathcal{H}_2}{\mathcal{H}_s - \mathcal{H}_1} = \frac{\theta_{G2} - \theta_s}{\theta_{G1} - \theta_s} \quad \dots (10.43)$$

SYSTEMS OTHER THAN AIR-WATER

Calculations relating to systems where the Lewis relation is not applicable are very much more complicated because the adiabatic saturation temperature and the wet bulb temperature do not coincide. Thus the significance of the adiabatic cooling lines on the temperature-humidity chart is very much restricted. They no longer represent the changes which take place in a gas as it is humidified by contact with liquid initially at the adiabatic saturation temperature of the gas, but simply give the compositions of all gases with the same adiabatic saturation temperatures.

Calculation of the change in the liquid and the gas in a humidification tower is rendered more difficult since equation 10.34a, which was derived for the air-water system, is no longer applicable. LEWIS and WHITE⁽¹³⁾ have developed a method of calculation based on the use of a "modified enthalpy" in place of the true enthalpy of the system.

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LIST OF SYMBOLS USED IN CHAPTER 10

A	Interfacial area	L^2
a	Interfacial area per unit height of column	L
C_a	Specific heat of gas at constant pressure	—
C_L	Specific heat of liquid	—
C_s	Specific heat of solid	—
C_p	Specific heat of gas and vapour mixture at constant pressure	—
C_w	Specific heat of vapour at constant pressure	—
c	Mass concentration of vapour	ML^{-3}
c_0	Mass concentration of vapour in saturated gas	ML^{-3}
G	Mass rate of flow of gas	MT^{-1}
H	Enthalpy of humid gas per unit mass of dry gas	θ
H_a	Enthalpy per unit mass, of dry gas	θ
H_w	Enthalpy per unit mass, of vapour	θ
H_1	Enthalpy of stream of gas, per unit mass of dry gas	θ
H_2	Enthalpy of another stream of gas, per unit mass of dry gas	θ
H_3	Enthalpy per unit mass of liquid or vapour	θ
h	Heat transfer coefficient	$ML^{-2}T^{-1}$
h_D	Mass transfer coefficient	LT^{-1}
h_G	Heat transfer coefficient for gas phase	$ML^{-2}T^{-1}$
h_L	Heat transfer coefficient for liquid phase	$ML^{-2}T^{-1}$
\mathcal{H}	Humidity	—
\mathcal{H}_0	Humidity of saturated gas	—
\mathcal{H}_s	Humidity of gas saturated at the adiabatic saturation temperature	—
\mathcal{H}_w	Humidity of gas saturated at the wet bulb temperature	—
\mathcal{H}_1	Humidity of a gas stream	—

\mathcal{H}_2	Humidity of second gas stream	—
L	Mass rate of flow of liquid	MT^{-1}
M_A	Molecular weight of gas	—
M_w	Molecular weight of vapour	—
m, m_1, m_2	Masses of dry gas	M
m_3	Mass of liquid or vapour	M
P	Total pressure	$\text{ML}^{-1}\text{T}^{-2}$
P_A	Mean partial pressure of gas	$\text{ML}^{-1}\text{T}^{-2}$
P_w	Partial pressure of vapour	$\text{ML}^{-1}\text{T}^{-2}$
P_{w0}	Partial pressure of vapour in saturated gas	$\text{ML}^{-1}\text{T}^{-2}$
Q	Rate of transfer of heat to liquid surface	$\text{MT}^{-1}\theta$
R	Universal gas constant	$\text{L}^2\text{T}^{-2}\theta^{-1}$
s	Humid heat of gas	—
T	Absolute temperature	θ
W	Rate of evaporation	MT^{-1}
z	Height from bottom of tower	L
Z	Percentage relative humidity	—
θ	Temperature of gas stream	θ
θ_0	Reference temperature, taken as the melting point of the material	θ
θ_s	Adiabatic saturation temperature	θ
θ_w	Wet bulb temperature	θ
λ	Latent heat of vaporisation per unit mass, at datum temperature	θ
λ_f	Latent heat of freezing per unit mass, at datum temperature	θ
ρ	Mean density of gas and vapour	ML^{-3}
ρ_A	Mean density of gas at partial pressure P_A	ML^{-3}
Pr	Prandtl Number	—
Sc	Schmidt Number	—

Suffixes 1, 2, f , L , G denote conditions at the bottom of the tower, the top of the tower, the interface, the liquid, and the gas, respectively.

APPENDIX

TABLES OF PHYSICAL PROPERTIES

Table		Page
1. Temperature conversion chart	336
2. Thermal conductivities of liquids	338
3. Latent heats of vaporisation	340
4. Specific heats of liquids	342
5. Specific heats C_p of gases and vapours at 1 atmosphere pressure	344
6. Viscosity of water	344
7. Thermal conductivities of gases and vapours	346
8. Viscosities of gases	348
9. Viscosities and densities of liquids	350

STEAM TABLES

Table		
10A. Properties of saturated steam	354
10B. Total heat of dry steam (superheated), Centigrade units per unit mass		356
10C. Total heat of dry steam (superheated), Fahrenheit units per unit mass		356
10D. Entropy of dry steam (superheated), Centigrade units per unit mass		358
10E. Entropy of dry steam (superheated), Fahrenheit units per unit mass		358

TABLE 1. Temperature conversion chart

General formula: °F = (°C × $\frac{9}{5}$) + 32; °C = (°F - 32) × $\frac{5}{9}$

C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.	C.	F.
-273.1	-459.7	-17.8	0	32	10.0	50	122.0	38	100	212	260	500	932	538	1000	1832	816	1500	2732	1093	2000	3632	1371	2500	4532
-268	-450	-17.2	1	33.8	10.6	51	123.8	43	110	230	266	510	950	543	1010	1850	821	1510	2750	1099	2010	3650	1377	2510	4550
-262	-440	-16.7	2	35.6	11.1	52	125.6	49	120	248	271	520	968	549	1020	1868	827	1520	2768	1104	2020	3668	1382	2520	4568
-257	-430	-16.1	3	37.4	11.7	53	127.4	54	130	266	277	530	986	554	1030	1886	832	1530	2786	1110	2030	3686	1388	2530	4586
-251	-420	-15.6	4	39.2	12.2	54	129.2	60	140	284	282	540	1004	560	1040	1904	838	1540	2804	1116	2040	3704	1393	2540	4604
-246	-410	-15.0	5	41.0	12.8	55	131.0	66	150	302	288	550	1022	566	1050	1922	843	1550	2822	1121	2050	3722	1399	2550	4622
-240	-400	-14.4	6	42.8	13.3	56	132.8	71	160	320	293	560	1040	571	1060	1940	849	1560	2840	1127	2060	3740	1404	2560	4640
-234	-390	-13.9	7	44.6	13.9	57	134.6	77	170	338	299	570	1058	577	1070	1958	854	1570	2858	1132	2070	3758	1410	2570	4658
-229	-380	-13.3	8	46.4	14.4	58	136.4	82	180	356	304	580	1076	582	1080	1976	860	1580	2876	1138	2080	3776	1416	2580	4676
-223	-370	-12.8	9	48.2	15.0	59	138.2	88	190	374	310	590	1094	588	1090	1994	866	1590	2894	1143	2090	3794	1421	2590	4694
-218	-360	-12.2	10	50.0	15.6	60	140.0	93	200	392	316	600	1112	593	1100	2012	871	1600	2912	1149	2100	3812	1427	2600	4712
-212	-350	-11.7	11	51.8	16.1	61	141.8	99	210	410	321	610	1130	599	1110	2030	877	1610	2930	1154	2110	3830	1432	2610	4730
-207	-340	-11.1	12	53.6	16.7	62	143.6	100	212	413	327	620	1148	604	1120	2048	882	1620	2948	1160	2120	3848	1438	2620	4748
-201	-330	-10.6	13	55.4	17.2	63	145.4	104	220	428	332	630	1166	610	1130	2066	888	1630	2966	1166	2130	3866	1443	2630	4766
-196	-320	-10.0	14	57.2	17.8	64	147.2	110	230	446	338	640	1184	616	1140	2084	893	1640	2984	1171	2140	3884	1449	2640	4784
-190	-310	-9.44	15	59.0	18.3	65	149.0	116	240	464	343	650	1202	621	1150	2102	899	1650	3002	1177	2150	3902	1454	2650	4802
-184	-300	-8.89	16	60.8	18.9	66	150.8	121	250	482	349	660	1220	627	1160	2120	904	1660	3020	1182	2160	3920	1460	2660	4820
-179	-290	-8.33	17	62.6	19.4	67	152.6	127	260	500	354	670	1238	632	1170	2138	910	1670	3038	1188	2170	3938	1466	2670	4838
-173	-280	-7.78	18	64.4	20.0	68	154.4	132	270	518	360	680	1256	638	1180	2156	916	1680	3056	1193	2180	3956	1471	2680	4856
-169.5	-273.1	-7.22	19	66.2	20.6	69	156.2	138	280	536	366	690	1274	643	1190	2174	921	1690	3074	1199	2190	3974	1477	2690	4874
-168	-270	-6.67	20	68.0	21.1	70	158.0	143	290	554	371	700	1292	649	1200	2192	927	1700	3092	1204	2200	3992	1482	2700	4892
-162	-260	-6.11	21	69.8	21.7	71	159.8	149	300	572	377	710	1310	654	1210	2210	932	1710	3110	1210	2210	4010	1488	2710	4910
-157	-250	-5.56	22	71.6	22.2	72	161.6	154	310	590	382	720	1328	660	1220	2228	938	1720	3128	1216	2220	4028	1493	2720	4928
-151	-240	-5.00	23	73.4	22.8	73	163.4	160	320	608	388	730	1346	666	1230	2246	943	1730	3146	1221	2230	4046	1499	2730	4946
-146	-230	-4.44	24	75.2	23.3	74	165.2	166	330	626	393	740	1364	671	1240	2264	949	1740	3164	1227	2240	4064	1504	2740	4964
-140	-220	-3.89	25	77.0	23.9	75	167.0	171	340	644	399	750	1382	677	1250	2282	954	1750	3182	1232	2250	4082	1510	2750	4982
-134	-210	-3.33	26	78.8	24.4	76	168.8	177	350	662	404	760	1400	682	1260	2300	960	1760	3200	1238	2260	4100	1516	2760	5000
-129	-200	-2.78	27	80.6	25.0	77	170.6	182	360	680	410	770	1418	688	1270	2318	966	1770	3218	1243	2270	4118	1521	2770	5018
-123	-190	-2.22	28	82.4	25.6	78	172.4	188	370	698	416	780	1436	693	1280	2336	971	1780	3236	1249	2280	4136	1527	2780	5036
-118	-180	-1.67	29	84.2	26.1	79	174.2	193	380	716	421	790	1454	699	1290	2354	977	1790	3254	1254	2290	4154	1532	2790	5054

— 112	— 170	— 274	—	1·11	30	86·0	26·7	80	176·0	199	390	734	427	800	1472	704	1300	2372	982	1800	3272	1260	2300	4172	1538	2800	5072
— 107	— 160	— 256	—	0·56	31	87·8	27·2	81	177·8	204	400	752	432	810	1490	710	1310	2390	988	1810	3290	1266	2310	4190	1543	2810	5090
— 101	— 150	— 238	—	0	32	89·6	27·8	82	179·6	210	410	770	438	820	1508	716	1320	2408	993	1820	3308	1271	2320	4208	1549	2820	5108
— 95·6	— 140	— 220	—	0·56	33	91·4	28·3	83	181·4	216	420	788	443	830	1526	721	1330	2426	999	1830	3326	1277	2330	4226	1554	2830	5126
— 90·0	— 130	— 202	—	1·11	34	93·2	28·9	84	183·2	221	430	806	449	840	1544	727	1340	2444	1004	1840	3344	1282	2340	4244	1560	2840	5144
—	—	—	—	1·67	35	95·0	29·4	85	185·0	227	440	824	454	850	1562	732	1350	2462	1010	1850	3362	1288	2350	4262	1566	2850	5162
—	—	—	—	2·22	36	96·8	30·0	86	186·8	232	450	842	460	860	1580	738	1360	2480	1016	1860	3380	1293	2360	4280	1571	2860	5180
—	—	—	—	2·78	37	98·6	30·6	87	188·6	238	460	860	466	870	1598	743	1370	2498	1021	1870	3398	1299	2370	4298	1577	2870	5198
—	—	—	—	3·33	38	100·4	31·1	88	190·4	243	470	878	471	880	1616	749	1380	2516	1027	1880	3416	1304	2380	4316	1582	2880	5216
—	—	—	—	3·89	39	102·2	31·7	89	192·2	249	480	896	477	890	1634	754	1390	2534	1032	1890	3434	1310	2390	4334	1588	2890	5234
—	—	—	—	4·44	40	104·0	32·2	90	194·0	254	490	914	482	900	1652	760	1400	2552	1038	1900	3452	1316	2400	4352	1593	2900	5252
—	—	—	—	5·00	41	105·8	32·8	91	195·8				488	910	1670	766	1410	2570	1043	1910	3470	1321	2410	4370	1599	2910	5270
—	—	—	—	5·56	42	107·6	33·3	92	197·6				493	920	1688	771	1420	2588	1049	1920	3488	1327	2420	4388	1604	2920	5288
—	—	—	—	6·11	43	109·4	33·9	93	199·4				499	930	1706	777	1430	2606	1054	1930	3506	1332	2430	4406	1610	2930	5306
—	—	—	—	6·67	44	111·2	34·4	94	201·2				504	940	1724	782	1440	2624	1060	1940	3524	1338	2440	4424	1616	2940	5324
—	—	—	—	7·22	45	113·0	35·0	95	203·0				510	950	1742	788	1450	2642	1066	1950	3542	1343	2450	4442	1621	2950	5342
—	—	—	—	7·78	46	114·8	35·6	96	204·8				516	960	1760	793	1460	2660	1071	1960	3560	1349	2460	4460	1627	2960	5360
—	—	—	—	8·33	47	116·6	36·1	97	206·6				521	970	1778	799	1470	2678	1077	1970	3578	1354	2470	4478	1632	2970	5378
—	—	—	—	8·89	48	118·4	36·7	98	208·4				527	980	1796	804	1480	2696	1082	1980	3596	1360	2480	4496	1638	2980	5396
—	—	—	—	9·44	49	120·2	37·2	99	210·2				532	990	1814	810	1490	2714	1088	1990	3614	1366	2490	4514	1643	2990	5414
—	—	—	—				37·8	100	212·0										1093	2000	3632				1649	3000	5432

Interpolation factors

C.	F.	C.	F.
0·56	1 1·8	3·33	6 10·8
1·11	2 3·6	3·89	7 12·6
1·67	3 5·4	4·44	8 14·4
2·22	4 7·2	5·00	9 16·2
2·78	5 9·0	5·56	10 18·0

Note.—The numbers in bold-face type refer to the temperature (in either Centigrade or Fahrenheit degrees) which it is desired to convert into the other scale. If converting from Fahrenheit degrees to Centigrade degrees the equivalent temperature is in the left column, while if converting from degrees Centigrade to degrees Fahrenheit, the equivalent temperature is in the column on the right. This table, made by Albert Sauveur, is published by permission of Mrs. Albert Sauveur.

TABLE 2. Thermal conductivities of liquids*

$k = \text{lb-calories}/(\text{hr})(\text{ft}^2)(^{\circ}\text{C per ft}) = \text{B.Th.U.}/(\text{hr})(\text{ft}^2)(^{\circ}\text{F per ft})$

A linear variation with temperature may be assumed. The extreme values given constitute also the temperature limits over which the data are recommended.

Liquid		°C	k	Liquid		°C	k
Acetic acid 100 %	.	20	0.099	Hexane (n-)	.	30	0.080
50 %	.	20	0.20	.	.	60	0.078
Acetone	.	30	0.102	Heptyl alcohol (n-)	.	30	0.094
.	.	75	0.095	.	.	75	0.091
Allyl alcohol	.	25-30	0.104	Hexyl alcohol (n-)	.	30	0.093
Ammonia	.	- 15 to + 30	0.29	.	.	75	0.090
Ammonia, aqueous	.	20	0.261	Kerosene	.	20	0.086
.	.	60	0.29	.	.	75	0.081
Amyl acetate	.	10	0.083	Mercury	.	28	4.83
Amyl alcohol (n-)	.	30	0.094	Methyl alcohol 100 %	.	20	0.124
.	.	100	0.089	80 %	.	20	0.154
Amyl alcohol (iso-)	.	30	0.088	60 %	.	20	0.190
.	.	75	0.087	40 %	.	20	0.234
Aniline	.	0-20	0.100	20 %	.	20	0.284
.	.	.	.	100 %	.	50	0.114
Benzene	.	30	0.092	Methyl Chloride	.	- 15	0.111
.	.	60	0.087	.	.	30	0.089
Bromobenzene	.	30	0.074	Nitrobenzene	.	30	0.095
.	.	100	0.070	.	.	100	0.088
Butyl acetate (n-)	.	25-30	0.085	Nitromethane	.	30	0.125
Butyl alcohol (n-)	.	30	0.097	.	.	60	0.120
.	.	75	0.095	Nonane (n-)	.	30	0.084
Butyl alcohol (iso-)	.	10	0.091	.	.	60	0.082
Calcium chloride brine 30 %	.	30	0.32	Octane (n-)	.	30	0.083
15 %	.	30	0.34	.	.	60	0.081
Carbon disulphide	.	30	0.093	Oils, Petroleum	.	0	0.08-0.09
.	.	75	0.088	Oil, Castor	.	20	0.104
Carbon tetrachloride	.	0	0.107	.	.	100	0.100
.	.	68	0.094	Oil, Olive	.	20	0.097
Chlorobenzene	.	10	0.083	.	.	100	0.095
Chloroform	.	30	0.080				
Cymene (para)	.	30	0.078				
.	.	60	0.079				

Decane (<i>n</i> -)	0-085	Paraldehyde	30	0-084
Dichlorodifluoromethane	0-083	Pentane (<i>n</i> -)	100	0-078
									0-057		30	0-078
									0-053		75	0-074
									0-048	Perchloroethylene.	50	0-092
									0-043	Petroleum ether	30	0-075
									0-038		75	0-073
Dichloroethane	0-082	Propyl alcohol (<i>n</i> -)	30	0-099
Dichloromethane	0-111		75	0-095
									0-096	Propyl alcohol (iso-)	30	0-091
											60	0-090
Ethyl acetate.	0-101			
Ethyl alcohol 100%	0-105	Sodium	100	49
80%	0-137		210	46
60%	0-176	Sodium chloride brine 25.0%.	30	0-33
40%	0-224	12.5%.	30	0-34
20%	0-211	Sulphuric acid 90%	30	0-21
100%	0-087	60%	30	0-25
Ethyl benzene	0-086	30%	30	0-30
	0-082	Sulphur dioxide	15	0-128
Ethyl bromide	0-070		30	0-111
Ethyl ether	0-080			
	0-078	Toluene	30	0-086
	0-064		75	0-084
Ethyl iodide	0-063	β -trichloroethane	50	0-077
	0-153	Trichloroethylene.	50	0-080
Ethylene glycol		Turpentine	15	0-074
		
Gasoline	0-078			
Glycerol 100%	0-164	Vaseline	15	0-106
80%	0-189			
60%	0-220	Water.	0	0-330
40%	0-259		30	0-356
20%	0-278		60	0-381
100%	0-164		80	0-398
		
Heptane (<i>n</i> -)	0-081	Xylene (ortho-)	20	0-090
	0-079	(meta-)	20	0-090

To convert to lb-calories/(ft²)(sec.)(°C per ft) multiply by 0-000278
To convert to g-calories/(cm²)(sec.)(°C per cm) multiply by 0-00414

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TABLE 3. Latent heats of vaporisation*

For water 100°C, $\theta_c - \theta = 374 - 100 = 274$,
and the latent heat of vaporisation is 540°C units.

No.	Compound	Range $\theta_c - \theta$ °C	θ_c °C
18	Acetic acid . . .	100-225	321
22	Acetone . . .	120-210	235
29	Ammonia . . .	50-200	133
13	Benzene . . .	10-400	289
16	Butane . . .	90-200	153
21	Carbon dioxide . .	10-100	31
4	Carbon disulphide .	140-275	273
2	Carbon tetrachloride .	30-250	283
7	Chloroform . . .	140-275	263
8	Dichloromethane . .	150-250	216
3	Diphenyl . . .	175-400	527
25	Ethane . . .	25-150	32
26	Ethyl alcohol . . .	20-140	243
28	Ethyl alcohol . . .	140-300	243
17	Ethyl chloride . . .	100-250	187
13	Ethyl ether . . .	10-400	194
2	Freon-11 (CCl ₃ F) . .	70-250	198
2	Freon-12 (CCl ₂ F ₂) . .	40-200	111
5	Freon-21 (CHCl ₂ F) . .	70-250	178
6	Freon-22 (CHClF ₂) . .	50-170	96
1	Freon-113 (CCl ₂ F-CClF ₂) .	90-250	214
10	Heptane . . .	20-300	267
11	Hexane . . .	50-225	235
15	Isobutane . . .	80-200	134
27	Methanol . . .	40-250	240
20	Methyl chloride . . .	70-250	143
19	Nitrous oxide . . .	25-150	36
9	Octane . . .	30-300	296
12	Pentane . . .	20-200	197
23	Propane . . .	40-200	96
24	Propyl alcohol . . .	20-200	264
14	Sulphur dioxide . . .	90-160	157
30	Water . . .	100-500	374

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Latent heats of vaporisation

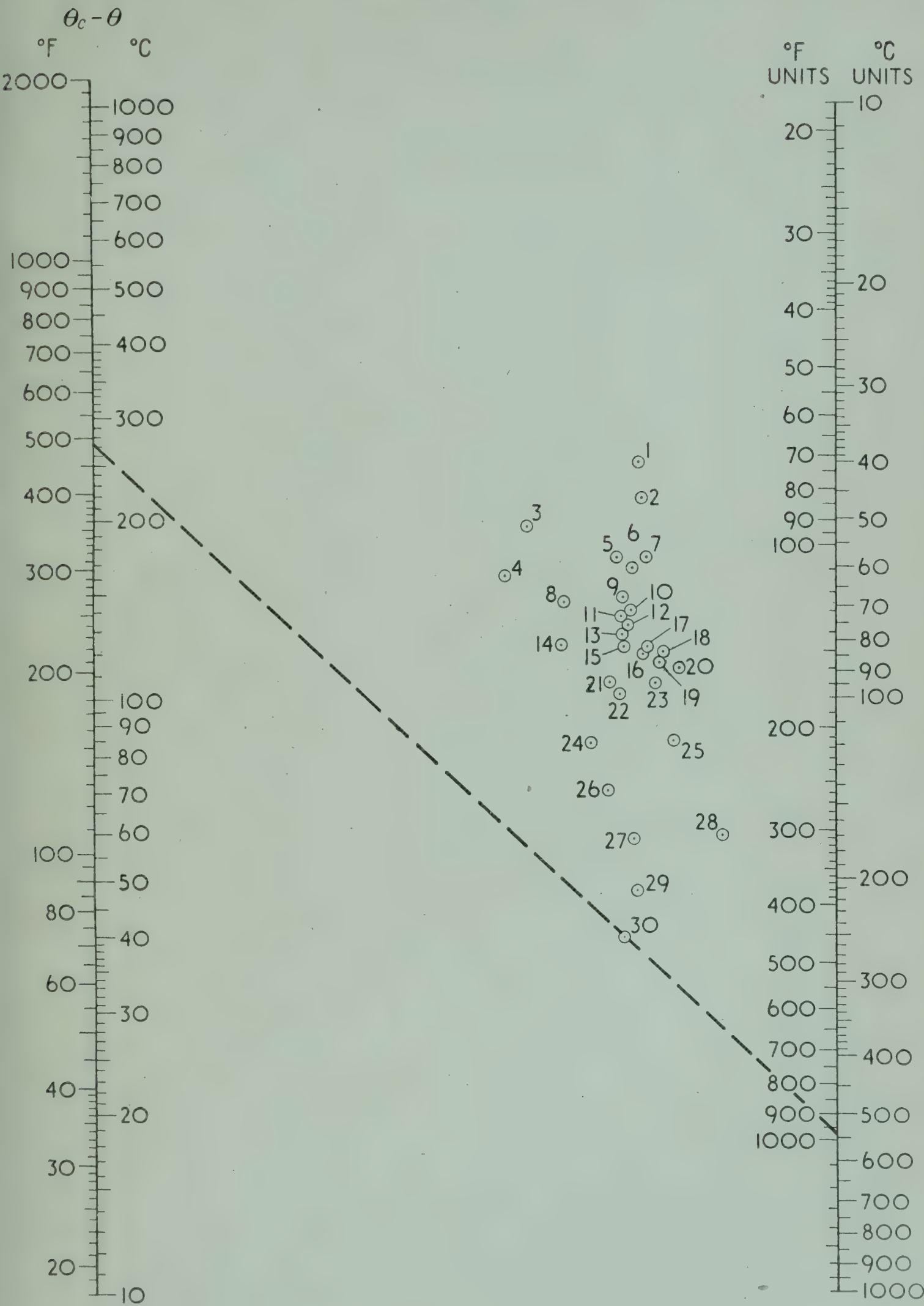


TABLE 4. Specific heats of liquids*

No.	Liquid	Range °C
29	Acetic acid, 100 % . . .	0- 80
32	Acetone	20- 50
52	Ammonia	- 70- 50
37	Amyl alcohol	- 50- 25
26	Amyl acetate	0-100
30	Aniline	0-130
23	Benzene	10- 80
27	Benzyl alcohol	- 20- 30
10	Benzyl chloride	- 30- 30
49	Brine, 25 % CaCl ₂	- 40- 20
51	Brine, 25 % NaCl	- 40- 20
44	Butyl alcohol	0-100
2	Carbon disulphide	- 100- 25
3	Carbon tetrachloride	10- 60
8	Chlorobenzene	0-100
4	Chloroform	0- 50
21	Decane	- 80- 25
6A	Dichloroethane	- 30- 60
5	Dichloromethane	- 40- 50
15	Diphenyl	80-120
22	Diphenylmethane	30-100
16	Diphenyl oxide	0-200
16	Dowtherm A	0-200
24	Ethyl acetate	- 50- 25
42	Ethyl alcohol, 100 %	30- 80
46	Ethyl alcohol, 95 %	20- 80
50	Ethyl alcohol, 50 %	20- 80
25	Ethyl benzene	0-100
1	Ethyl bromide	5- 25
13	Ethyl chloride	- 30- 40
36	Ethyl ether	- 100- 25
7	Ethyl iodide	0-100
39	Ethylene glycol	- 40-200
2A	Freon-11 (CCl ₃ F)	- 20- 70
6	Freon-12 (CCl ₂ F ₂)	- 40- 15
4A	Freon-21 (CHCl ₂ F)	- 20- 70
7A	Freon-22 (CHClF ₂)	- 20- 60
3A	Freon-113 (CCl ₂ F-CClF ₂)	- 20- 70
38	Glycerol	- 40- 20
28	Heptane	0- 60
35	Hexane	- 80- 20
48	Hydrochloric acid, 30 %	20-100
41	Isoamyl alcohol	10-100
43	Isobutyl alcohol	0-100
47	Isopropyl alcohol	- 20- 50
31	Isopropyl ether	- 80- 20
40	Methyl alcohol	- 40- 20
13A	Methyl chloride	- 80- 20
14	Naphthalene	90-200
12	Nitrobenzene	0-100
34	Nonane	- 50- 25
33	Octane	- 50- 25
3	Perchlorethylene	- 30-140
45	Propyl alcohol	- 20-100
20	Pyridine	- 50- 25
9	Sulphuric acid, 98 %	10- 45
11	Sulphur dioxide	- 20-100
23	Toluene	0- 60
53	Water	10-200
19	Xylene ortho	0-100
18	Xylene meta	0-100
17	Xylene para	0-100

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Specific heats of liquids

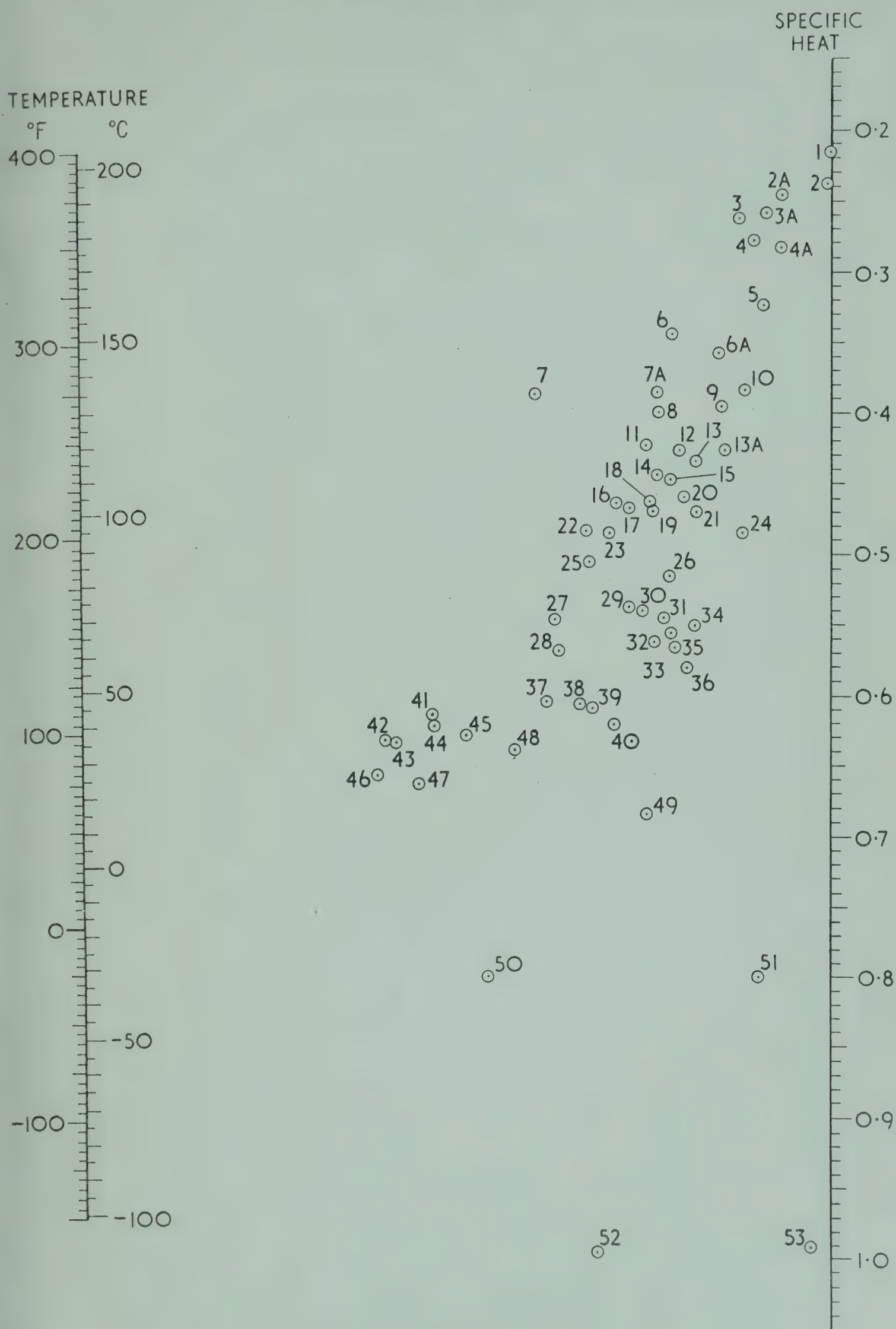


TABLE 5. Specific heats C_p of gases and vapours at 1 atmosphere pressure*

No.	Gas	Range °C	No.	Gas	Range °C
10	Acetylene . . .	0- 200	1	Hydrogen . . .	0- 600
15	Acetylene . . .	200- 400	2	Hydrogen . . .	600-1400
16	Acetylene . . .	400-1400	35	Hydrogen bromide . . .	0-1400
27	Air	0-1400	30	Hydrogen chloride . . .	0-1400
12	Ammonia	0- 600	20	Hydrogen fluoride . . .	0-1400
14	Ammonia	600-1400	36	Hydrogen iodide . . .	0-1400
18	Carbon dioxide . . .	0- 400	19	Hydrogen sulphide . . .	0- 700
24	Carbon dioxide . . .	400-1400	21	Hydrogen sulphide . . .	700-1400
26	Carbon monoxide . . .	0-1400	5	Methane	0- 300
32	Chlorine	0- 200	6	Methane	300- 700
34	Chlorine	200-1400	7	Methane	700-1400
3	Ethane	0- 200	25	Nitric oxide	0- 700
9	Ethane	200- 600	28	Nitric oxide	700-1400
8	Ethane	600-1400	26	Nitrogen	0-1400
4	Ethylene	0- 200	23	Oxygen	0- 500
11	Ethylene	200- 600	29	Oxygen	500-1400
13	Ethylene	600-1400	33	Sulphur	300-1400
17B	Freon-11 (CCl_3F) . . .	0- 150	22	Sulphur dioxide	0- 400
17C	Freon-21 (CHCl_2F) . . .	0- 150	31	Sulphur dioxide	400-1400
17A	Freon-22 (CHClF_2) . . .	0- 150	17	Water	0-1400
17D	Freon-113 ($\text{CCl}_2\text{F-CClF}_2$) .	0- 150			

* (From McADAMS, W. H.: *Heat Transmission* (McGraw-Hill, 1942).)

TABLE 6. Viscosity of water*

Temp. °C	Viscosity, centipoises	Temp. °C	Viscosity, centipoises	Temp. °C	Viscosity, centipoises
0	1.7921	33	0.7523	67	0.4233
1	1.7313	34	0.7371	68	0.4174
2	1.6728	35	0.7225	69	0.4117
3	1.6191	36	0.7085	70	0.4061
4	1.5674	37	0.6947	71	0.4006
5	1.5188	38	0.6814	72	0.3952
6	1.4728	39	0.6685	73	0.3900
7	1.4284	40	0.6560	74	0.3849
8	1.3860	41	0.6439	75	0.3799
9	1.3462	42	0.6321	76	0.3750
10	1.3077	43	0.6207	77	0.3702
11	1.2713	44	0.6097	78	0.3655
12	1.2363	45	0.5988	79	0.3610
13	1.2028	46	0.5883	80	0.3565
14	1.1709	47	0.5782	81	0.3521
15	1.1404	48	0.5683	82	0.3478
16	1.1111	49	0.5588	83	0.3436
17	1.0828	50	0.5494	84	0.3395
18	1.0559	51	0.5404	85	0.3355
19	1.0299	52	0.5315	86	0.3315
20	1.0050	53	0.5229	87	0.3276
20.20	1.0000	54	0.5146	88	0.3239
21	0.9810	55	0.5064	89	0.3202
22	0.9579	56	0.4985	90	0.3165
23	0.9358	57	0.4907	91	0.3130
24	0.9142	58	0.4832	92	0.3095
25	0.8937	59	0.4759	93	0.3060
26	0.8737	60	0.4688	94	0.3027
27	0.8545	61	0.4618	95	0.2994
28	0.8360	62	0.4550	96	0.2962
29	0.8180	63	0.4483	97	0.2930
30	0.8007	64	0.4418	98	0.2899
31	0.7840	65	0.4355	99	0.2868
32	0.7679	66	0.4293	100	0.2838

* Calculated by the formula: $1/\mu = 2.1482[(\theta - 8.435) + \sqrt{8078.4 + (\theta - 8.435)^2}] - 120$
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Specific heats of gases at constant pressures

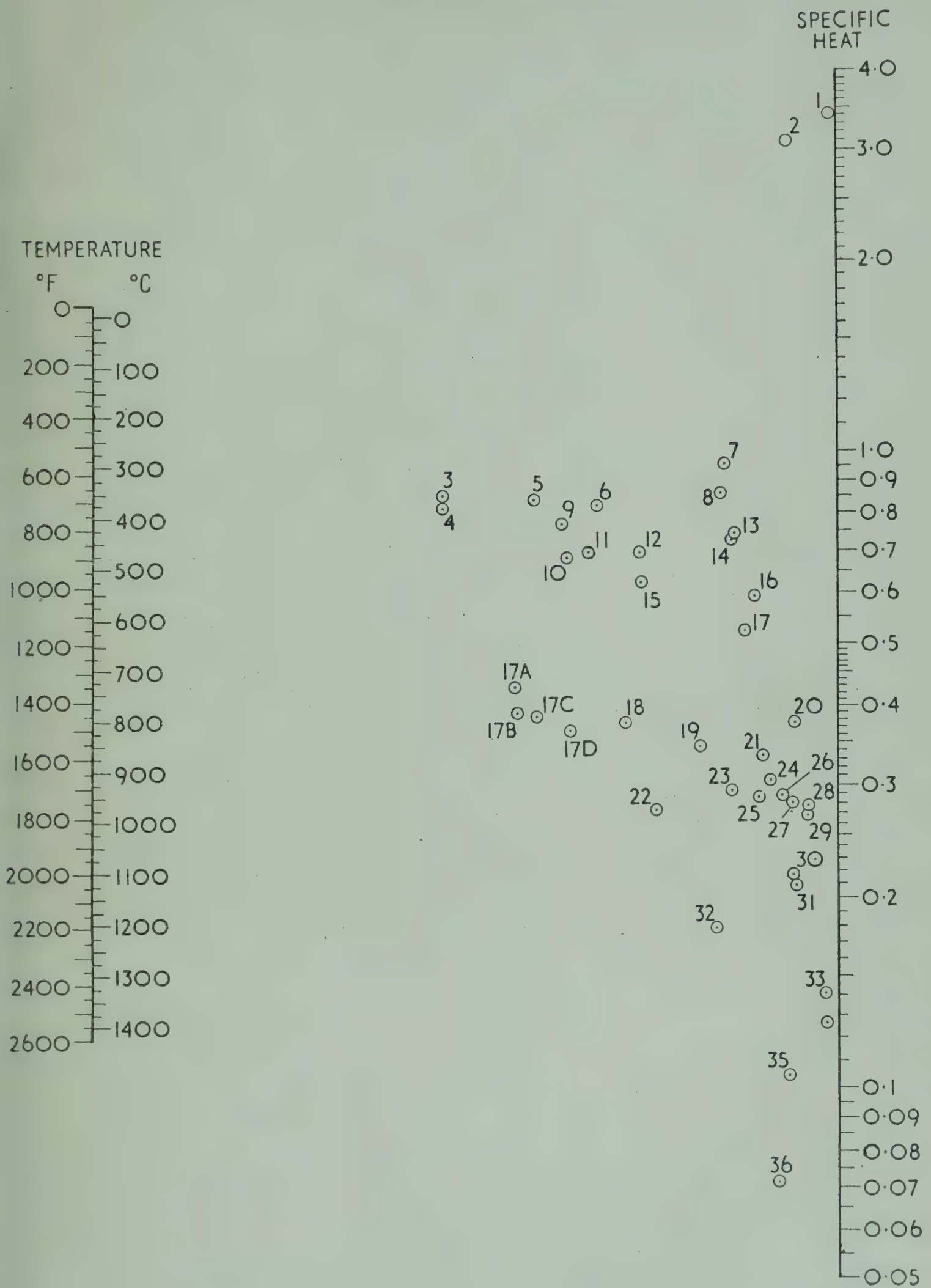


TABLE 7. Thermal conductivities of gases and vapours*

$k = \text{lb-calories}/(\text{hr})(\text{ft}^2)(^\circ\text{C per ft}) = \text{B.Th.U.}/(\text{hr})(\text{ft}^2)(^\circ\text{F per ft})$

The extreme temperature values given constitute the experimental range. For extrapolation to other temperatures, it is suggested that the data given be plotted as $\log k$ vs. $\log T$, or that use be made of the assumption that the ratio $C_p\mu/k$ is practically independent of temperature (or of pressure, within moderate limits).

Substance	°C	<i>k</i>	Substance	°C	<i>k</i>
Acetone	0	0.0057	Dichlorodifluoromethane .	0	0.0048
	46	0.0074		50	0.0064
	100	0.0099		100	0.0080
	184	0.0147		150	0.0097
Acetylene	− 75	0.0068	Ethane	− 70	0.0066
	0	0.0108		− 34	0.0086
	50	0.0140		0	0.0106
	100	0.0172		100	0.0175
Air	− 100	0.0095	Ethyl acetate	46	0.0072
	0	0.0140		100	0.0096
	100	0.0183		184	0.0141
	200	0.0226	Alcohol	20	0.0089
Ammonia	300	0.0265		100	0.0124
	− 60	0.0095		0	0.0055
	0	0.0128		100	0.0095
	50	0.0157	Chloride	184	0.0135
	100	0.0185		212	0.0152
	0	0.0052		0	0.0077
	46	0.0073		46	0.0099
Benzene	100	0.0103	Ether	100	0.0131
	184	0.0152		184	0.0189
	212	0.0176		212	0.0209
	0	0.0078	Ethylene	− 71	0.0064
Butane (<i>n</i> -)	100	0.0135		0	0.0101
	0	0.0080		50	0.0131
	100	0.0139		100	0.0161
(iso-)	0	0.0040	Heptane(<i>n</i> -)	200	0.0112
	7	0.0042		100	0.0103
	− 191	0.0041		0	0.0072
	− 181	0.0046		20	0.0080
Carbon dioxide	0	0.0135	Hexane (<i>n</i> -)	0	0.0061
	46	0.0041		100	0.0109
	100	0.0052		− 100	0.065
	184	0.0065		− 50	0.083
Disulphide	0	0.0043	Hexene	0	0.100
	0	0.0038		50	0.115
	46	0.0046		100	0.129
	100	0.0058		300	0.178
Monoxide	184	0.0077	Hydrogen	0	0.0083
	102	0.0095		20%	0.0165
	0	0.0043		40%	0.0270
	0	0.0038		60%	0.0410
Tetrachloride	46	0.0046		80%	0.0620
	100	0.0058		100%	0.10
	184	0.0077	Hydrogen and carbon dioxide		
	102	0.0095		0%	0.0083
Chlorine	0	0.0043		20%	0.0165
	0	0.0038		40%	0.0270
	46	0.0046		60%	0.0410
	100	0.0058		80%	0.0620
Chloroform	184	0.0077		100%	0.10
	102	0.0095			
	0	0.0043			
	0	0.0038			
Cyclohexane	46	0.0046			
	100	0.0058			
	184	0.0077			
	102	0.0095			

To convert to lb-calories/(ft²)(sec.)(°C per ft) multiply by 0.000278
To convert to g.-calories/(cm²)(sec.)(°C per cm) multiply by 0.00414

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TABLE 7. Thermal conductivities of gases and vapours—*contd.*

Substance	°C	<i>k</i>	Substance	°C	<i>k</i>
Hydrogen and nitrogen .	0		Nitric oxide . . .	— 70	0·0103
0 % H ₂ . . .		0·0133		0	0·0138
20 % . . .		0·0212	Nitrogen. . . .	— 100	0·0095
40 % . . .		0·0313		0	0·0140
60 % . . .		0·0438		50	0·0160
80 % . . .		0·0635		100	0·0180
Hydrogen and nitrous oxide	0		Nitrous oxide . . .	— 72	0·0067
0 % H ₂ . . .		0·0092		0	0·0087
20 % . . .		0·0170		100	0·0128
40 % . . .		0·0270			
60 % . . .		0·0410	Oxygen	— 100	0·0095
80 % . . .		0·0650		— 50	0·0119
Hydrogen sulphide . .	0	0·0076		0	0·0142
				50	0·0164
Mercury	200	0·0197		100	0·0185
Methane	— 100	0·0100			
	— 50	0·0145	Pentane (<i>n</i> -) . . .	0	0·0074
	0	0·0175		20	0·0083
	50	0·0215	(<i>iso</i> -)	0	0·0072
Methyl alcohol . . .	0	0·0083		100	0·0127
	100	0·0128	Propane	0	0·0087
Acetate	0	0·0059		100	0·0151
	20	0·0068			
Chloride	0	0·0053	Sulphur dioxide . .	0	0·0050
	46	0·0072		100	0·0069
	100	0·0094			
	184	0·0130	Water vapour . . .	46	0·0120
	212	0·0148		100	0·0137
Methylene chloride . .	0	0·0039		200	0·0187
	46	0·0049		300	0·0248
	100	0·0063		400	0·0315
	212	0·0095		500	0·0441

TABLE 8. Viscosities of gases*

Co-ordinates for use with graph on opposite page

No.	Gas	X	Y
1	Acetic acid . . .	7.7	14.3
2	Acetone . . .	8.9	13.0
3	Acetylene . . .	9.8	14.9
4	Air . . .	11.0	20.0
5	Ammonia . . .	8.4	16.0
6	Argon . . .	10.5	22.4
7	Benzene . . .	8.5	13.2
8	Bromine . . .	8.9	19.2
9	Butene . . .	9.2	13.7
10	Butylene. . .	8.9	13.0
11	Carbon dioxide . .	9.5	18.7
12	Carbon disulphide .	8.0	16.0
13	Carbon monoxide . .	11.0	20.0
14	Chlorine. . .	9.0	18.4
15	Chloroform . . .	8.9	15.7
16	Cyanogen . . .	9.2	15.2
17	Cyclohexane . . .	9.2	12.0
18	Ethane . . .	9.1	14.5
19	Ethyl acetate . . .	8.5	13.2
20	Ethyl alcohol . . .	9.2	14.2
21	Ethyl chloride . . .	8.5	15.6
22	Ethyl ether . . .	8.9	13.0
23	Ethylene. . .	9.5	15.1
24	Fluorine. . .	7.3	23.8
25	Freon-11 (CCl ₃ F) . .	10.6	15.1
26	Freon-12 (CCl ₂ F ₂) . .	11.1	16.0
27	Freon-21 (CHCl ₂ F) . .	10.8	15.3
28	Freon-22 (CHClF ₂) . .	10.1	17.0
29	Freon-113 (CCl ₂ F-CClF ₂). .	11.3	14.0
30	Helium . . .	10.9	20.5
31	Hexane . . .	8.6	11.8
32	Hydrogen . . .	11.2	12.4
33	3H ₂ + 1N ₂ . . .	11.2	17.2
34	Hydrogen bromide . .	8.8	20.9
35	Hydrogen chloride . .	8.8	18.7
36	Hydrogen cyanide . .	9.8	14.9
37	Hydrogen iodide . . .	9.0	21.3
38	Hydrogen sulphide . .	8.6	18.0
39	Iodine . . .	9.0	18.4
40	Mercury . . .	5.3	22.9
41	Methane . . .	9.9	15.5
42	Methyl alcohol . . .	8.5	15.6
43	Nitric oxide . . .	10.9	20.5
44	Nitrogen . . .	10.6	20.0
45	Nitrosyl chloride . .	8.0	17.6
46	Nitrous oxide . . .	8.8	19.0
47	Oxygen . . .	11.0	21.3
48	Pentane . . .	7.0	12.8
49	Propane . . .	9.7	12.9
50	Propyl alcohol . . .	8.4	13.4
51	Propylene . . .	9.0	13.8
52	Sulphur dioxide . . .	9.6	17.0
53	Toluene . . .	8.6	12.4
54	2, 3, 3-trimethylbutane . .	9.5	10.5
55	Water . . .	8.0	16.0
56	Xenon . . .	9.3	23.0

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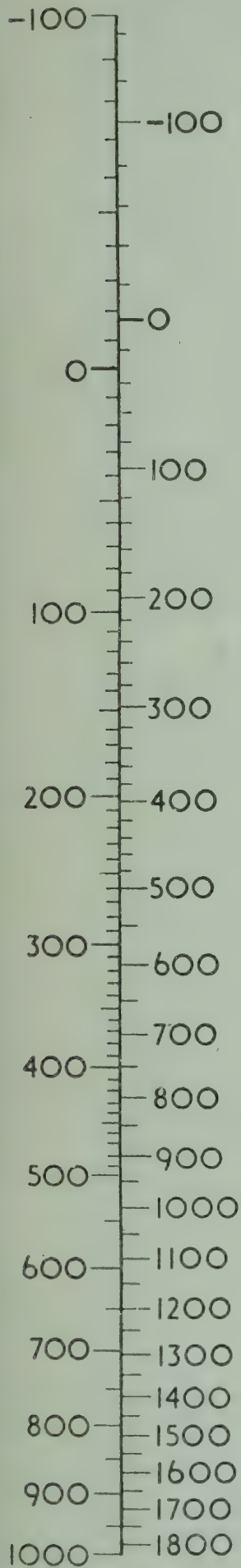
To convert to lb/ft-sec. multiply by 0.000672
To convert to lb/ft-hr multiply by 2.42

Viscosities of gases

TEMPERATURE

°C

°F



VISCOSITY

cp

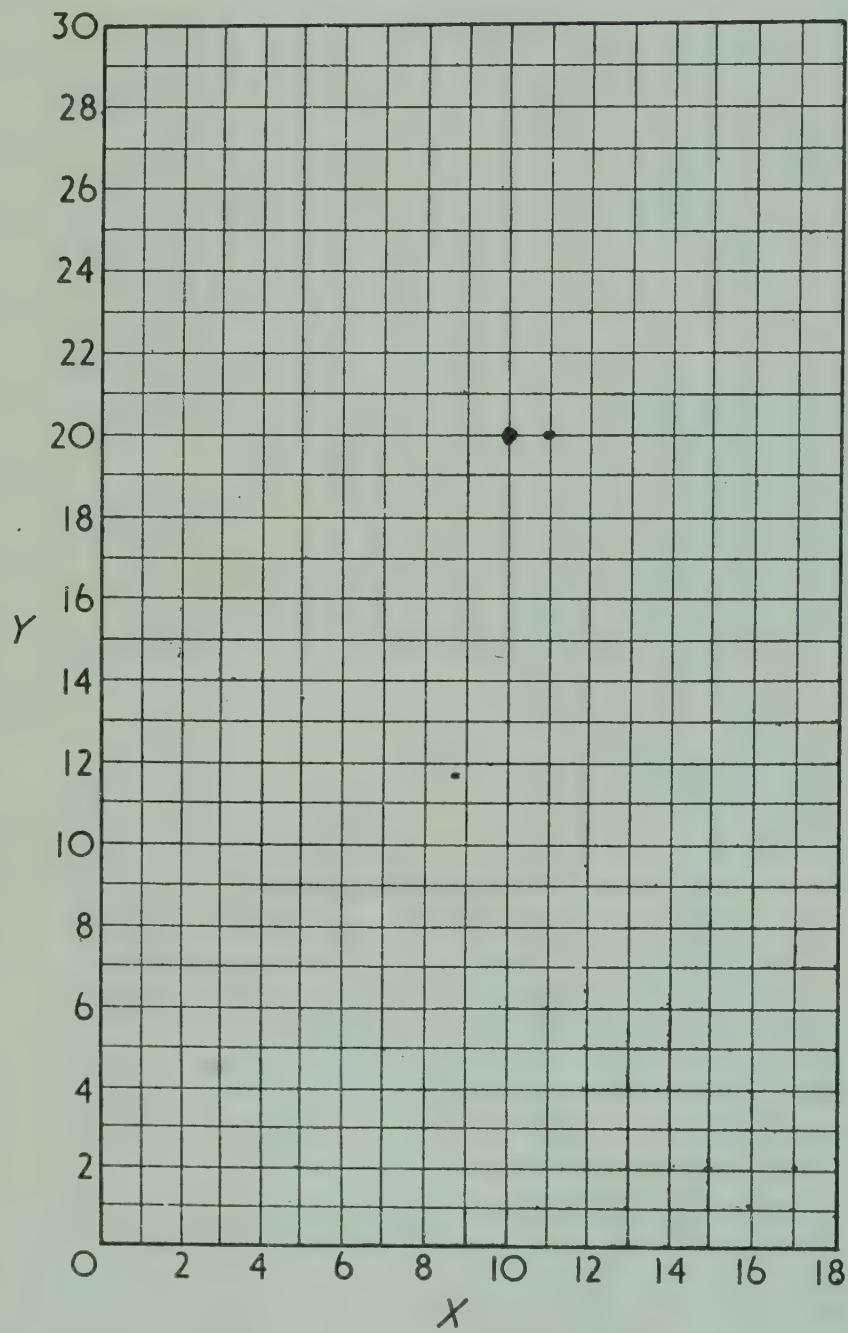
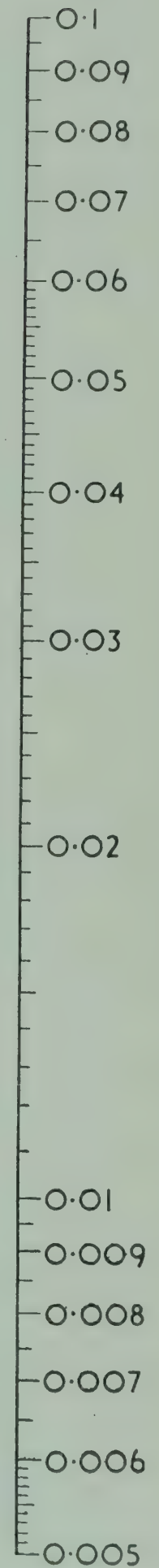


TABLE 9. Viscosities and densities of liquids*

Co-ordinates for graph on page 352

No.	Liquid	X	Y	Density at 20°C g./cc.
1	Acetaldehyde	15.2	4.8	0.783 (18°)
2	Acetic acid, 100 %	12.1	14.2	1.049
3	Acetic acid, 70 %	9.5	17.0	1.069
4	Acetic anhydride	12.7	12.8	1.832
5	Acetone, 100 %	14.5	7.2	0.792
6	Acetone, 35 %	7.9	15.0	0.948
7	Allyl alcohol	10.2	14.3	0.854
8	Ammonia, 100 %	12.6	2.0	0.817 (— 79°)
9	Ammonia, 26 %	10.1	13.9	0.904
10	Amyl acetate	11.8	12.5	0.879
11	Amyl alcohol	7.5	18.4	0.817
12	Aniline	8.1	18.7	1.022
13	Anisole	12.3	13.5	0.990
14	Arsenic trichloride	13.9	14.5	2.163
15	Benzene	12.5	10.9	0.879
16	Brine, CaCl ₂ , 25 %	6.6	15.9	1.228
17	Brine, NaCl, 25 %	10.2	16.6	1.186 (25°)
18	Bromine	14.2	13.2	3.119
19	Bromotoluene	20.0	15.9	1.41
20	Butyl acetate	12.3	11.0	0.882
21	Butyl alcohol	8.6	17.2	0.810
22	Butyric acid	12.1	15.3	0.964
23	Carbon dioxide	11.6	0.3	1.101 (— 37°)
24	Carbon disulphide	16.1	7.5	1.263
25	Carbon tetrachloride	12.7	13.1	1.595
26	Chlorobenzene	12.3	12.4	1.107
27	Chloroform	14.4	10.2	1.489
28	Chlorosulphonic acid	11.2	18.1	1.787 (25°)
29	Chlorotoluene, ortho	13.0	13.3	1.082
30	Chlorotoluene, meta	13.3	12.5	1.072
31	Chlorotoluene para	13.3	12.5	1.070
32	Cresol, meta	2.5	20.8	1.034
33	Cyclohexanol	2.9	24.3	0.962
34	Dibromoethane	12.7	15.8	2.495
35	Dichloroethane	13.2	12.2	1.256
36	Dichloromethane	14.6	8.9	1.336
37	Diethyl oxalate	11.0	16.4	1.079
38	Dimethyl oxalate	12.3	15.8	1.148 (54°)
39	Diphenyl	12.0	18.3	0.992 (73°)
40	Dipropyl oxalate	10.3	17.7	1.038 (0°)
41	Ethyl acetate	13.7	9.1	0.901
42	Ethyl alcohol, 100 %	10.5	13.8	0.789
43	Ethyl alcohol, 95 %	9.8	14.3	0.804
44	Ethyl alcohol, 40 %	6.5	16.6	0.935
45	Ethyl benzene	13.2	11.5	0.867
46	Ethyl bromide	14.5	8.1	1.431
47	Ethyl chloride	14.8	6.0	0.917 (6°)
48	Ethyl ether	14.5	5.3	0.708 (25°)
49	Ethyl formate	14.2	8.4	0.923
50	Ethyl iodide	14.7	10.3	1.933
51	Ethylene glycol	6.0	23.6	1.113
52	Formic acid	10.7	15.8	1.220
53	Freon-11 (CCl ₃ F)	14.4	9.0	1.494 (17°)
54	Freon-12 (CCl ₂ F ₂)	16.8	5.6	1.486 (— 30°)
55	Freon-21 (CHCl ₂ F)	15.7	7.5	1.426 (0°)

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TABLE 9. Viscosities and densities of liquids—*contd.*

No.	Liquid	X	Y	Density at 20°C g./cc.
56	Freon-22 (CHClF ₂)	17.2	4.7	3.87 (0°)
57	Freon-113 (CCl ₂ F-CClF ₂)	12.5	11.4	1.576
58	Glycerol, 100%	2.0	30.0	1.261
59	Glycerol, 50%	6.9	19.6	1.126
60	Heptane	14.1	8.4	0.684
61	Hexane	14.7	7.0	0.659
62	Hydrochloric acid, 31.5%	13.0	16.6	1.157
63	Isobutyl alcohol	7.1	18.0	0.779 (26°)
64	Isobutyric acid	12.2	14.4	0.949
65	Isopropyl alcohol	8.2	16.0	0.789
66	Kerosene	10.2	16.9	0.78–0.82
67	Linseed oil, raw	7.5	27.2	0.930–0.938 (15°)
68	Mercury	18.4	16.4	13.546
69	Methanol, 100%	12.4	10.5	0.792
70	Methanol, 90%	12.3	11.8	0.820
71	Methanol, 40%	7.8	15.5	0.935
72	Methyl acetate	14.2	8.2	0.924
73	Methyl chloride	15.0	3.8	0.952 (0°)
74	Methyl ethyl ketone	13.9	8.6	0.805
75	Naphthalene	7.9	18.1	1.145
76	Nitric acid, 95%	12.8	13.8	1.493
77	Nitric acid, 60%	10.8	17.0	1.367
78	Nitrobenzene	10.6	16.2	1.205 (18°)
79	Nitrotoluene	11.0	17.0	1.16
80	Octane	13.7	10.0	0.703
81	Octyl alcohol	6.6	21.1	0.827
82	Pentachloroethane	10.9	17.3	1.671 (25°)
83	Pentane	14.9	5.2	0.630 (18°)
84	Phenol	6.9	20.8	1.071 (25°)
85	Phosphorus tribromide	13.8	16.7	2.852 (15°)
86	Phosphorus trichloride	16.2	10.9	1.574
87	Propionic acid	12.8	13.8	0.992
88	Propyl alcohol	9.1	16.5	0.804
89	Propyl bromide	14.5	9.6	1.353
90	Propyl chloride	14.4	7.5	0.890
91	Propyl iodide	14.1	11.6	1.747
92	Sodium	16.4	13.9	0.97
93	Sodium hydroxide, 50%	3.2	25.8	1.525
94	Stannic chloride	13.5	12.8	2.226
95	Sulphur dioxide	15.2	7.1	1.434 (0°)
96	Sulphuric acid, 110%	7.2	27.4	1.98
97	Sulphuric acid, 98%	7.0	24.8	1.836
98	Sulphuric acid, 60%	10.2	21.3	1.498
99	Sulphuryl chloride	15.2	12.4	1.667
100	Tetrachloroethane	11.9	15.7	1.600
101	Tetrachloroethylene	14.2	12.7	1.624 (15°)
102	Titanium tetrachloride	14.4	12.3	1.726
103	Toluene	13.7	10.4	0.866
104	Trichloroethylene	14.8	10.5	1.466
105	Turpentine	11.5	14.9	0.861–0.867
106	Vinyl acetate	14.0	8.8	0.932
107	Water	10.2	13.0	0.998
108	Xylene, ortho	13.5	12.1	0.881
109	Xylene, meta	13.9	10.6	0.867
110	Xylene, para	13.9	10.9	0.861

To convert lb/ft-sec. multiply by 0.000672

To convert to lb/ft-hr multiply by 2.42

Viscosities of liquids

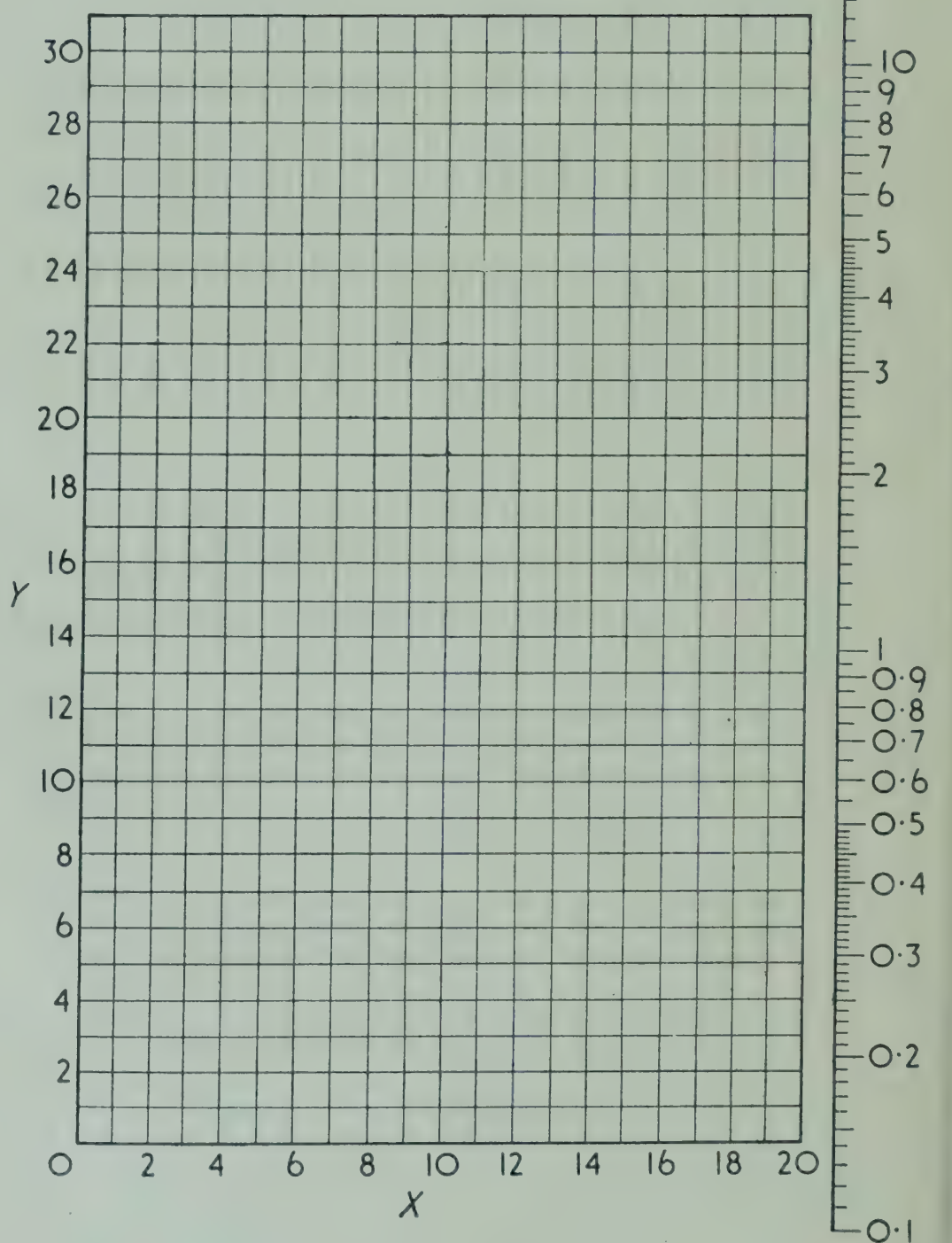
TEMPERATURE

°C °F

200	390
190	380
180	370
170	360
160	350
150	340
140	330
130	320
120	310
110	300
100	290
90	280
80	270
70	260
60	250
50	240
40	230
30	220
20	210
10	200
0	190
-10	180
-20	170
-30	160
-40	150
-50	140
-60	130
-70	120
-80	110
-90	100
-100	90
-110	80
-120	70
-130	60
-140	50
-150	40
-160	30
-170	20
-180	10
-190	0
-200	-10
-210	-20

VISCOSITY

cp



Steam Tables

Tables 10A, 10B, 10C, 10D and 10E are reproduced from the

Abridged Callendar Steam Tables

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TABLE 10A. Properties of saturated steam

Pressure		Temperature		Heat per unit mass						Entropy per unit mass (heat units)		Steam
lb/in. ²	Vacuum (in.)	°C	°F	Centrigrade units			Fahrenheit units			Water	Steam	Specific volume ft ³ /lb
				Water	Latent	Steam	Water	Latent	Steam			
0.5	28.99	26.42	79.6	26.45	582.50	608.95	47.6	1048.5	1096.1	0.0924	2.0367	643.0
0.6	28.79	29.57	85.3	29.58	580.76	610.34	53.2	1045.4	1098.6	0.1028	2.0214	540.6
0.7	28.58	32.28	90.1	32.28	579.27	611.55	58.1	1042.7	1100.8	0.1117	2.0082	466.6
0.8	28.38	34.67	94.4	34.66	577.95	612.61	62.4	1040.3	1102.7	0.1196	1.9970	411.7
0.9	28.17	36.80	98.2	36.80	576.74	613.54	66.2	1038.1	1104.3	0.1264	1.9871	368.7
1.0	27.97	38.74	101.7	38.74	575.60	614.34	69.7	1036.1	1105.8	0.1326	1.9783	334.0
1.1	27.76	40.52	104.9	40.52	574.57	615.09	72.9	1034.3	1107.2	0.1381	1.9702	305.2
1.2	27.56	42.17	107.9	42.17	573.63	615.80	75.9	1032.5	1108.4	0.1433	1.9630	281.1
1.3	27.35	43.70	110.7	43.70	572.75	616.45	78.7	1030.9	1109.6	0.1484	1.9563	260.5
1.4	27.15	45.14	113.3	45.12	571.94	617.06	81.3	1029.5	1110.8	0.1527	1.9501	243.0
1.5	26.95	46.49	115.7	46.45	571.16	617.61	83.7	1028.1	1111.8	0.1569	1.9442	228.0
1.6	26.74	47.77	118.0	47.73	570.41	618.14	86.0	1026.8	1112.8	0.1609	1.9387	214.3
1.7	26.54	48.98	120.2	48.94	569.71	618.65	88.2	1025.5	1113.7	0.1646	1.9336	202.5
1.8	26.33	50.13	122.2	50.08	569.06	619.14	90.2	1024.4	1114.6	0.1681	1.9288	191.8
1.9	26.13	51.22	124.2	51.16	568.47	619.63	92.1	1023.3	1115.4	0.1715	1.9243	182.3
2.0	25.92	52.27	126.1	52.22	567.89	620.11	94.0	1022.2	1116.2	0.1749	1.9200	173.7
3.0	23.88	60.83	141.5	60.78	562.89	623.67	109.4	1013.2	1122.6	0.2008	1.8869	118.7
4.0	21.84	67.23	153.0	67.20	559.29	626.49	121.0	1006.7	1127.7	0.2199	1.8632	90.63
5.0	19.80	72.38	162.3	72.36	556.24	628.60	130.2	1001.6	1131.8	0.2348	1.8449	73.52
6.0	17.76	76.72	170.1	76.71	553.62	630.33	138.1	996.6	1134.7	0.2473	1.8299	61.98
7.0	15.71	80.49	176.9	80.52	551.20	631.72	144.9	992.2	1137.1	0.2582	1.8176	53.64
8.0	13.67	83.84	182.9	83.89	549.16	633.05	151.0	988.5	1139.5	0.2676	1.8065	47.35
9.0	11.63	86.84	188.3	86.88	547.42	634.30	156.5	985.2	1141.7	0.2762	1.7968	42.40
10.0	9.59	89.58	193.2	89.61	545.82	635.43	161.3	982.5	1143.8	0.2836	1.7884	38.42
11.0	7.55	92.10	197.8	92.15	544.26	636.41	165.9	979.6	1145.5	0.2906	1.7807	35.14
12.0	5.50	94.44	202.0	94.50	542.75	637.25	170.1	976.9	1147.0	0.2970	1.7735	32.40
13.0	3.46	96.62	205.9	96.69	541.34	638.03	173.9	974.6	1148.5	0.3029	1.7672	30.05
14.0	1.42	98.65	209.6	98.73	540.06	638.79	177.7	972.2	1149.9	0.3086	1.7613	28.03
14.696	Gauge lb/in. ²	100.00	212.0	100.06	539.22	639.28	180.1	970.6	1150.7	0.3122	1.7574	26.80
15	0.3	100.57	213.0	100.65	538.9	639.5	181.2	970.0	1151.2	0.3137	1.7556	26.28
16	1.3	102.40	216.3	102.51	537.7	640.2	184.5	967.9	1152.4	0.3187	1.7505	24.74
17	2.3	104.13	219.5	104.27	536.5	640.8	187.6	965.9	1153.5	0.3231	1.7456	23.38
18	3.3	105.78	222.4	105.94	535.5	641.4	190.6	964.0	1154.6	0.3276	1.7411	22.17
19	4.3	107.36	225.2	107.53	534.5	642.0	193.5	962.2	1155.7	0.3319	1.7368	21.07
20	5.3	108.87	228.0	109.05	533.6	642.6	196.3	960.4	1156.7	0.3358	1.7327	20.09
21	6.3	110.32	230.6	110.53	532.6	643.1	198.9	958.8	1157.7	0.3396	1.7287	19.19
22	7.3	111.71	233.1	111.94	531.7	643.6	201.4	957.2	1158.6	0.3433	1.7250	18.38
23	8.3	113.05	235.5	113.30	530.8	644.1	203.9	955.6	1159.5	0.3468	1.7215	17.63
24	9.3	114.34	237.8	114.61	530.0	644.6	206.3	954.0	1160.3	0.3502	1.7181	16.94
25	10.3	115.59	240.1	115.87	529.2	645.1	208.6	952.5	1161.1	0.3534	1.7148	16.30
26	11.3	116.80	242.2	117.11	528.4	645.5	210.8	951.1	1161.9	0.3565	1.7118	15.72
27	12.3	117.97	244.4	118.31	527.6	645.9	212.9	949.7	1162.6	0.3595	1.7089	15.17
28	13.3	119.11	246.4	119.47	526.8	646.3	215.0	948.3	1163.3	0.3625	1.7060	14.67
29	14.3	120.21	248.4	120.58	526.1	646.7	217.0	947.0	1164.0	0.3654	1.7032	14.19
30	15.3	121.3	250.3	121.7	525.4	647.1	219.0	945.6	1164.6	0.3682	1.7004	13.73

TABLE 10A. Properties of saturated steam

Pressure		Temperature		Heat per unit mass						Entropy per unit mass (heat units)		Steam
b/in. ²	Gauge lb/in. ²	°C	°F	Centigrade units			Fahrenheit units			Water	Steam	Specific volume ft ³ /lb
				Water	Latent	Steam	Water	Latent	Steam			
32	17.3	123.3	254.0	123.8	524.1	647.9	222.7	943.1	1165.8	0.3735	1.6952	12.93
34	19.3	125.3	257.6	125.8	522.8	648.6	226.3	940.7	1167.0	0.3785	1.6905	12.21
36	21.3	127.2	260.9	127.7	521.5	649.2	229.7	938.5	1168.2	0.3833	1.6860	11.58
38	23.3	128.9	264.1	129.5	520.3	649.8	233.0	936.4	1169.4	0.3879	1.6817	11.02
40	25.3	130.7	267.2	131.2	519.2	650.4	236.1	934.4	1170.5	0.3923	1.6776	10.50
42	27.3	132.3	270.3	132.9	518.0	650.9	239.1	932.3	1171.4	0.3964	1.6737	10.30
44	29.3	133.9	273.1	134.5	516.9	651.4	242.0	930.3	1172.3	0.4003	1.6700	9.600
46	31.3	135.4	275.8	136.0	515.9	651.9	244.9	928.3	1173.2	0.4041	1.6664	9.209
48	33.3	136.9	278.5	137.5	514.8	652.3	247.6	926.4	1174.0	0.4077	1.6630	8.848
50	35.3	138.3	281.0	139.0	513.8	652.8	250.2	924.6	1174.8	0.4112	1.6597	8.516
52	37.3	139.7	283.5	140.4	512.8	653.2	252.7	922.9	1175.6	0.4146	1.6566	8.208
54	39.3	141.0	285.9	141.8	511.8	653.6	255.2	921.1	1176.3	0.4179	1.6536	7.922
56	41.3	142.3	288.3	143.1	510.9	654.0	257.6	919.4	1177.0	0.4211	1.6507	7.656
58	43.3	143.6	290.5	144.4	510.0	654.4	259.9	917.8	1177.7	0.4242	1.6478	7.407
60	45.3	144.9	292.7	145.6	509.2	654.8	262.2	916.2	1178.4	0.4272	1.6450	7.175
62	47.3	146.1	294.9	146.8	508.4	655.2	264.4	914.6	1179.0	0.4302	1.6423	6.957
64	49.3	147.3	296.9	148.0	507.6	655.6	266.5	913.1	1179.6	0.4331	1.6398	6.752
66	51.3	148.4	299.0	149.2	506.7	655.9	268.6	911.6	1180.2	0.4359	1.6374	6.560
68	53.3	149.5	301.0	150.3	505.9	656.2	270.7	910.1	1180.8	0.4386	1.6350	6.378
70	55.3	150.6	302.9	151.5	505.0	656.5	272.7	908.7	1181.4	0.4412	1.6327	6.206
72	57.3	151.6	304.8	152.6	504.2	656.8	274.6	907.4	1182.0	0.4437	1.6304	6.044
74	59.3	152.6	306.7	153.6	503.4	657.0	276.5	906.0	1182.5	0.4462	1.6282	5.890
76	61.3	153.6	308.5	154.7	502.6	657.3	278.4	904.6	1183.0	0.4486	1.6261	5.743
78	63.3	154.6	310.3	155.7	501.8	657.5	280.3	903.2	1183.5	0.4510	1.6240	5.604
80	65.3	155.6	312.0	156.7	501.1	657.8	282.1	901.9	1184.0	0.4533	1.6219	5.472
82	67.3	156.5	313.7	157.7	500.3	658.0	283.9	900.6	1184.5	0.4556	1.6199	5.346
84	69.3	157.5	315.4	158.6	499.6	658.2	285.6	899.4	1185.0	0.4579	1.6180	5.226
86	71.3	158.4	317.1	159.6	498.9	658.5	287.3	898.1	1185.4	0.4601	1.6161	5.110
88	73.3	159.4	318.7	160.5	498.3	658.8	289.0	896.8	1185.8	0.4622	1.6142	5.000
90	75.3	160.3	320.3	161.5	497.6	659.1	290.7	895.5	1186.2	0.4643	1.6124	4.896
92	77.3	161.2	321.9	162.4	496.9	659.3	292.3	894.3	1186.6	0.4664	1.6106	4.796
94	79.3	162.0	323.3	163.3	496.3	659.6	293.9	893.1	1187.0	0.4684	1.6088	4.699
96	81.3	162.8	324.8	164.1	495.7	659.8	295.5	891.9	1187.4	0.4704	1.6071	4.607
98	83.3	163.6	326.6	165.0	495.0	660.0	297.0	890.8	1187.8	0.4723	1.6054	4.519
100	85.3	164.4	327.8	165.8	494.3	660.1	298.5	889.7	1188.2	0.4742	1.6038	4.434
105	90.3	166.4	331.3	167.9	492.7	660.6	302.2	886.9	1189.1	0.4789	1.6000	4.230
110	95.3	168.2	334.8	169.8	491.2	661.0	305.7	884.2	1189.9	0.4833	1.5963	4.046
115	100.3	170.0	338.1	171.7	489.8	661.5	309.2	881.5	1190.7	0.4876	1.5927	3.880
120	105.3	171.8	341.3	173.6	488.3	661.9	312.5	878.9	1191.4	0.4918	1.5891	3.729
125	110.3	173.5	344.4	175.4	486.9	662.3	315.7	876.4	1192.1	0.4958	1.5856	3.587
130	115.3	175.2	347.3	177.1	485.6	662.7	318.8	874.0	1192.8	0.4997	1.5823	3.456
135	120.3	176.8	350.2	178.8	484.2	663.0	321.9	871.5	1193.4	0.5035	1.5792	3.335
140	125.3	178.3	353.0	180.5	482.9	663.4	324.9	869.1	1194.0	0.5071	1.5763	3.222
145	130.3	179.8	355.8	182.1	481.6	663.7	327.8	866.8	1194.6	0.5106	1.5733	3.116
150	135.3	181.3	358.4	183.7	480.3	664.0	330.6	864.5	1195.1	0.5140	1.5705	3.015

TABLE 10B. Total heat *H* of dry

P. abs. lb/in. ²	Saturation		Degrees of superheat							P. abs. lb/in. ²
	<i>θ</i>	<i>H_s</i>	10°	20°	30°	40°	50°	60°	70°	
15	100.6	639.5	645.6	649.4	654.3	659.0	663.7	668.5	673.2	15
20	108.9	642.6	647.8	652.6	657.4	662.2	667.0	671.9	676.7	20
30	121.3	647.1	652.4	657.2	662.0	666.9	671.9	676.8	681.6	30
40	130.7	650.4	655.6	660.5	665.4	670.4	675.4	680.3	685.2	40
50	138.3	652.8	658.0	663.0	668.1	673.1	678.1	683.1	688.0	50
60	144.9	654.8	660.0	665.2	670.3	675.4	680.4	685.4	690.4	60
70	150.6	656.5	661.7	667.0	672.2	677.3	682.4	687.4	692.4	70
80	155.6	657.8	663.2	668.5	673.8	679.0	684.1	689.2	694.3	80
90	160.3	659.1	664.5	669.9	675.2	680.5	685.7	690.9	696.1	90
100	164.4	660.1	665.7	671.1	676.5	681.8	687.0	692.2	697.4	100
120	171.8	661.9	667.6	673.3	678.7	684.1	689.5	694.8	700.0	120
140	178.3	663.4	669.2	675.0	680.5	686.0	691.5	696.9	702.2	140
160	184.2	664.6	670.6	676.5	682.1	687.7	693.2	698.6	703.9	160
180	189.5	665.6	671.8	677.7	683.5	689.1	694.6	700.1	705.5	180
200	194.3	666.4	672.7	678.7	684.6	690.3	695.9	701.5	707.0	200
250	204.9	668.2	674.5	680.8	687.0	692.9	698.7	704.4	710.0	250
300	214.1	668.9	675.6	682.2	688.7	694.8	700.8	706.6	712.4	300
400	229.2	669.6	677.0	684.1	691.0	697.4	703.7	709.9	715.9	400
500	241.7	669.7	677.2	685.0	692.0	698.9	705.6	712.0	718.3	500
600	252.3	669.0	677.3	685.2	692.7	700.0	707.0	713.6	720.1	600
700	261.7	667.9	677.0	685.2	693.0	700.6	707.7	714.7	721.4	700
800	270.1	666.4	676.2	684.9	693.1	700.9	708.3	715.5	722.4	800
1000	284.8	662.7	673.5	683.4	692.5	701.0	709.0	716.7	724.0	1000
2000	335.4	631.2	651.6	667.2	680.7	692.7	703.6	713.6	722.9	2000

TABLE 10C. Total heat *H* of dry

P. abs. lb/in. ²	Saturation		Degrees of superheat							P. abs. lb/in. ²
	<i>θ</i>	<i>H_s</i>	20°	40°	60°	80°	100°	120°	140°	
15	213.0	1151.2	1161.2	1170.9	1180.5	1190.0	1199.5	1208.9	1218.4	15
20	228.0	1156.7	1167.0	1176.7	1186.3	1195.8	1205.3	1214.8	1224.3	20
30	250.3	1164.6	1175.1	1184.9	1194.7	1204.4	1214.0	1223.6	1233.2	30
40	267.2	1170.5	1181.0	1190.9	1200.8	1210.7	1220.6	1230.3	1240.0	40
50	281.0	1174.8	1185.5	1195.7	1205.7	1215.6	1225.5	1235.3	1245.1	50
60	292.7	1178.4	1189.3	1199.5	1209.7	1219.8	1229.7	1239.6	1249.5	60
70	302.9	1181.4	1192.5	1202.8	1213.0	1223.2	1233.3	1243.3	1253.4	70
80	312.0	1184.0	1195.2	1205.6	1215.9	1226.2	1236.5	1246.7	1256.8	80
90	320.3	1186.2	1197.5	1208.0	1218.5	1228.9	1239.3	1249.6	1259.8	90
100	327.8	1188.2	1199.3	1210.2	1220.9	1231.4	1241.8	1252.2	1262.5	100
120	341.3	1191.4	1202.8	1214.0	1224.9	1235.7	1246.3	1256.8	1267.2	120
140	353.0	1194.0	1205.8	1217.1	1228.2	1239.1	1249.9	1260.6	1271.2	140
160	363.6	1196.1	1208.3	1219.7	1231.0	1242.1	1253.1	1263.9	1274.6	160
180	373.1	1198.0	1210.3	1222.0	1233.5	1244.7	1255.9	1266.8	1277.7	180
200	381.8	1199.5	1212.1	1224.0	1235.6	1247.0	1258.2	1269.3	1280.2	200
250	401.0	1202.1	1215.3	1227.6	1239.7	1251.7	1263.4	1274.9	1286.2	250
300	417.3	1203.8	1217.3	1230.3	1242.9	1255.1	1267.2	1278.9	1290.3	300
400	444.6	1205.5	1219.9	1234.1	1247.5	1260.5	1273.1	1285.3	1297.2	400
500	467.0	1205.4	1220.8	1235.7	1250.0	1263.6	1277.0	1289.7	1302.1	500
600	486.2	1204.2	1220.7	1236.5	1251.7	1266.0	1279.7	1292.7	1305.5	600
700	503.1	1202.2	1220.0	1236.6	1252.2	1267.0	1281.3	1295.0	1308.1	700
800	518.2	1199.6	1218.7	1236.1	1252.2	1267.4	1282.1	1296.3	1309.9	800
1000	544.6	1192.8	1214.5	1233.9	1251.6	1268.4	1283.9	1298.9	1313.2	1000
2000	635.8	1136.1	1176.1	1206.6	1232.9	1256.0	1276.7	1295.6	1313.5	2000

steam (superheated), Centigrade units per unit mass

P. abs. lb/in. ²	Degrees of superheat								P. abs. lb/in. ²
	80°	90°	100°	120°	140°	160°	180°	200°	
15	677.9	682.6	687.2	696.7	706.2	715.7	725.2	734.7	15
20	681.4	686.1	690.7	700.2	709.8	719.3	728.9	738.5	20
30	686.4	691.1	695.8	705.4	715.1	724.7	734.4	744.1	30
40	690.0	694.8	699.5	709.2	718.9	728.6	738.4	748.2	40
50	692.9	697.8	702.6	712.4	722.2	731.9	741.7	751.6	50
60	695.3	700.2	705.1	714.9	724.7	734.5	744.4	754.3	60
70	697.4	702.3	707.3	717.1	727.0	736.9	746.8	756.7	70
80	699.4	704.4	709.3	719.2	729.1	739.0	748.9	758.9	80
90	701.2	706.2	711.1	721.0	730.9	740.8	750.7	760.7	90
100	702.6	707.7	712.7	722.8	732.8	742.7	752.5	762.5	100
120	705.2	710.3	715.4	725.5	735.6	745.7	755.7	765.7	120
140	707.4	712.6	717.7	727.9	738.1	748.2	758.3	768.4	140
160	709.2	714.5	719.8	730.1	740.3	750.5	760.7	770.9	160
180	710.9	716.3	721.6	732.0	742.3	752.6	762.9	773.1	180
200	712.4	717.8	723.1	733.5	744.0	754.4	764.8	775.0	200
250	715.5	721.1	726.6	737.3	747.9	758.4	768.9	779.2	250
300	718.0	723.6	729.3	740.1	750.9	761.5	772.2	782.7	300
400	722.0	727.7	733.5	744.7	755.8	766.7	777.5	788.4	400
500	724.6	730.6	736.6	748.1	759.5	770.7	781.8	792.8	500
600	726.5	732.8	739.0	750.6	762.2	773.7	785.0	796.3	600
700	728.0	734.6	741.0	753.0	764.9	776.5	788.0	799.3	700
800	729.2	735.9	742.4	755.0	767.2	779.2	790.9	802.2	800
1000	731.1	737.9	744.6	757.4	770.0	782.2	794.1	805.8	1000
2000	731.6	739.8	747.8	762.9	777.1	790.7	803.9	816.6	2000

steam (superheated), Fahrenheit units per unit mass

P. abs. lb/in. ²	Degrees of superheat								P. abs. lb/in. ²
	160°	180°	200°	240°	280°	320°	360°	400°	
15	1227.7	1237.0	1246.2	1265.0	1284.0	1303.1	1322.3	1341.5	15
20	1233.7	1243.1	1252.5	1271.5	1290.6	1309.8	1329.1	1348.5	20
30	1242.8	1252.3	1261.8	1281.1	1300.5	1319.9	1339.3	1358.7	30
40	1249.7	1259.3	1268.9	1288.3	1307.7	1327.2	1346.7	1366.3	40
50	1254.9	1264.7	1274.4	1293.9	1313.4	1332.9	1352.6	1372.4	50
60	1259.4	1269.3	1279.1	1298.7	1318.3	1338.0	1357.8	1377.6	60
70	1263.4	1273.3	1283.1	1302.8	1322.5	1342.3	1362.1	1382.0	70
80	1266.9	1276.9	1286.8	1306.5	1326.2	1346.0	1365.8	1385.7	80
90	1270.0	1280.1	1290.1	1309.9	1329.7	1349.5	1369.3	1389.2	90
100	1272.7	1282.9	1293.0	1313.0	1332.9	1352.7	1372.6	1392.5	100
120	1277.5	1287.8	1298.0	1318.2	1338.2	1358.1	1378.1	1398.0	120
140	1281.7	1292.0	1302.3	1322.6	1342.8	1362.9	1383.0	1403.0	140
160	1285.2	1295.7	1306.0	1326.3	1346.6	1366.9	1387.2	1407.5	160
180	1288.3	1298.8	1309.2	1329.7	1350.2	1370.7	1391.1	1411.5	180
200	1291.0	1301.6	1312.1	1332.9	1353.7	1374.3	1394.9	1415.3	200
250	1297.1	1307.7	1318.5	1340.0	1361.2	1382.0	1402.6	1423.1	250
300	1301.6	1312.6	1323.4	1345.1	1366.5	1387.6	1408.6	1429.4	300
400	1308.9	1320.3	1331.6	1353.9	1375.9	1397.5	1418.7	1439.8	400
500	1314.0	1325.8	1337.5	1360.5	1383.4	1405.3	1427.0	1448.3	500
600	1317.9	1330.1	1342.1	1365.5	1388.6	1411.2	1433.5	1455.3	600
700	1320.9	1333.5	1345.9	1369.9	1393.4	1416.3	1438.8	1460.9	700
800	1323.2	1336.2	1349.0	1373.6	1397.5	1420.9	1443.7	1465.8	800
1000	1326.8	1340.0	1353.2	1378.5	1402.8	1426.9	1450.2	1473.1	1000
2000	1330.1	1346.0	1361.4	1390.4	1417.9	1444.4	1469.7	1494.0	2000

TABLE 10D. Entropy *S* of dry

P. abs. lb/in. ²	Saturation		Degrees of superheat							P. abs. lb/in. ²
	<i>θ</i>	<i>S</i>	10°	20°	30°	40°	50°	60°	70°	
15	100.6	1.7556	1.7692	1.7811	1.7930	1.8049	1.8166	1.8279	1.8388	15
20	108.9	1.7327	1.7460	1.7578	1.7698	1.7815	1.7930	1.8040	1.8149	20
30	121.3	1.7004	1.7133	1.7252	1.7370	1.7484	1.7596	1.7704	1.7809	30
40	130.7	1.6776	1.6900	1.7019	1.7135	1.7250	1.7358	1.7464	1.7568	40
50	138.3	1.6597	1.6716	1.6834	1.6950	1.7063	1.7172	1.7277	1.7379	50
60	144.9	1.6450	1.6570	1.6688	1.6803	1.6916	1.7026	1.7129	1.7233	60
70	150.6	1.6327	1.6448	1.6566	1.6681	1.6794	1.6905	1.7009	1.7112	70
80	155.6	1.6219	1.6341	1.6460	1.6575	1.6688	1.6798	1.6902	1.7005	80
90	160.3	1.6124	1.6248	1.6368	1.6484	1.6597	1.6705	1.6809	1.6912	90
100	164.4	1.6038	1.6163	1.6285	1.6401	1.6513	1.6621	1.6728	1.6830	100
120	171.8	1.5891	1.6014	1.6137	1.6253	1.6366	1.6474	1.6579	1.6680	120
140	178.3	1.5763	1.5886	1.6010	1.6127	1.6241	1.6348	1.6453	1.6554	140
160	184.2	1.5652	1.5773	1.5898	1.6015	1.6131	1.6240	1.6345	1.6447	160
180	189.5	1.5554	1.5674	1.5799	1.5917	1.6032	1.6142	1.6247	1.6349	180
200	194.3	1.5466	1.5585	1.5710	1.5827	1.5942	1.6052	1.6158	1.6261	200
250	204.9	1.5276	1.5403	1.5531	1.5651	1.5767	1.5879	1.5987	1.6091	250
300	214.1	1.5117	1.5250	1.5381	1.5503	1.5622	1.5737	1.5846	1.5951	300
400	229.2	1.4857	1.5002	1.5135	1.5261	1.5382	1.5498	1.5608	1.5714	400
500	241.7	1.4646	1.4796	1.4934	1.5066	1.5189	1.5310	1.5416	1.5528	500
600	252.3	1.4466	1.4616	1.4762	1.4899	1.5030	1.5151	1.5264	1.5377	600
700	261.7	1.4308	1.4469	1.4621	1.4763	1.4894	1.5019	1.5135	1.5248	700
800	270.1	1.4165	1.4337	1.4494	1.4641	1.4776	1.4904	1.5021	1.5135	800
1000	284.8	1.3909	1.4100	1.4274	1.4429	1.4572	1.4703	1.4828	1.4948	1000
2000	335.4	1.2857	1.3174	1.3430	1.3645	1.3833	1.4000	1.4148	1.4286	2000

TABLE 10E. Entropy *S* of dry

P. abs. lb/in. ²	Saturation		Degrees of superheat							P. abs. lb/in. ²
	<i>θ</i>	<i>S</i>	20°	40°	60°	80°	100°	120°	140°	
15	213.0	1.7556	1.7706	1.7844	1.7969	1.8101	1.8230	1.8353	1.8470	15
20	228.0	1.7327	1.7475	1.7610	1.7742	1.7867	1.7991	1.8110	1.8227	20
30	250.3	1.7004	1.7148	1.7281	1.7410	1.7534	1.7655	1.7773	1.7887	30
40	267.2	1.6776	1.6912	1.7045	1.7172	1.7295	1.7416	1.7531	1.7644	40
50	280.9	1.6597	1.6729	1.6861	1.6987	1.7109	1.7228	1.7344	1.7457	50
60	292.7	1.6450	1.6580	1.6713	1.6839	1.6961	1.7084	1.7200	1.7312	60
70	302.9	1.6327	1.6459	1.6593	1.6719	1.6840	1.6962	1.7078	1.7189	70
80	312.0	1.6219	1.6353	1.6488	1.6615	1.6736	1.6855	1.6971	1.7082	80
90	320.3	1.6124	1.6261	1.6396	1.6524	1.6645	1.6763	1.6878	1.6989	90
100	327.8	1.6038	1.6176	1.6311	1.6440	1.6561	1.6680	1.6795	1.6906	100
120	341.3	1.5891	1.6031	1.6166	1.6296	1.6417	1.6537	1.6652	1.6762	120
140	353.0	1.5762	1.5903	1.6038	1.6169	1.6290	1.6411	1.6526	1.6635	140
160	363.6	1.5652	1.5791	1.5926	1.6058	1.6179	1.6301	1.6416	1.6527	160
180	373.1	1.5554	1.5690	1.5825	1.5958	1.6080	1.6203	1.6318	1.6429	180
200	381.8	1.5466	1.5598	1.5735	1.5865	1.5990	1.6111	1.6228	1.6339	200
250	401.0	1.5276	1.5417	1.5557	1.5690	1.5818	1.5939	1.6057	1.6169	250
300	417.3	1.5117	1.5264	1.5408	1.5545	1.5675	1.5798	1.5918	1.6031	300
400	444.6	1.4857	1.5015	1.5164	1.5302	1.5434	1.5560	1.5680	1.5794	400
500	467.0	1.4646	1.4810	1.4959	1.5107	1.5240	1.5372	1.5594	1.5612	500
600	486.2	1.4466	1.4632	1.4793	1.4943	1.5084	1.5216	1.5341	1.5459	600
700	503.1	1.4308	1.4486	1.4654	1.4807	1.4950	1.5085	1.5211	1.5333	700
800	518.2	1.4165	1.4353	1.4529	1.4677	1.4832	1.4969	1.5099	1.5221	800
1000	544.6	1.3909	1.4119	1.4307	1.4477	1.4630	1.4772	1.4906	1.5031	1000
2000	635.8	1.2857	1.3206	1.3481	1.3710	1.3908	1.4082	1.4241	1.4387	2000

steam (superheated), Centigrade units per unit mass

P. abs. lb/in. ²	Degrees of superheat								P. abs. lb/in. ²
	80°	90°	100°	120°	140°	160°	180°	200°	
15	1.8493	1.8596	1.8694	1.8891	1.9079	1.9261	1.9435	1.9604	15
20	1.8252	1.8352	1.8451	1.8646	1.8832	1.9011	1.9184	1.9350	20
30	1.7912	1.8011	1.8109	1.8300	1.8482	1.8658	1.8830	1.8995	30
40	1.7670	1.7768	1.7865	1.8053	1.8233	1.8408	1.8577	1.8741	40
50	1.7480	1.7578	1.7675	1.7860	1.8040	1.8215	1.8383	1.8546	50
60	1.7334	1.7432	1.7528	1.7712	1.7890	1.8063	1.8230	1.8394	60
70	1.7213	1.7310	1.7404	1.7587	1.7764	1.7936	1.8102	1.8265	70
80	1.7106	1.7202	1.7297	1.7480	1.7655	1.7826	1.7991	1.8153	80
90	1.7013	1.7109	1.7203	1.7384	1.7559	1.7730	1.7894	1.8057	90
100	1.6930	1.7026	1.7119	1.7302	1.7477	1.7645	1.7808	1.7969	100
120	1.6780	1.6877	1.6971	1.7154	1.7328	1.7496	1.7660	1.7817	120
140	1.6654	1.6751	1.6846	1.7028	1.7201	1.7369	1.7533	1.7689	140
160	1.6548	1.6646	1.6738	1.6921	1.7094	1.7261	1.7424	1.7579	160
180	1.6450	1.6548	1.6643	1.6825	1.7000	1.7165	1.7327	1.7482	180
200	1.6362	1.6460	1.6555	1.6737	1.6913	1.7080	1.7241	1.7396	200
250	1.6192	1.6290	1.6385	1.6568	1.6742	1.6909	1.7069	1.7224	250
300	1.6052	1.6150	1.6247	1.6428	1.6603	1.6770	1.6931	1.7085	300
400	1.5815	1.5916	1.6015	1.6196	1.6370	1.6537	1.6696	1.6850	400
500	1.5633	1.5735	1.5834	1.6016	1.6191	1.6358	1.6520	1.6675	500
600	1.5484	1.5585	1.5683	1.5869	1.6043	1.6212	1.6375	1.6533	600
700	1.5356	1.5458	1.5578	1.5745	1.5922	1.6093	1.6252	1.6414	700
800	1.5245	1.5348	1.5450	1.5638	1.5816	1.5997	1.6148	1.6308	800
1000	1.5058	1.5164	1.5266	1.5458	1.5638	1.5809	1.5972	1.6129	1000
2000	1.4416	1.4536	1.4646	1.4848	1.5041	1.5220	1.5389	1.5548	2000

steam (superheated), Fahrenheit units per unit mass

P. abs. lb/in. ²	Degrees of superheat								P. abs. lb/in. ²
	160°	180°	200°	240°	280°	320°	360°	400°	
15	1.8584	1.8694	1.8802	1.9015	1.9220	1.9415	1.9604	1.9786	15
20	1.8340	1.8450	1.8556	1.8770	1.8973	1.9166	1.9352	1.9529	20
30	1.8000	1.8108	1.8213	1.8422	1.8621	1.8812	1.8996	1.9173	30
40	1.7755	1.7865	1.7969	1.8176	1.8374	1.8563	1.8745	1.8920	40
50	1.7567	1.7675	1.7778	1.7980	1.8175	1.8363	1.8546	1.8724	50
60	1.7422	1.7529	1.7628	1.7829	1.8023	1.8210	1.8392	1.8569	60
70	1.7299	1.7406	1.7503	1.7702	1.7895	1.8080	1.8261	1.8437	70
80	1.7192	1.7298	1.7397	1.7595	1.7787	1.7970	1.8150	1.8325	80
90	1.7099	1.7204	1.7302	1.7500	1.7691	1.7874	1.8054	1.8228	90
100	1.7015	1.7120	1.7220	1.7418	1.7607	1.7789	1.7969	1.8142	100
120	1.6870	1.6974	1.7073	1.7270	1.7460	1.7639	1.7818	1.7990	120
140	1.6744	1.6848	1.6947	1.7144	1.7333	1.7512	1.7690	1.7861	140
160	1.6636	1.6740	1.6840	1.7036	1.7224	1.7402	1.7579	1.7749	160
180	1.6539	1.6643	1.6743	1.6939	1.7128	1.7305	1.7482	1.7652	180
200	1.6449	1.6554	1.6655	1.6854	1.7042	1.7220	1.7395	1.7563	200
250	1.6280	1.6384	1.6485	1.6684	1.6872	1.7050	1.7225	1.7390	250
300	1.6142	1.6246	1.6349	1.6547	1.6734	1.6914	1.7086	1.7250	300
400	1.5905	1.6011	1.6114	1.6313	1.6501	1.6679	1.6851	1.7016	400
500	1.5724	1.5831	1.5935	1.6133	1.6320	1.6502	1.6675	1.6842	500
600	1.5574	1.5682	1.5785	1.5986	1.6176	1.6358	1.6532	1.6698	600
700	1.5448	1.5558	1.5664	1.5864	1.6056	1.6236	1.6410	1.6578	700
800	1.5338	1.5449	1.5557	1.5759	1.5951	1.6133	1.6306	1.6472	800
1000	1.5151	1.5264	1.5374	1.5579	1.5771	1.5954	1.6129	1.6296	1000
2000	1.4522	1.4648	1.4768	1.4988	1.5191	1.5380	1.5557	1.5720	2000

Problems

*A number of these questions are taken from
examination papers
of the University of London*

Problems

1. Calculate the ideal available energy produced by the discharge to atmosphere through a nozzle, of air stored in a cylinder of capacity 3 ft^3 , at a pressure of 50 atmospheres. The initial temperature of the air is 20°C and the ratio of the specific heats is 1.4.

2. Obtain expressions for the variation of

- (a) internal energy with change of volume,
- (b) internal energy with change of pressure, and
- (c) enthalpy with change of pressure,

all at constant temperature, for a gas whose equation of state is given by Van der Waals' law.

3. Calculate the energy stored in 60 cubic inches of gas at 800 atmospheres pressure.

4. One thousand gallons of water per hour are to be pumped through a steel pipe, 1 in. diameter and 100 ft long, to a tank 40 ft higher than its reservoir. Calculate approximately the power required. What type of pump would you instal for the purpose and what H.P. motor would you provide?

Viscosity of water = 1.30 centipoises.

Density of water = 62.4 lb/ft^3 .

5. Calculate the pressure drop in, and the horse power required to operate, a condenser consisting of 400 tubes, 15 ft long and 0.5 in. internal diameter. The coefficient of contraction at the entrance of the tubes is 0.6, and 500 gallons of water per minute are to be pumped through the condenser.

6. 75% sulphuric acid, of density 103 lb/ft^3 and viscosity 8.6 centipoises, is to be pumped for half a mile along a 2 in. internal diameter pipe at the rate of 10 tons per hour, and then raised vertically 50 ft by the pump. If the pump is electrically driven and has an efficiency of 50%, what power will be required? What type of pump would you use and of what material would you construct the pump and pipe?

7. 60% sulphuric acid is to be pumped at the rate of 3,000 gallons per hour through a lead pipe 1 in. diameter and raised to a height of 80 ft. The pipe is 90 ft long and includes two right-angled bends. Calculate the theoretical horse-power required.

The specific gravity of the acid is 1.531 and its kinematic viscosity is $0.0000458 \text{ ft}^2/\text{sec}$. The density of water may be taken as 62.4 lb/ft^3 .

8. 10,000 lb per hour of 98% sulphuric acid is to be pumped through a 1 in. diameter pipe, 100 ft long, to a tank 40 ft higher than its reservoir. Calculate the power required and indicate the type of pump and the material of construction of the line that you would choose.

Viscosity of acid = 25 centipoises

Specific gravity = 1.84.

9. Calculate the hydraulic mean diameter of the annular space between a $1\frac{1}{2}$ in. and a 2 in. tube.

10. $30 \text{ ft}^3/\text{min}$ of acetic acid are pumped through a 3 in. diameter horizontal pipe 240 ft long. What is the pressure drop in the pipe?

Viscosity of acid = 2.5 centipoises.

Density of acid = 66 lb/ft^3 .

Roughness of pipe surface = $2 \times 10^{-6} \text{ in.}$

11. A cylindrical tank, 16 ft in diameter, discharges through a mild steel pipe 300 ft long and 9 in. diameter connected to the base of the tank. Find the time taken for the water level in the tank to drop from 9 ft to 4 ft above the bottom. Take the viscosity of water as 1 centipoise.

12. Two storage tanks A and B containing a petroleum product discharge through pipes, each 1 ft in diameter and 1 mile long to a junction at D. From D, the benzene is carried by an 18 in. diameter pipe to a third storage tank C, $\frac{1}{2}$ mile away. The surface of the liquid in A is initially 30 ft above that in C and the liquid level in B is 20 ft higher than that in A. Calculate the initial rate of discharge of liquid if the pipes are of mild steel. Take the density of benzene as 0.87 gm/cc and the viscosity as 0.7 centipoise.

13. Find the drop in pressure due to friction in a pipe 1,000 ft long and 6 in. diameter when water is flowing at the rate of 600 gallons per minute. The pipe is of glazed porcelain.

14. Two tanks, the bottoms of which are at the same level, are connected with one another by a horizontal pipe 3 in. diameter and 1,000 ft long. The pipe is bell mouthed at each end so that losses on entry and exit are negligible. One tank is 20 ft diameter and contains water to a depth of 20 ft. The other tank is 15 ft diameter and contains water to a depth of 10 ft.

If the tanks are put into communication with one another by means of the pipe, how long will it take before the water level in the larger tank has fallen to 17 ft? Assume the pipe to be an old mild steel pipe.

15. Two immiscible fluids A and B, of viscosities μ_A and μ_B , flow under streamline conditions between two horizontal parallel planes of width b , situated a distance $2a$ apart (where a is much less than b), as two distinct layers one above the other, each of depth a .

Show that the volumetric rate of flow of A is

$$\frac{\Delta P a^3 b}{12 \mu_A l} \times \frac{7 \mu_A + \mu_B}{\mu_A + \mu_B}$$

where ΔP is the pressure drop over a length l in the direction of flow.

16. Coal-gas, having a molecular weight of 13, and a kinematic viscosity of 0.000269 f.p.s. unit, is flowing through a pipe 9 in. internal diameter and 3 miles long, at the rate of 50,000 ft³ per hour. Calculate the pressure required to maintain this rate of flow.

The pound-molecular volume at 60°F and 30 in. mercury may be taken as 385.

What effect on the pressure required would result if the gas were delivered at a height of 500 ft (i) above and (ii) below its point of entry into the pipe?

17. Nitrogen at 120 atmospheres gauge pressure is fed through a 1 in. diameter mild steel pipe to a synthetic ammonia plant at the rate of 10,000 lb per hour. What will be the drop in pressure over a 100 ft length of pipe for isothermal flow of the gas?

Absolute roughness of the pipe surface = 0.0002 in.

Pound molecular volume = 359 ft³.

Viscosity of nitrogen = 0.02 centipoise.

18. Hydrogen is pumped from a reservoir at 20 atmospheres pressure absolute through a clean horizontal mild steel pipe 2 in. diameter and 1,600 ft long. The downstream pressure is also 20 atmospheres and the pressure of the gas is raised to 25 atmospheres by a pump at the upstream end of the pipe. The conditions of flow are isothermal and the temperature of the gas is 20°C. What is the flow rate and what is the effective rate of working of the pump?

Viscosity of hydrogen = 0.009 centipoise at 20°C.

19. In a synthetic ammonia plant, the hydrogen is fed through a 2 in. steel pipe to the converters. The pressure drop over the 100 ft length of pipe is 5 atmospheres, the pressure at the downstream end being 75 atmospheres gauge. What power is required in order to overcome friction losses in the pipe? Assume an isothermal expansion of the gas. What error is introduced by assuming the gas to be an incompressible fluid of density equal to that at the mean pressure in the pipe?

20. A vacuum distillation plant operating at 50 mm Hg pressure at the top has a boil-up rate of 1,000 lb of xylene per hr. Calculate the pressure drop along a 6 in. bore vapour pipe used to connect the column to the condenser.

The pipe length may be taken as equivalent to 20 ft.

21. Nitrogen at 120 atmospheres gauge pressure is fed through a 1 in. diameter mild steel pipe to a synthetic ammonia plant at the rate of 3,000 lb/hr. What will be the drop in pressure over a 100 ft length of pipe assuming an isothermal expansion of the gas? What is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions?

What would be the pressure drop in the pipe if it were perfectly lagged?

22. A petroleum fraction is pumped a mile and a quarter from the distillation plant to the storage tanks through a mild steel pipe-line, 6 in. diameter, at the rate of 500 gallons per minute. What is the pressure drop along the pipe, and the power supplied to the pumping unit if it has an efficiency of 50%?

The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced?

Specific gravity of liquid:	0.705
Viscosity of liquid:	0.5 centipoise
Roughness of pipe surface:	0.00015 in.

23. Glycerol is pumped from storage tanks to rail cars through a single 2 in. diameter main 40 ft long, which must be used for all grades of glycerol. After the line has been used for commercial material, how much pure glycerol must be pumped before the issuing liquid contains not more than 1% of the commercial material? The flow in the pipeline is streamline and the two grades of glycerol have identical densities and viscosities.

24. Sulphuric acid of specific gravity 1.3 is flowing through a pipe 2 in. internal diameter. A thin-lipped orifice, 0.4 in. diameter, is fitted in the pipe and the differential pressure shown by a mercury manometer is 4 in. Assuming that the leads to the manometer are filled with the acid, calculate (a) the weight of acid flowing per hour, and (b) the approximate loss of pressure (in lb/in.²) caused by the orifice.

The coefficient of discharge of the orifice may be taken as 0.61, the specific gravity of mercury as 13.55 and the density of water as 62.4 lb/ft³.

25. The rate of discharge of water from a tank is measured by means of a notch, for which the flow rate is directly proportional to the height of liquid above the bottom of the notch. Calculate and plot the profile of the notch if the flow rate is 100 gallons per minute when the liquid level is 6 in. above the bottom of the notch.

26. Water flows at between 40 and 50 gallons per minute through a 2 in. pipe and is metered by means of an orifice. Suggest a suitable size of orifice if the pressure difference is to be measured with a simple water manometer. What approximately is the pressure difference recorded at the maximum flow rate?

27. The rate of flow of water in a 6 in. diameter pipe is measured with a venturi meter with a 2 in. diameter throat. When the pressure drop over the converging section is 4.2 in. of water, the flow rate is 6.4 lb/sec. What is the coefficient for the converging cone of the meter at that flow rate and what is the head lost due to friction? If the total loss of head over the meter is 0.6 in. water, what is the coefficient for the diverging cone?

28. A venturi meter with a 2 in. throat is used to measure the flow of slightly salt water in a pipe of inside diameter 4 in. The meter is checked by adding 1 litre per minute of normal sodium chloride solution above the meter and analysing a sample of water downstream from the meter. Before addition of the salt, 1 litre of water requires 10 cc of N/10 silver nitrate solution in a titration. 1 litre of the downstream sample requires 23.5 cc N/10 silver nitrate. If a mercury under water manometer connected to the meter gives a reading of 6.5 in., what is the discharge coefficient of the meter? Assume that the density of the liquid is not appreciably affected by the salt.

29. A gas cylinder containing 1,000 ft³ of air at 60 atmospheres gauge pressure discharges to the atmosphere through a valve which may be taken as equivalent to a sharp edged orifice

of $\frac{1}{4}$ in. diameter (coefficient of discharge = 0.6). Plot the rate of discharge against the pressure in the cylinder. How long will it take for the pressure in the cylinder to fall to

- (a) 10 atmospheres gauge and
- (b) $\frac{1}{2}$ atmosphere gauge?

Assume an adiabatic expansion of the gas through the valve and that the contents of the cylinder remain at 0°C .

30. Air at a pressure of 200 lb/in.^2 gauge and a temperature of 200°F flows through an orifice of 0.05 in.^2 to atmospheric pressure.

If the coefficient of discharge is 0.65, the critical pressure ratio is 0.527 and the ratio of the specific heats is 1.4, calculate the weight flowing per hour.

31. Water flows through an orifice of 1 in. diameter situated in a 3 in. pipe, at the rate of 4 gallons per minute. What will be the difference in level on a water manometer connected across the meter? Take the viscosity of water as 1 centipoise.

32. A three-stage compressor is required to compress air from 20 lb/in.^2 absolute, and 50°F to 600 lb/in.^2 absolute. Calculate the ideal intermediate pressures, the work required per pound of gas and the isothermal efficiency of the process. Assume the compression to be adiabatic and the interstage cooling to cool the air to the initial temperature. Show qualitatively, by means of temperature-entropy diagrams, the effect of unequal work distribution and imperfect intercooling, on the performance of the compressor.

33. A twin-cylinder, single acting compressor, working at 300 r.p.m., delivers air at 60 lb/in.^2 gauge pressure, at the rate of 300 ft^3 per minute. If the diameter of the cylinder is 9 in., the cylinder clearance ratio 5% and the temperature of the inlet air 10°C , calculate the length of stroke of the piston.

34. A single stage double-acting compressor, running at 200 r.p.m., is used to compress air from 1 lb/in.^2 gauge pressure and 45°F to 150 lb/in.^2 gauge pressure. If the internal diameter of the cylinder is 8 in., the length of stroke 10 in. and the piston clearance 5%, calculate:

- (a) The maximum capacity of the machine, referred to air at the initial temperature and pressure; and
- (b) the theoretical horse power requirements under isentropic conditions.

35. Methane is to be compressed from atmospheric pressure to 300 atmospheres in four stages.

Calculate the ideal intermediate pressures and the work required per pound of gas. Assume compression to be isentropic and the gas to behave as an ideal gas. Indicate on a temperature-entropy diagram the effect of imperfect intercooling on the work done at each stage.

36. An air-lift raises 160 gallons of water per minute from a well 320 ft deep through a 4 in. diameter pipe. The level of the water is 120 ft below the surface. The air consumed is 200 ft^3 of free air per minute compressed to 100 lb/in.^2 (gauge).

Calculate the efficiency of the pump and the mean velocity of the mixture in the pipe.

37. In a single-stage compressor:

Suction pressure	= 14.7 lb/in.^2
Suction temperature	= 10°C
Final pressure	= 55 lb/in.^2 absolute.
Clearance	= 5%
Compression is adiabatic.	

If each new charge is heated 10°F by contact with the clearance gases, calculate the maximum temperature attained in the cylinder.

38. A single acting reciprocating pump has a cylinder diameter of $4\frac{1}{2}$ in. and a stroke of 9 in. The suction line is 20 ft long and 2 in. diameter and the level of the water in the suction tank is 10 ft below the cylinder of the pump. What is the maximum speed at which the pump can run without an air vessel if separation is not to occur in the suction line? The

piston undergoes approximately simple harmonic motion. Atmospheric pressure is equivalent to a head of 34 ft of water and separation occurs at an absolute pressure corresponding to a head of 4 ft of water.

39. An air lift pump is used for raising 10 gallons per minute of a liquid of specific gravity 1.2 to a height of 60 ft. Air is available at 50 lb/in.² gauge pressure. If the efficiency of the pump is 30%, calculate the power requirement, assuming isentropic compression of the air ($\gamma = 1.4$).

40. A single acting air compressor supplies 200 ft³ of air (at S.T.P.) per minute compressed to 55 lb/in.² absolute from 14.7 lb/in.² absolute pressure. If the suction temperature is 15.5°C, the stroke is 10 in. and the speed is 250 r.p.m., find the cylinder diameter. Assume the cylinder clearance is 4% and compression and re-expansion are isentropic ($\gamma = 1.4$). What is the theoretical horse power required for the compression?

41. Air at 20°C is compressed from 14.7 to 300 lb/in.² absolute pressure in a two stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of the clearance gas is $PV^{1.25} = \text{constant}$. The compression ratio in each of the two cylinders is the same and the interstage cooler may be taken as perfectly efficient. If the clearances in the two cylinders are 4% and 5% respectively, calculate,

- the work of compression per unit mass of gas compressed,
- the isothermal efficiency,
- the isentropic efficiency ($\gamma = 1.4$),
- the ratio of the swept volumes in the two cylinders.

42. Calculate the time taken for the distant face of a brick wall, of thermal diffusivity, $D_H = 0.0042 \text{ cm}^2/\text{sec}$ and thickness, $l = 18 \text{ in.}$, initially at 20°C, to rise to 200°C if the near face is suddenly raised to a temperature of $\theta' = 600^\circ\text{C}$ and maintained at that temperature. Assume that all the heat flow is perpendicular to the faces of the wall and that the distant face is perfectly insulated.

43. Calculate the time for the distant face to reach 200°C under the same conditions, except that the distant face is not perfectly lagged. Instead, a very large thickness of material of the same thermal properties as the brickwork is stacked against it.

Note:

$$p^{-1} \cdot e^{-k\sqrt{p}} \text{ is the Laplace Transform of } \text{erfc} \frac{k}{2/t}$$

Tables of $\text{erfc } x$ for various values of x are given on p. 373 of *Conduction of Heat in Solids* by CARSLAW and JAEGER.

44. Benzene vapour, at atmospheric pressure, condenses on a plane surface, 6 ft long and 3 ft wide, maintained at 25°C and inclined at an angle of 45° to the horizontal. Plot the thickness of the condensate film and the point heat transfer coefficient against distance from the top of the surface.

45. It is desired to warm 7,000 lb/hr of air from 50°F to 200°F by passing it through the pipes of a bank consisting of 20 rows with 20 pipes in each row. The arrangement is in line with centre to centre spacing, in both directions, equal to twice the pipe diameter. Flue gases, entering at 200°F with a free flow mass velocity of 800 lb/ft²-hr are passed across the outside of the pipes.

Neglecting gas radiation, how long should the pipes be?

For simplicity, outer or inner pipe diameter may be taken as $\frac{1}{2}$ in.

Values of k and μ , which may be used for both air and flue gases, are given below. The specific heat of air and flue gases is 0.24 B.Th.U./lb-°F.

Thermal Conductivity		Viscosity
k in B.Th.U./ft-hr-°F		μ in lb/ft-hr
0°F	0.013	0.040
500°F	0.023	0.067
1000°F	0.032	0.089

46. A cooling coil, consisting of a single length of tubing through which water is circulated, is provided in a reaction vessel, the contents of which are kept uniformly at 90°C by means of a stirrer. The inlet and outlet temperatures of the cooling water are 10°C and 50°C respectively. What would the outlet water temperature become if the length of the cooling coil were increased 5 times? Assume the overall heat transfer coefficient to be constant over the length of the tube, and independent of the water temperature.

47. In an oil cooler 8 lb of hot oil a minute enters a thin metal pipe of diameter 1 in. An equal mass of cooling water flows through the annular space between the pipe and a larger concentric pipe, the oil and water moving in opposite directions. The oil enters at 300°F and is to be cooled to 120°F . If the water enters at 60°F , what length of pipe will be required? Take coefficients of 280 B.Th.U./ft²-hr- $^{\circ}\text{F}$ on the oil side and 640 B.Th.U./ft²-hr- $^{\circ}\text{F}$ on the water side and 0.5 B.Th.U./lb- $^{\circ}\text{F}$ for the specific heat of the oil.

48. The walls of a furnace are built up of 6 in. thickness of a refractory of thermal conductivity 0.50 lb-cal/ft-hr- $^{\circ}\text{C}$. The surface temperatures of the inner and outer faces of the refractory are 1100°C and 240°C respectively.

If a layer of insulating material 1 in. thick, of thermal conductivity 0.10 lb-cal/ft-hr- $^{\circ}\text{C}$, is added, what temperatures will its surfaces attain, assuming the inner surface of the furnace to remain at 1100°C ? The coefficient of heat transfer from the outer surface of the insulation to the surroundings, which are at 20°C , may be taken as 2.4, 2.9, 3.5 and 4.1 lb-cal/ft²-hr- $^{\circ}\text{C}$, for surface temperatures of 100°C , 150°C , 200°C and 250°C respectively. What will be the reduction in heat loss?

49. A pipe of outer diameter 2 in., maintained at $1,500^{\circ}\text{F}$, is covered with 2 in. of insulation, of thermal conductivity 0.10 B.Th.U./ft-hr- $^{\circ}\text{F}$.

Would it be feasible to use a magnesia insulation, which will not stand temperatures above 650°F and has thermal conductivity 0.05 B.Th.U./ft-hr- $^{\circ}\text{F}$, for an additional layer thick enough to reduce the outer surface temperature to 200°F in surroundings at 50°F ? Take the surface coefficient of heat transfer by radiation and convection as 2.0 B.Th.U./ft²-hr- $^{\circ}\text{F}$.

50. In order to warm 3400 lb/hr of a heavy oil from 38°C to 54°C it is passed through tubes of inside diameter $\frac{3}{4}$ in. and length 5 ft, forming a bank, on the outside of which steam is condensing at 100°C . How many tubes will be needed?

In calculating Nu ., Pr ., and Re ., the thermal conductivity of the oil may be taken as 0.08 lb-cal/ft-hr- $^{\circ}\text{C}$, and the specific heat as 0.50 lb-cal/lb- $^{\circ}\text{C}$, irrespective of temperature. The viscosity is to be taken at the mean oil temperature. Viscosity of the oil at 46°C is 372, and at 100°C 46.5 lb/ft-hr.

51. A metal pipe of $\frac{1}{2}$ in. outer diameter is maintained at 300°F . Calculate the rate of heat loss per foot run in surroundings uniformly at 60°F , (a) when the pipe is covered with $\frac{1}{2}$ in. thickness of a material of thermal conductivity 0.20 B.Th.U./ft-hr- $^{\circ}\text{F}$ and surface emissivity 0.95, and (b) when the thickness of the covering material is reduced to $\frac{1}{4}$ in., but the outer surface is so treated as to reduce its emissivity to 0.10.

The coefficients of radiation from a perfectly black surface in surroundings at 60°F are 1.10, 1.44, and 1.88 B.Th.U./ft²-hr- $^{\circ}\text{F}$ at 100°F , 200°F and 300°F respectively.

The coefficients of convection may be taken as $0.25 (\theta/d)^{\frac{1}{2}}$ B.Th.U./ft²-hr- $^{\circ}\text{F}$, where θ is the temperature difference between the surface and the surrounding air, and d is the outer diameter in feet.

52. A condenser consists of 30 rows of parallel pipes of outer diameter $\frac{3}{4}$ in. and thickness 0.05 in., with 40 pipes, each 6 ft long, per row. Water, inlet temperature 10°C , flows through the pipes at 3 ft per second, and steam at 99°C condenses on the outside of the pipes. There is a layer of scale 0.01 in. thick, of thermal conductivity 1.2 lb-cal/ft-hr- $^{\circ}\text{C}$, on the inside of the pipes.

Taking the coefficients of heat transfer on the water side as 700, and on the steam side as 1500 lb-cal/ft²-hr- $^{\circ}\text{C}$, calculate the outlet water temperature and the total weight of steam condensed per hour. The latent heat of steam at 99°C is 538 lb-cal per lb. 1 ft³ water weighs 62.4 lb.

53. In an oil cooler, water flows at the rate of 800 lb/hr per tube through metal tubes of outer diameter $\frac{3}{4}$ in. and thickness 0.05 in., along the outside of which oil flows in the opposite direction at the rate of 600 lb/hr per tube.

If the tubes are 6 ft long, and the inlet temperatures of the oil and water are respectively 100°C and 10°C, what will be the outlet oil temperature? The coefficient of heat transfer on the oil side is 300, and on the water side 450 lb-cal/ft²-hr-°C, and the specific heat of the oil is 0.45.

54. Waste gases flowing across the outside of a bank of pipes are being used to warm air which flows through the pipes. The bank consists of 12 rows of pipes with 20 pipes, each 2 ft long, per row. They are arranged in-line, with centre-to-centre spacing equal in both directions to one-and-a-half times the pipe diameter. Both inner and outer diameter may be taken as 0.5 in.

Air, with mass velocity 6000 lb/ft²-hr, enters the pipes at 60°F. The initial gas temperature is 400°F and the total weight of the gases crossing the pipes hourly is the same as the total weight of the air flowing through them.

Neglecting gas radiation, estimate the outlet temperature of the air. The physical constants for the waste gases may be assumed the same as for air, given below—

<i>Temperature</i>	<i>Thermal conductivity</i>	<i>Viscosity</i>
°F	B.Th.U./ft-hr-°F	lb/ft-hr
0	0.0133	0.040
100	0.0154	0.046
200	0.0174	0.052
300	0.0193	0.058
400	0.0212	0.063

Specific heat = 0.24 B.Th.U./lb-°F.

55. Oil is to be warmed from 80°F to 160°F by passing it at 3 ft/sec through the pipes of a shell-and-tube heat exchanger. Steam at 220°F condenses on the outside of the pipes, which have outer and inner diameters of 1.90 and 1.60 in. respectively; but, owing to fouling, the inside diameter has been reduced to 1.50 in., and the resistance to heat transfer of the pipe wall and dirt together, based on this diameter, is 0.005 ft²-hr-°F/B.Th.U.

It is known from previous measurements under similar conditions that the oil side coefficients of heat transfer for a velocity of 3 ft/sec, based on a diameter of 1.50 in., vary with the temperature of the oil according to the table below—

Oil temperature, °F	80	100	120	140	160
Oil side coefficient of heat transfer, B.Th.U./ft ² -hr-°F	13	14	17	24	43

The specific heat and density of the oil may be assumed constant at 0.45 B.Th.U./lb-°F and 56 lb/ft³ respectively, and any resistance to heat transfer on the steam side neglected.

Find the length of tube bundle required.

56. It is proposed to construct a heat exchanger to condense 60,000 lb/hr of *n*-hexane at a pressure of 8 lb/in.²-G., involving a heat load of 8.5 million lb-cal/hr. The hexane is to reach the condenser from the top of a fractionating column at its condensing temperature of 83°C.

From experience it is anticipated that the overall heat transfer coefficient will be 80 lb-cal/ft²-hr-°C. The available cooling water is at 15°C.

Outline the proposals that you would make for the type and size of the exchanger, and explain the details of the mechanical construction that you consider require special attention.

57. A heat exchanger is to be mounted at the top of a fractionating column about 50 ft high to condense 30,000 lb/hr of *n*-pentane at 15 lb/in.²-G., corresponding to a condensing temperature of 60°C. Give an outline of the calculations you would make to obtain an approximate idea of the size and construction of the exchanger required.

For purposes of standardisation the company will use $\frac{3}{4}$ in. outer diameter tubes of 16 B.W.G. and these may be 8, 12 or 16 ft in length. The film coefficient for condensing pentane on the outside of a horizontal tube bundle may be taken as 200 lb-cal/hr-ft²-°C. The condensation is effected by pumping water through the tubes, the initial water temperature being 15°C.

The latent head of condensation of pentane is 80 lb-cal/lb.

For these $\frac{3}{4}$ in. tubes, a water velocity of 1 ft/sec corresponds to a flow rate of 470 lb of water per hour.

58. An organic liquid is boiling at 150°F on the inside of a metal surface of thermal conductivity 24 B.Th.U./ft-hr-°F and thickness $\frac{1}{8}$ in. The outside of the surface is heated by condensing steam. Assuming that the heat transfer coefficient from steam to the outer metal surface is constant at 2000 B.Th.U./ft²-hr-°F, irrespective of the steam temperature, find what value of the steam temperature would give a maximum rate of evaporation.

The coefficients of heat transfer from the inner metal surface to the boiling liquid depend upon the temperature difference as shown below—

<i>Temp. difference metal surface to boiling liquid</i>	<i>Heat transfer coefficient metal surface to boiling liquid</i>
°F	B.Th.U./ft ² -hr-°F
40	780
50	1040
60	1300
65	1285
70	1200
75	1120
80	1010
90	800

59. It is desired to warm an oil of specific heat 0.5 from 25°C to 50°C by passing it through a tubular heat exchanger with metal tubes of inner diameter 0.5 in. Along the outside of the tubes flows water, inlet temperature 99°C and outlet temperature 88°C.

The overall heat transfer coefficient from water to oil, reckoned on the inside area of the tubes, may be assumed constant at 40 lb-cal/ft²-hr-°C, and 600 lb of oil per hour is to be passed through each tube.

The oil is to make two passes through the heater. The water makes one pass along the outside of the tubes. Calculate the length of the tubes required.

60. A condenser consists of a number of metal pipes of outer diameter 1 in., and thickness 0.10 in. Water, flowing at 2 ft/sec, enters the pipes at 65°F, and it is not permissible that it should be discharged at a temperature in excess of 95°F.

If 10,000 lb of a hydrocarbon vapour are to be condensed hourly at 160°F on the outside of the pipes, how long should each pipe be, and how many pipes would be needed?

Take the coefficient of heat transfer on the water side as 450, and on the vapour side as 140, B.Th.U./ft²-hr-°F and assume that the overall coefficient of heat transfer from vapour to water, based upon these figures, is reduced 20% by the effects of the pipe walls, dirt and scale.

The latent heat of the hydrocarbon vapour at 160°F is 135 B.Th.U./lb.

61. An organic vapour is being condensed at 80°C on the outside of a nest of pipes through which water flows at 2 ft/sec, its inlet temperature being 20°C. The outer and inner diameters of the pipes are 0.75 and 0.58 in. respectively, but a layer of scale 1/100 in. thick and thermal conductivity 1.2 lb-cal/ft-hr-°C, has formed on the inside of the pipes.

If the coefficients of heat transfer on the vapour and water sides respectively are 300 and 560 lb-cal/ft²-hr-°C and it is required to condense 200 lb of vapour per hour on each of the pipes, how long should these be, and what will be the outlet temperature of the water?

The latent heat of condensation is 78 lb-cal/lb.

Neglect any small resistance to heat transfer in the pipe walls.

62. A heat exchanger is required to cool continuously 160,000 lb/hr of warm water from 90°C to 65°C by means of 200,000 lb/hr of cold water, inlet temperature 30°C.

Assuming that the water velocities are such as to give an overall coefficient of heat transfer of 350 lb-cal/ft²-hr-°C, assumed constant, calculate the total area of surface required (a) in a counterflow heat exchanger, i.e., one in which the hot and cold fluids flow in opposite directions, and (b) in a multipass heat exchanger, with the cold water making two passes through the tubes, and the hot water making one pass along the outside of the tubes. In case (b) assume that the hot water flows in the same direction as the inlet cold water, and that its temperature over any cross-section is uniform.

63. Find the heat loss per ft² of surface through a brick wall 18 in. thick when the inner surface is at 120°C and the outside at 40°C: the thermal conductivity of the brick may be taken as 0.4 lb-cal/hr-ft²-°C/ft.

64. A furnace is constructed with 9 in. of firebrick, 4½ in. of insulating brick and 9 in. of building brick. The inside temperature is 930°C and the outside temperature 55°C. If the thermal conductivities are 0.8, 0.12 and 0.4 lb-cal/hr-ft²-°C/ft, find the heat loss per unit area, and the temperature at the junction of the firebrick and insulating brick.

65. Calculate the total heat loss by radiation and convection from an unlagged horizontal steam pipe of 2 in. outside diameter at 140°C to air at 10°C.

66. Toluene is continuously nitrated to mononitrotoluene in a cast iron vessel of 3 ft diameter fitted with a propeller agitator of 1 ft diameter driven at 150 r.p.m. The temperature is maintained at 35°C by circulating cooling water at 3500 lb/hr through a stainless steel coil of 1 in. o.d. and 0.37 in. i.d. wound in the form of a helix of 32 in. diameter. The conditions are such that the reacting material may be considered to have the same physical properties as 75% sulphuric acid. If the mean water temperature is 17°C, what is the overall heat transfer coefficient?

67. 60,000 lb/hr of pure iso-butane are to be condensed at a temperature of 58.5°C in a horizontal tubular exchanger using a water inlet temperature of 28°C. It is proposed to use ¾ in. outside diameter tubes of 16 B.W.G. arranged on a 1 in. triangular pitch. Under these conditions the resistance of the scale may be taken as 0.003 ft²-hr-°C/lb-cal. It is required to determine the number and arrangement of the tubes in the shell.

68. 300,000 lb/hr of crude oil are to be heated from 70 to 136°F by heat exchange with the bottom product from a distillation column. The product at 257,500 lb/hr is to be cooled from 295 to 225°F. There is available a tubular exchanger with an inside shell diameter of 23¼ in., having one pass on the shell side and two passes on the tube side. It has 324 tubes, ¾ in. o.d. by 14 B.W.G. and 12 ft long, arranged on a 1 in. square pitch and supported by baffles with a 25% cut, spaced at 9 in. intervals. Would this exchanger be suitable?

69. A 6 in. internal diameter steam pipe is carrying steam at 171°C and is lagged with 2 in. of 85% magnesia. What will be the heat loss to the air at 21°C?

70. Ammonia gas is diffusing at a constant rate through a layer of stagnant air 1 mm thick. Conditions are fixed so that the gas contains 50% by volume of ammonia at one boundary of the stagnant layer. The ammonia diffusing to the other boundary is quickly absorbed and the concentration is negligible at that plane. The temperature is 20°C and the pressure atmospheric and under these conditions the diffusivity of ammonia in air is 0.18 cm²/sec. Calculate the rate of diffusion of ammonia through the layer.

71. A simple rectifying column consists of a tube, arranged vertically and supplied at the bottom with a mixture of benzene and toluene as vapour. At the top a condenser returns some of the product as a reflux which flows in a thin film down the inner wall of the tube. The tube is insulated and heat losses can be neglected. At one point in the column, the vapour contains 70 mol% benzene and the adjacent liquid reflux contains 59 mol% benzene. The temperature at this point is 90°C. Assuming the diffusional resistance to vapour transfer to be equivalent to the diffusional resistance of a stagnant vapour layer 0.2 mm thick, calculate

the rate of interchange of benzene and toluene between vapour and liquid. The molar latent heats of the two materials can be taken as equal. The vapour pressure of toluene at 90°C is 404.6 mm and the diffusivity of the vapours is $0.051\text{ cm}^2/\text{sec}$.

72. By what percentage would the rate of absorption be increased or decreased by increasing the total pressure from 1 to 2 atmospheres in the following cases?

(a) The absorption of ammonia from a mixture of ammonia and air containing 10% of ammonia by volume, using pure water as solvent. Assume that all the resistance to mass transfer lies within the gas phase.

(b) The same conditions as (a) but the absorbing solution exerts a partial vapour pressure of ammonia of 0.05 atmospheres.

The diffusivity can be assumed to be inversely proportional to the absolute pressure.

73. If the temperature rise per foot length along a pipe carrying air at 40 ft/sec is 20°C , what will be the corresponding pressure drop in inches of water gauge, for a pipe temperature of 150°C and an air temperature of 40°C ?

The density of air at 40°C is 0.071 lb/ft^3 .

74. It is required to warm a quantity of air from 16°C to 40°C by passing it through a number of parallel metal tubes of inner diameter 2 in. maintained at 100°C . The pressure drop must not exceed 1 in. water gauge.

How long should the individual tubes be?

The density of air at 28°C is 0.074 lb/ft^3 .

The coefficients of heat transfer by convection from tube to air are 8, 11 and $13\frac{1}{2}\text{ lb-cal/ft}^2\text{-hr-}^{\circ}\text{C}$ respectively, for velocities of 60, 80 and 100 ft/sec at 28°C .

75. Air at 60°C , flowing at 30 ft/sec, enters a pipe of inner diameter 1 in., maintained at 140°C . The drop of static pressure along the pipe is 0.10 in. of water gauge per foot length. Using the Reynolds analogy between heat transfer and friction, estimate the temperature of the air 2 ft along the pipe.

76. Air flows at 40 ft/sec through a pipe of inside diameter 1 in. The rate of heat transfer by convection between the pipe and the air is $10\text{ lb-cals/ft}^2\text{-hr-}^{\circ}\text{C}$. Neglecting the effects of temperature variation, estimate the pressure drop per foot length of pipe in inches of water gauge.

77. Calculate the thickness of the boundary layer at a distance of 3 in. from the leading edge of a plane surface over which water is flowing at a rate of 10 ft/sec. Assume that the flow in the boundary layer is streamline and that the velocity u of the fluid at a distance y from the surface can be represented by the relation $u = a + by + cy^2 + dy^3$ (where the coefficients a , b , c , and d are independent of y). Take the viscosity of water as 1 centipoise.

78. Water flows at a velocity of 3 ft/sec over a plane surface 2 ft wide and 3 ft long. Calculate the total drag force acting on the surface, if the transition from streamline to turbulent flow in the boundary layer occurs when the Reynolds Group Re_x equals 10^5 .

79. Calculate the thickness of the boundary layer at a distance of 6 in. from the leading edge of a surface over which oil, of viscosity 50 centipoises and density 62 lb/ft^3 , flows with a velocity of 1 ft/sec. What is the displacement thickness of the boundary layer?

80. Calculate the thickness of the laminar sub-layer when benzene flows through a pipe, 2 in. diameter, at 30 gallons per minute. What is the velocity of the benzene at the edge of the laminar sub-layer? Assume fully developed flow exists within the pipe.

81. Calculate the rise in temperature of water passed at 12 ft/sec through a smooth 1 in. diameter pipe, 20 ft long. The water enters at 30°C and the temperature of the wall of the tube can be taken as approximately constant at 60°C . Use

(a) The simple Reynolds Analogy.

(b) The Taylor-Prandtl modification.

(c) Equation 9.83.

(d) Equation 8.30.

Comment on the differences in the results so obtained.

82. Calculate the rise in temperature of a stream of air, entering at 20°C and passing at 12 ft/sec through the tube maintained at 80°C , other conditions remaining the same as in the previous question.

83. In a process in which benzene is used as a solvent, it is evaporated into dry nitrogen. The resulting mixture at a temperature of 24°C and a pressure of 14.7 lb/in.^2 (absolute) has a relative humidity of 60%. It is desired to recover 80% of the benzene present by cooling to 10°C and compressing to a suitable pressure. What must this pressure be?

Vapour pressures of benzene: at $24^{\circ}\text{C} = 91.5\text{ mm Hg}$, at $10^{\circ}\text{C} = 45.4\text{ mm Hg}$.

84. 2 million ft^3 of gas are to be dried per day from a dew point of 21 to a dew point of 4.5°C . How much water must be removed and what will be the volume of the gas after drying?

Vapour pressure of water at 21°C 0.74 in. mercury .

Vapour pressure of water at 4.5°C 0.25 in. mercury .

85. Wet material, containing 70% moisture, is to be dried at the rate of 1200 lb/hr in a countercurrent dryer to give a product containing 5% moisture (both on the wet basis). The drying medium consists of air heated to 212°F and containing water vapour equivalent to a partial pressure of 7.7 mm mercury. The air leaves the dryer at 104°F and 70% saturated. Calculate how much air will be required to remove the moisture. The vapour pressure of water at 104°F may be taken as 55.3 mm mercury.

86. A million ft^3 of coal gas (measured at 60°F and 30 in. mercury, saturated with water vapour) are compressed to 20 lb/in.^2 gauge pressure, cooled to 60°F and the condensed water is drained off. Subsequently, the pressure is reduced to 10 lb/in.^2 gauge, and the gas is distributed at this pressure and 60°F . What is the percentage humidity of the gas after the above treatment?

The vapour pressure of water at 60°F is 0.522 in. mercury , and 1 lb/in.^2 pressure corresponds to 2.04 in. mercury .

87. A rotary countercurrent dryer is fed with ammonium nitrate containing 5% moisture, at the rate of 200 lb/min., and discharges the nitrate with 0.2% moisture. The air enters at 132°C and leaves at 82°C , the humidity of the entering air being 0.007 lb of moisture per lb of dry air. The nitrate enters at 21°C and leaves at 66°C .

Neglecting radiation losses, calculate the pounds of dry air passing through the dryer, and the humidity of the air leaving the dryer.

Latent heat of water at $21^{\circ}\text{C} = 585\text{ lb-cal/lb}$

Specific heat of ammonium nitrate $= 0.45$

Specific heat of dry air $= 0.238$

Specific heat of water vapour $= 0.48$

88. Material is fed to a dryer at the rate of 1 ton/hr and the moisture removed is 35% of the wet charge. The stock enters and leaves the dryer at 83°F . The air temperature falls from 155 to 100°F , its humidity rising from 0.01 to 0.02.

Calculate the heat loss to the surroundings in B.Th.U./hr.

Latent heat of water at 83°F is 1045 B.Th.U./lb .

Specific heat of dry air is 0.238.

Specific heat of water vapour is 0.48.

89. A rotary dryer is fed with sand at a rate of 120 lb/min. The feed is 50% wet and the sand is discharged with 3% moisture. The entering air is at 220°F and has an absolute humidity of 0.007 lb/lb. The wet sand enters at 70°F and leaves at 97°F , the air leaving at 100°F .

Calculate the mass of air passing through the dryer and the humidity of the air leaving the dryer. Allow radiation loss of 10 B.Th.U./lb of dry air.

Latent heat of water at 70°F is 1053 B.Th.U./lb .

Specific heat of sand 0.21.

Specific heat of dry air 0.238.

Specific heat of vapour 0.48.

90. Water is to be cooled in a packed tower from 55°C to 20°C by means of air flowing countercurrently. The liquid flows at a rate of 20 gallons/ $\text{ft}^2\text{-hr}$ and the air at 8000 $\text{ft}^3/\text{ft}^2\text{-hr}$. The entering air has a temperature of 20°C and a relative humidity of 20%. Calculate the required height of tower and the condition of the air leaving at the top.

The whole of the resistance to heat and mass transfer can be considered as being within the gas phase and the product of the mass transfer coefficient and the transfer surface per unit volume of column ($h_D \cdot a$) can be taken as 0.2 sec^{-1} .

91. Water is to be cooled in a small packed column from a temperature of 55°C to 10°C by means of air flowing countercurrently. The rate of flow of liquid is 100 gallons/ $\text{ft}^2\text{-hr}$. and the flow rate of the air, which enters at a temperature of 20°C and a relative humidity of 60%, is 25,000 $\text{ft}^3/\text{hr-ft}^2$. Calculate the required height of tower if the whole of the resistance to heat and mass transfer can be considered as being in the gas phase and the product of the mass transfer coefficient and the transfer surface per unit volume of column is 2 sec^{-1} .

What is the condition of the air which leaves at the top?

92. Air containing 0.005 lb of water vapour per lb of dry air is heated to 52°C in a dryer and passed to the lower shelves. It leaves these shelves at 60% relative humidity and is reheated to 52°C and passed over another set of shelves, again leaving at 60% relative humidity. This is again reheated for the third and fourth sets of shelves, after which the air leaves the dryer. On the assumption that the material in each shelf has reached the wet bulb temperature and that heat losses from the dryer can be neglected, determine

- (a) the temperature of the material on each tray,
- (b) the amount of water removed, in lb/hr, if 10,000 ft^3/min of moist air leaves the dryer, and
- (c) the temperature to which the inlet air would have to be raised to carry out the drying in a single stage.

8

Index

- Absorption 233, 234, 236, 244
- Absorptivity 178
 - of gas 182
 - of water vapour 183
- Acceleration 1
 - due to gravity 23
- Acid egg 123
 - — efficiency 124
 - — energy losses 125
- ACKERMAN G. 175
- Addition of liquid or vapour to gas 317
- Adiabatic cooling line 309, 313
 - — — systems other than air-water 331
 - flow of gas in pipe 55, 59
 - process 16
 - saturation temperature 308, 313
 - — — compared with wet bulb temperature 309
- Air, condition of, in water cooling tower 327
 - humidification 330
 - lift pump 126
 - — — advantages and disadvantages 131
 - — — air : liquid ratio 127
 - — — efficiency 127
 - — — energy losses 130
 - — — footpiece 129
 - — — operation 129
 - — — pressure distribution 128
 - : liquid ratio in air lift pump 127
 - vessels for reciprocating pumps 108
 - -water system 311
- Ampere 5
- Anemometer, hot wire 97
 - vane 98
- Annular sections, heat transfer 173
- Annulus, streamline flow in 44
- Area 1
- ATKIN G. A. 199
- Atomic volume 241
- Automatic control of humidifying plant 321
 - valve for acid egg 124, 125
- Average velocity, streamline flow in pipe 42
 - — turbulent flow in pipe 46
- AWBERY J. H. 170
- BADGER W. L. 190
- Baffles 206, 209
 - effect on heat transfer coefficients 172
 - for heat exchangers 172
- BAILEY A. 173
- Ball and tube meter 93
- Banks of tubes, pressure drop in flow over 50
- Basic units 1
- Bernoulli's equation 24
- Black body 178
- Blade angle of impeller 118
- Blasius equation 45, 253, 266, 273, 279, 290, 291
- BLASIUS H. 45, 265, 266
- BLISS H. 196, 199
- Blowers 132
- Boiling at high pressures 198
 - in horizontal surface 195
 - liquids, heat transfer 192
 - on submerged surfaces 195
 - on vertical surface 194
- BONILLA C. F. 196, 197, 198
- BOSNJAKOVIC F. 192
- Boundary layer 259
 - — for flow in pipe 271, 273
 - — — heat transfer 280
 - — — — (see Thermal boundary layer)
 - — — mass transfer 288
- Bourdon gauge 77
- BOWMAN R. A. 210
- BRAS G. H. 192
- BRIDGMAN P. W. 7
- British engineering system of units 2
 - Thermal Unit 5
- BROWN R. W. 204
- BS 1042 83
- BS 1500 209
- Bubble formation 192
- Buffer layer 245, 259, 276
- Bulk flow in mass transfer 236
 - modulus of elasticity 57
 - temperature 287
- Calories 5
- Cam pump 113
- CARPENTER E. F. 188, 189
- CARPENTER F. G. 174
- Cavitation in centrifugal pump 120
- Centigrade units 4
- Centimetre 1, 4, 5
- Centipoise 6
- Centrifugal blowers and compressors 133
 - pump 105, 115
 - — advantages and disadvantages 122
 - — characteristics 119
 - — corrosive liquids and suspensions 121
 - — diffusion ring 115
 - — end thrust 120
 - — head developed 118
 - — impeller 115

- Centrifugal pump losses 119
 - materials of construction 121
 - power 118
 - self-priming 120
 - specific speed 119
 - throughput 118
 - turbine type 115
 - virtual head 116
 - volute type 115
- C.G.S. system 1, 5
- Characteristic curves of centrifugal pump 119, 121
- Chemical methods of determining humidity 318
- Chezy equation 68
- CHILTON T. H. 201, 203, 204, 253, 254
- CICHELLI M. T. 198
- CLARK J. A. 193
- Clearance volume 138
- Coefficient of contraction, nozzle 90
 - orifice meter 84
 - of discharge, notch 96
 - nozzle 90
 - orifice meter 84
 - rotameter 94
 - venturi meter 91
- Coefficients, heat transfer (*see* Heat transfer coefficients)
 - mass transfer (*see* Mass transfer coefficients)
- Coils, heat transfer in 201
- Coil heaters 200
- COLBURN A. P. 167, 188, 189, 192, 253, 254, 297
- COMINGS E. W. 309
- Composite walls, heat flow in 153
- Compressed air for pumping 123
- Compressible fluids 18
 - flow through constriction 82
 - friction losses in pipes 51
 - adiabatic flow 55, 59
 - isothermal and adiabatic flow 57
 - isothermal flow in horizontal pipe 52
 - isothermal flow in inclined pipe 63
 - non-ideal gas 57
 - non-isothermal flow 55
 - small pressure drops 53
 - impact pressure 77
 - orifice meter 85
 - static pressure 74
- Compressibility factor 20
- Compressors 132
 - efficiency 143
- Concentric tubes, heat transfer 173
- Condensation inside tubes 189
 - of mixed vapours 191
 - of vapours 183
 - on inclined surface 185
- Conduction of heat 151, 246
 - through thick walled tube 154
- Conductivities of solids, liquids, and gases 152
- Contraction, coefficient of, nozzle 90
 - orifice meter 84
- Convection 151
 - in gases 166
- Converging-diverging nozzles 64
- Conversion chart for temperature 336
 - factors 4
 - examples 6
- Cooling of gas shown on enthalpy-humidity diagram 316
 - of liquids 305
- COOPER H. B. H. 196
- COPE W. F. 173
- Correction factor for kinetic energy 23
 - for mean temperature difference in multi-pass exchangers 210, 211
- COULSON J. M. 212
- Covers for heat exchangers 209
- Critical depth 71
 - flow 67, 71
 - pressure ratio in orifice meter 88
 - state 21
 - velocities for streamline and turbulent flow 28, 29
- CRYDER D. S. 196, 197, 198, 199
- CUMMINGS G. H. 202, 204, 207
- Current (electrical) 5
- Curved pipes, frictional losses 47
 - type of flow 48
- Cycloidal blower 133
- Darcy's friction factor 31
- DAVIS A. H. 170, 175
- DAVIES E. S. 174
- Dehumidification 305, 320
 - by cooling 316, 320
- Densities of liquids 350
- Density 3
- Derived units 1, 3
- Determination of humidity 318
 - chemical methods 318
 - dew-point method 319
 - torsion method 319
 - wet bulb method 318
- Development of laminar flow 272
- Dew point 306
 - for determination of humidity 319
- Diaphragm pump 109
- Diffusion 233
 - coefficient (*see* diffusivity)
 - Graham's Law 15
 - in the gas phase 234
 - in the liquid phase 241
 - pumps 135
 - ring 115
 - through stationary gas 236
- Diffusivity 233, 235, 247
 - in liquid phase 241, 242

- Diffusivity in liquid phase, calculation of 242
 - in terms of molecular motion 249
 - of vapour 239, 240
 - — — calculation of 240
 - — — determination of 240
- Dimensional analysis 7
 - — of heat transfer 164
- Dimensionless groups for heat transfer 165
 - — for mass transfer 243
- Dimensions 1, 5
- Discharge coefficient for notch 96
 - — — nozzle 90
 - — — orifice meter 84, 87
 - — — rotameter 94
 - — — venturi meter 91
- Displacement thickness of boundary layer 264, 268
- Distillation 233, 234, 244
- DODGE B. F. 16
- DONOHUE D. A. 172
- Double acting pump 106
- DREW T. B. 190, 201
- Dropwise condensation 190
- Drying 244
- Duplex pump 106
- Dynamic similarity 38
- Dyne 2, 4, 5

- EAGLE A. 167
- Economic pipe diameter 61
 - thickness of lagging 223
- Eddy currents 38, 250
 - diffusion 234, 250
 - diffusivity 234, 251
 - kinematic viscosity 38, 251, 275
 - thermal diffusivity 251
 - transfer 245, 250, 251
 - velocity 251
- Efficiency of acid egg 124
 - — air lift pump 127
- Ejectors 135
- Elbows, friction losses in 50
- Electric charge 4
- Electrical analogue for unsteady heat transfer 162
 - susceptibility 4
 - units 4, 5
- Emission as function of temperature 179
 - from a black body 179
- Emissive power 178, 181
- Emissivity 178, 179
 - of water vapour 183
- End contractions of notch 96
 - thrust in centrifugal pump 120
- Energy 3, 5
 - balance equation 23
 - — — ideal gas, irreversible process 27
 - — — — isentropic conditions 25
 - — — — isothermal conditions 25
 - — — incompressible fluid 24
 - — — in terms of fluid head 27
- Energy balance equation in terms of pressure 27
 - internal 22
 - kinetic 22
 - losses in air lift pump 130
 - — in acid egg 125
 - pressure 22
 - potential 22
 - relationships 13
- Enthalpy 17
 - difference in water cooling tower 326
 - -humidity chart 311, 312, 313
 - — — addition of liquid or vapour 317
 - — — isothermals 314
 - — — mixing of gases 317
 - — — oblique axes 316
 - — — rectangular axes 315
 - — — relations 314
 - of dry saturated steam 354
 - of dry steam (superheated) 356
 - of water 354
- Entropy 4, 5, 16
 - of dry saturated steam 354
 - of dry steam (superheated) 358
 - of water 354
- Entry conditions for flow in pipe 271
- Equation of state of ideal gas 14
- Equilibrium of element of fluid 39
- Equimolecular counterdiffusion 234
 - — and diffusion through stationary gas compared 237
- Erg 5
- Evaporation 233, 244, 256
- Ewing ball and tube meter 93
- Examples, boundary layer 270, 271, 274
 - fluid flow 32, 62, 88, 92
 - heat transfer 153, 154, 161, 181, 202, 214, 216, 222, 294, 296
 - humidification and drying 321
 - pumps 110, 144
 - water cooling 328
- Externally packed floating heads 206, 207

- Fahrenheit units 4
- Fanning 31
- Fanno line 59
- FERGUSON R. M. 167
- Ferrari's acid pump 109
- FICK A. 233
- Fick's Law 233, 246
- Film boiling 194
 - coefficients, determination of 211
 - — for air and water in tubes 168
 - — — coil 201
 - — — condensing vapours 184, 188
 - — — convection 164
 - — — finned tube 219
 - — in jacketted vessel 203
 - — (tables) 214
 - heat transfer coefficient for cylinder 170
- FINALBORG A. C. 196, 197, 198, 199

- Finite difference equation for unsteady heat transfer 162
 Finned tube heat exchangers 218
 Floating heads 206
 Flow gauge 98
 — in pipe 271
 — — — entry conditions 271
 — of fluids 10
 — of liquid over surface 184
 — rate, calculation from pressure drop 32
 Flowing temperature 78
 Fluid flow 10
 — head 27
 — pressure 74
 Foot 2, 4, 5
 Footpieces for air lift pump 129
 Force 1, 4, 5
 Forced convection heat transfer 151, 165, 252
 — — — inside tubes 166
 — — — — streamline flow 168
 — — — — turbulent flow of gases 167
 — — — — — of liquids 167
 — — — — viscous liquids 169
 — — mass transfer 243
 — — outside tubes 169
 — — — single cylinders 170
 — — — tube bundles 170
 Form drag 258, 279
 — — effect on heat and mass transfer 258
 F.P.S. system 2, 4, 5
 Francis formula for notch 96
 Friction chart for flow in pipes 30, 279
 — — — pipe flow $\left(\frac{R}{\rho u^2} \text{ v. } Re.\right)$ fold-out
 — — — — $\left(\frac{R}{\rho u^2} \cdot Re^2 \text{ v. } Re.\right)$ fold-out
 — factor, equations for 30
 — head 27, 31
 — loss in pipes 28
 — — — calculation of 31
 — — — — compressible fluids 51
 — — — — curved pipes 47
 — — — — over banks of tubes 50
 — — — — pipe fittings 49
 — — — — sudden contraction 48
 — — — — sudden enlargement 47
 FRITZ W. 192, 193
 Froude Number 71
 Fully developed flow 271
 GAMSON B. W. 258
 Gas radiation 182
 Gases, specific heats 344
 — thermal conductivities 346
 — viscosities 348
 Gaseous state 13
 Gear pump 112
 — — internal type 113
 Geometric factor for radiation 181
 GILLILAND E. R. 197, 240, 255
 Glandless centrifugal pump 121
 — plunger pump 110
 Graham's Law of Diffusion 15
 Gram 1, 4, 5
 Grashof Number 165, 243
 Gravitational force 1
 Grey body 179
 GRIFFITHS E. 170, 256
 GRIMISON E. D. 172
 HAGEN G. 42
 Hairpin tubes 206, 208
 HASELDEN G. G. 188
 Heat and mass transfer at liquid surface 308
 — exchangers 171, 205
 — loss from lagged pipe 222
 — transfer 151, 246, 249, 252
 — — between two fluids 155
 — — by conduction 151
 — — by convection 151, 164
 — — by radiation 151, 178
 — — coefficient 156
 — — — for boiling liquids 194, 199
 — — — — condensing vapours 186
 — — — — radiation 181
 — — — — streamline flow over plane surface 284
 — — — in water cooling tower 327
 — — — overall (tables) 213
 — — in condensation 183
 — — in reaction vessels 200
 — — streamline flow over plane surface 281
 — — — — through tube 285
 — — to boiling liquids 192
 — — to turbulent fluid, circular tube 291
 — — — — plane surface 290
 — — — — Reynolds analogy 289
 — — — — Taylor-Prandtl modification 289
 — — — — Universal velocity profile 293
 Heating liquids in tanks 204
 Height of water cooling tower 323
 — — — — construction 325
 Helical coils, heat transfer in 201
 HILPERT R. 170
 HIMUS G. W. 256
 HINCHLEY J. W. 256
 Horizontal tubes, condensation on 187
 Horse-power 6
 Hot wire anemometer 79, 97
 HOTTEL H. C. 183
 Humid heat 306, 310, 313
 — volume 306
 Humidification 244, 305, 319
 — definitions 305
 — of gas 310
 — systems other than air-water 331
 Humidifying tower design 330
 Humidity 305, 306, 311
 — chart 311, 312, fold out
 — determination 318

- Hydraulic gradient 28
 - jump 72
 - — energy dissipated 73
 - — maximum depth for 73
 - mean diameter 47, 173
 - — — of annulus 47
 - — — of circular section 47
 - — — of rectangular section 47
- Hydrodynamically smooth surface 30, 45
- Hypodermic tubing for pitot tubes 81
- Hytor pump 133, 134
- Hyvac pump 134
- Ideal gas 14, 18
- Impact pressure 77
- Impeller of centrifugal pump 115
- Inclined pipe, flow of gas 63
- Incompressible fluid, energy balance 24
 - — flow through constriction 81
 - — friction loss in pipes 28
 - — impact pressure 79
 - — nature of 18
 - — orifice meter 84
 - — static pressure 74
- Increasing humidity, methods 319
- Incremental heights in water cooling tower 327
- Inertia forces 38, 40
- Inlet length of pipe 271
 - to pipe, fall in pressure 272
- INSINGER T. H. 196, 199
- Insulating materials 153, 178
- Insulation 221
- Internal energy 15, 22
 - — as function of temperature and volume 18
 - — of ideal gas 19
 - — variation with temperature 17
- gear pump 113
- Interstage cooling 140
- Irreversible change 15
- Isentropic process 16
 - — pressure-volume relation for ideal gas 26
- Isothermal flow of ideal gas, heat flow required 54
 - — — — horizontal pipe 52
 - — — — inclined pipe 63
- Isothermals on enthalpy-humidity diagram 314
- Isotropic turbulence 250
- Jacketed pans 200
 - vessel 203
- JAKOB M. 184, 192, 193, 194, 197, 199
- JEBENS R. H. 201
- JESCHKE D. 201
- j*-factor for heat transfer 253, 255, 257
 - — mass transfer 254, 255, 257
- Joule 5
- Joule-Thomson effect 21
- KARMAN T. VON 275
- KELVIN 192
- Kelvin scale of temperature 3
- KERN D. Q. 200
- Kettle boilers 200
- Kinematic viscosity 3, 38
- Kinetic energy, correction factor 23
 - — — — streamline flow 43
 - — — — turbulent flow 47
 - — fluid 22
 - — molecules 14
 - — open channel 69
 - — streamline flow 42
 - — turbulent flow 46
- Kirchhoff's Law 178
- KIRKBRIDE G. C. 190
- Knock 108
- KOPP H. 241
- Kopp's law of additive volumes 241
- KRAUSSOLD H. 177
- Laboratory flow gauge 98
- Lagged pipe 222
- Lagging materials 222
- Laminar sub-layer 245, 246, 259, 268, 276
 - — thickness 269, 274
 - — velocity 269, 274
- LANDER C. H. 29
- Laplace transform 158, 244
- LAPPLE C. E. 57
- Latent heat of vaporisation 313
 - heats of vaporisation 340
- Laval nozzles 64
- LAWRENCE A. 167
- Length 1, 4, 5
- LEWIS J. G. 331
- Lewis relation 257, 310, 324, 327, 328
- Linear notch 96
- LINFORD A. 99
- LINKE W. 197, 199
- LINTON W. H. 256
- Liquid-liquid extraction 244
- Liquid state 13
 - as incompressible fluid 18
- Liquids, densities 350
 - specific heats 342
 - thermal conductivities 338
 - viscosities 350
- Lobe pump 113
- Logarithmic mean radius 155
 - — temperature difference 157
- MCADAMS W. H. 172, 188, 199
- Mach Number 56
- Magnetic permeability 4, 5
- Manometer 75
 - inclined 76
 - inverted 75
 - simple 75

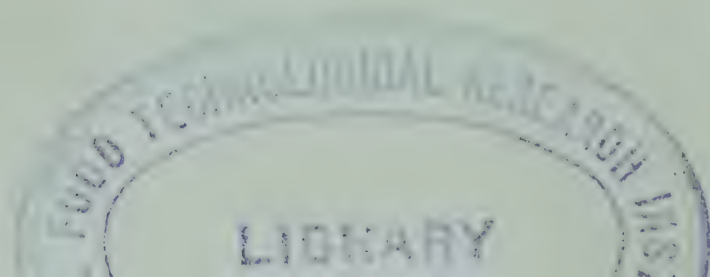
- Manometer, two-liquid 76
 MARKS W. M. 173
 Mass 1, 4, 5
 — rate of flow 23
 — transfer 233, 246
 — — applications of 244
 — — boundary layer 288
 — — by forced convection 243
 — — by natural convection 243
 — — coefficient 237, 238, 243, 254, 257
 — — — in water cooling tower 327
 — — in vertical tubes 254
 — — in wetted-wall columns 255
 — — through stationary gas 236
 — — to plane surfaces 256
 — — to turbulent fluid 296
 Materials of construction, piston pumps 108
 Maximum entropy 59, 60
 — flow, adiabatic flow in pipe 56, 59
 — — isothermal flow in pipe 53, 54
 — — orifice 87
 — heat flux to boiling liquids 196, 199
 — velocity, streamline in flow pipe 41
 MAXWELL J. C. 238, 240
 Maxwell's Law of Diffusion 238
 Mean free path 248, 249, 250
 — temperature difference 156
 — — — in multipass exchangers 209
 — velocity 23
 — — in pipe 273
 Measurement of flow rate 78
 — of pressure 74
 Mechanical energy 3, 5
 — equivalent of heat 3, 5
 MEHTA R. R. 212
 Metering pumps 123
 Minimum perimeter for rectangular channel 69
 Mixed vapours, condensation of 191
 Mixing length 250, 251, 275
 — of two humid gases 317
 Modified enthalpy 331
 Molecular diffusion 234
 — flow 37, 51
 — motion 14
 — transfer 245, 246
 — velocity 247, 248, 249, 250
 — volume 241, 243
 Momentum and heat transfer 253
 — equation 260
 — heat and mass transfer 232, 245
 — transfer 38, 246, 248
 Mono pump 114
 MOODY L. F. 29
 MUELLER A. C. 189, 210
 Multipass exchangers, mean temperature difference 209
 Multistage compression 140
 — — number of stages 142
 NAGLE W. M. 190, 210
 Natural convection 151
 — — heat transfer 165, 174
 — — — — air 176
 — — — — concentric cylinders 177
 — — — — horizontal surfaces 175
 — — — — streamline and turbulent flow 176, 177
 — — — — two surfaces 177
 — — — — various surfaces 176
 — — — — vertical surfaces 175
 — — mass transfer 243
 Newtonian fluids 11
 Non-circular ducts 47
 — sections, heat transfer in 173
 Non-condensable gas, effect on heat transfer 192
 Non-ideal gas 20
 — — flow in pipes 57
 Non-isothermal flow of gas in pipe 55
 Non-uniform flow in open channels 69
 Notch 79, 95
 — coefficient of discharge 96
 — end contractions 96
 — Francis formula 96
 — linear 96
 — rectangular 95
 — triangular 96
 Nozzles, flow measurement 79, 81
 — gas flow 64
 — Laval 64
 — minimum area 64
 — standard 90
 — subsonic flow 65
 — supersonic flow 65
 — throat pressure 64
 Nucleate boiling 194
 NUKIYAMA S. 194
 Number of stages 142
 Nusselt equation 186
 — — dimensionless form 187
 — Number 165, 177
 — — for flow over plane surface 285
 — — — mass transfer 243
 NUSSELT W. 184, 187
 Ohm 5
 Opaque material 178
 Open channels 66
 — — critical depth 71
 — — energy of fluid 69
 — — Froude number 71
 — — hydraulic jump 72
 — — non-uniform flow 69
 — — Reynolds Number 67
 — — specific energy 70
 — — types of flow 67
 — — uniform flow 67
 — — wave transmission 71
 Operating cost of heat exchanger 207, 208
 — line for water cooling tower 325
 Orifice meter 79, 81, 83

- Orifice meter, coefficient of contraction 84
 — — coefficient of discharge 84
 — — disadvantages 84
 — — ideal gas, critical pressure ratio 86
 — — — — high pressure ratios 86
 — — — — isothermal flow 85
 — — — — maximum flow 87
 — — — — non-isothermal flow 85
 — — incompressible fluid 84
 — — pressure tappings 83
 — — standard types 83
 Ower E. 99
- PANNELL J. 29
 Parallel plates, streamline flow 43
 Partial pressures of gas and vapour 306
 PASQUILL F. 256
 Passes in heat exchangers 206
 Percentage humidity 305, 306
 — relative humidity 306
 PERRY C. H. 196, 197, 198
 PIERSON O. L. 172
 Piezometer ring 75
 — tube 74
 PIGFORD R. L. 255
 Pipe fittings, friction loss in 50
 — flow 30
 Piston pump 106
 — — air vessel 108
 — — delivery from 106
 — — materials of construction 108
 — — method of driving 107
 — — theoretical delivery 107
 — — velocity of piston 106
 — — volumetric efficiency 107
 Pitot tube 78, 79
 — — alignment 80
 — — errors in 79, 80
 — — standard instruments 79, 80
 Plate heat exchangers 220
 Poise 6
 POISEUILLE J. 42
 Poiseuille's Law 42
 Positive displacement pumps 105
 — — rotary pumps 111
 Potential energy 22, 69
 Pound 2, 4, 5
 — weight 2, 4, 5
 Poundal 2, 4, 5
 POWELL R. W. 256
 Power 5
 — for compression 135
 — for pumping 60
 — — — gases 61
 — — — liquids 60
 PRANDTL L. 250, 251, 275, 289
 Prandtl Number 153, 165, 243, 249, 283
 — one-seventh power law 46, 266, 273
 PRATT H. R. C. 258
 PRATT N. H. 201, 202, 212
 Pressure 3, 13
 Pressure at pump delivery 107
 — distribution in air lift pump 128
 — drop due to friction in pipes 30
 — — — — — compressible fluids 51
 — — — — — incompressible fluid 31
 — effect on boiling 198
 — energy 22, 69
 — measurement 75
 — wave, velocity of transmission 53
 Problems 361
 PROSAD S. 188
 Pumping 60
 — with compressed air 123
 Pumps 104
 — for gases 131
 — — — and liquids 104, 132
 — — — centrifugal blowers 133
 — — — characteristics 133
 — — — efficiencies 133
 — — — maximum pressures 133
 — — — — throughputs 133
 — — — reciprocating piston compressor 132
 — — — rotary blowers 132
 — — — water cooling 133
 — — liquids, cam 113
 — — — centrifugal 115
 — — — factors affecting choice 105
 — — — gear 111
 — — — lobe 113
 — — — metering 123
 — — — Mono 114
 — — — piston 106
 — — — positive displacement rotary 111
 — — — reciprocating 106
 — — — types of 105
 — — — vane 113
 — function 104
 — work done 104
 Quantity meters 79, 98
 — — dry gas meter 98
 — — for gases 98
 — — — liquids 99
 — — wet gas meter 98
 Radiation 151, 178
 — between concentric cylinders 180
 — — parallel plates 180
 — from gases 182
 — intensity 180
 Rankine scale of temperature 3
 Rapid flow 67, 71, 72
 Reaction vessel with jacket 203
 — vessels, heat transfer in 200
 Reciprocating piston compressor 132
 — pumps 106
 — — diaphragm pump 109
 — — Ferrari's acid pump 109
 — — Kestner glandless pump 110
 — — piston pump 106
 — — plunger or ram pump 108

- Rectangular notch 95
- Reduced co-ordinates 20
 - pressure 20, 199
 - temperature 20
 - volume 21
- References, fluid flow (Chapters 2, 3, 4) 99
 - heat transfer (Chapter 6) 224
 - humidification and water cooling (Chapter 10) 332
 - momentum, heat, and mass transfer (Chapters 7, 8, 9) 297
 - pumps (Chapter 5) 146
 - units and dimensions (Chapter 1) 7
- REICHARDT H. 275
- REIHER M. 170
- Relative humidity 306
 - roughness 32
 - — of channel 68
 - — of pipe 29, 32
- Relaxation method 164
- Resistances, electrical 5
 - thermal 212
- Reversible change 15
- Reynolds analogy 252, 289
 - Number 28, 165, 248
 - — compressible fluid in pipe 52
 - — granular bed 39
 - — open channel 67
 - — orifice meter 87
 - — particle 39
 - — pitot tube 80
 - — plane surface 260
 - — roughness 278
 - — significance 38
- REYNOLDS O. 28, 252
- RHODES F. H. 212
- ROHSENOW W. M. 193
- Rootes blower 133
- Rosenblad heat exchanger 221
- Rotameters 93 (*see* Variable area meters)
- Rotary blowers 132
 - vane pump 134
- Roughness Reynolds Number 278
 - of surface of channel 68
 - — — effect of age 32
 - — — effect on shear stress 278
 - — — effect on velocity gradient 277
 - — — of pipe 32
- Saturated steam, enthalpy 354
 - — entropy 354
 - — specific volume 354
 - — temperature 354
 - volume 306, 313
- Saturation of gas 310
- SAUER E. T. 196
- SAUNDERS O. A. 175
- SAUVEUR A. 337
- Scale resistances 213
- SCHMIDT E. 163, 174
- Schmidt Number 239, 240, 242, 243, 250
- Schmidt's method for unsteady heat transfer 163
 - Second 1, 2, 4, 5
 - law of thermodynamics 13
- Self-priming centrifugal pump 120
- Separation 51, 108
- Shear stress at surface, mean value over whole surface 270
 - — — — streamline flow 265
 - — — — turbulent flow 266, 268, 269 275
 - — at walls of pipe 29
 - — — — effect of roughness 279
 - — — — turbulent flow 273
 - — in fluid 38, 275
- Shearing stress velocity 275, 278
- Shell and tube heat exchangers 205
 - — — — baffles 206, 209
 - — — — covers 209
 - — — — economic conditions 208
 - — — — externally packed floating head 206, 207
 - — — — fixed tube plates 205, 206
 - — — — flanges 209
 - — — — floating head 206
 - — — — hairpin tubes 206, 208
 - — — — operating cost 207, 208
 - — — — passes 206
 - — — — removal of tube bundle 206
 - — — — shell 208
 - — — — tie rods 209
 - — — — tube pitch 208
 - — — — tube sheets 209
 - — — — tube supports 209
 - — — — tubes 208
- SHERWOOD T. K. 167, 255, 256, 258, 309
- SHORT B. E. 172
- SIEDER E. N. 167, 201
- Simplex pump 106
- Single acting pump 106
 - stage compression 137
 - — — with clearance 138
- Skin friction 258, 279
 - — effect on heat and mass transfer 258
- Slug 2, 4, 5
- SMITH W. Q. 190
- Solid state 13
- Sonic velocity 56
- SOUTHWELL R. 164
- Specific energy of fluid in open channel 70
 - heat 3, 4
 - — at constant pressure 18
 - — — volume 18
 - heats of gases 344
 - — of liquids 342
 - speed of pump 119
 - volume of dry gas 313
- Spiral plate heat exchanger 221
- Spray ponds 322
- Stagnation temperature 78
- Standard nozzle 90

- Standard orifice meter 33
 - venturi meter 90
- Stanton Number 253, 254, 290, 291, 292
- STANTON T. 29
- Static pressure 74
 - — measurement of 74
- Steam jet ejectors 135
 - tables 353
- Stefan-Boltzmann constant 179
 - — law 179
- STEFAN J. 237
- Stefan's Law of Diffusion 237
- Stirring action of bubbles 193
 - effect on boiling 193
- Streamline boundary layer 259, 262
 - — — displacement thickness 264
 - — — thickness 264
 - flow 29
 - — annulus 44
 - — circular pipe 41
 - — kinetic energy 42
 - — mean velocity 42
 - — nature 38
 - — parallel plates 43
 - — pressure drop 40
 - — velocity distribution 42
 - — unstable 29
- Sudden contraction, friction loss 48
 - enlargement, friction loss 48
- Superheated steam, enthalpy 356
 - — entropy 358
- Surface, effect on boiling liquid 197
 - influence on boiling 193
- Surge wave, velocity of 71
- Symbols, fluid flow (Chapters 2, 3, 4) 101
 - heat transfer (Chapter 6) 226
 - humidification and water cooling (Chapter 10) 332
 - momentum, heat, and mass transfer (Chapters 7, 8, 9) 298
 - pumps (Chapter 5) 147
 - units and dimensions (Chapter 1) 7
- Tanks, heating liquids in 204
- TATE G. E. 167, 201
- Taylor-Prandtl modification of Reynolds analogy 289
 - — — — for gases 292
 - — — — for highly turbulent fluid 291
- TAYLOR G. I. 250, 289
- T.E.M.A. Code 209
- Temperature 3, 13, 4
 - change resulting from addition of liquid or vapour to gas 318
 - conversion chart 336
 - difference, effect on boiling liquid 194, 195, 196, 197
 - distribution in boiling liquid 192
 - flowing 78
 - gradient near surface 280
 - Temperature gradient near surface, streamline flow through heated pipe 286
 - — — — streamline flow over plane surface 282
 - -humidity chart 311, fold-out
 - of dry saturated steam 352
 - rise in terms of pressure drop for flow through tube 292
 - stagnation 78
 - thermodynamic scale 16
 - total 78
- TEN BOSCH M. 188
- TEPE J. B. 189
- Theoretical delivery from piston pump 107
- Thermal boundary layer 280
 - — — streamline flow over plane surface 281
 - — — streamline flow through pipe 285
 - — — thickness 284
 - conductivities of gases 346
 - — of liquids 338
 - conductivity 151
 - — in terms of molecular motion 248
 - — of solids, liquids, and gases 152
 - diffusivity 246, 247, 281
 - energy 3, 5
 - insulation 221
 - resistance 151, 153
 - resistances in series 153
 - — of metals 212
 - — — scale 213
 - units 4, 5
- Thermodynamic scale of temperature 16
- Thermodynamics of gas compression 135
 - — — — isentropic compression 136
 - — — — isothermal compression 136
 - — — — multistage 140
 - — — — reversible compression 135
 - — — — single stage 137
 - second law of 13
- Thick-walled tube, conduction through 154
- Thomas meter 98
- Tie rods 209
- Time 1, 45
- TINKER T. 172
- Torsion in fibre for measurement of humidity 319
- Total heat of dry saturated steam 354
 - — — steam (superheated) 356
 - — of water 354
 - temperature 78
- Tranquil flow 67, 71, 72
- Transition flow 29
- Triangular notch 96
- Tube bundles 170, 171
 - — heat transfer 173
 - — pressure drop 172
- Tubes for heat exchangers 208
- Turbine pump 115
- Turbocompressors 133

- Turbulence, effect on condensation 190
 - promoters 258
- Turbulent boundary layer 259, 265
 - — — displacement thickness 268
 - — — pipe flow 273
 - — — thickness 267
 - flow 29
 - — in pipe 273
 - — mean velocity 46
 - — nature of 38
 - — shear stress 45
 - — velocity distribution 45
 - Prandtl Number 251
 - Schmidt Number 252
- Types of boiling 193
 - of flow 37
 - of pumps 105
- Uniform flow in open channel 67
- Units 1, 4, 5
- Universal gas constant 14, 18
 - velocity profile 275, 277
 - — — effect of roughness 277
 - — — use for calculation of heat transfer 293
- Unsteady heat transfer 157, 205
 - mass transfer 244
- Vacuum pumps 134
- Vacseal pump 121
- Valves, friction loss in 50
- Van der Waals' equation 20, 57
- Vane pump 113
 - type blower 133
- Vaporisation in air stream 256
 - latent heats 340
- Vapour pressure at curved surface 192
 - velocity, effect on condensation 188, 189
- Variable area meters 93
 - — — coefficient of discharge 94
 - — — Ewing ball and tube meter 93
 - — — rotameters 93
- Velocity 1
 - distribution, effect of heat transfer 168
 - — in open channel 68
 - — in streamline flow in pipe 42
 - — in turbulent flow in pipe 45
 - gradient near surface 259
 - — — from universal velocity profile 277
 - — — — streamline flow 262
 - — — — turbulent flow 266
 - head 27
 - of light 4
 - of propagation of pressure wave 57
- Vena contracta 49, 84, 273
- Venturi meter 79, 81, 90
 - — coefficient of discharge 91
 - — friction losses 91
 - — standard 90
 - — with inclined axis 92
- Virtual head of centrifugal pump 116
- Viscosities of gases 348
 - of liquids 350
- Viscosity 3, 38
 - in terms of molecular motion 247
 - of water 344
- Viscous forces 38, 40
- V-notch 96
- Volt 5
- Volume 1
- Volumetric efficiency of piston pump 107
 - flow rate, streamline flow in pipe 41
- Volute pump 115
- WADE S. H. 256
- WASHINGTON L. 173
- Water cooling 244, 305, 322
 - — of compressors 133
 - — open ponds 322
 - — spray ponds 322
 - — tower 305, 322
 - — — approximate calculations 328
 - — — calculation of height 323
 - — — change in condition of air 326
 - — — construction for height 325
 - — — forced draught 322
 - — — heat and mass transfer coefficient 327
 - — — natural draught 322
 - enthalpy 354
 - entropy 354
 - latent heat of vaporisation 354
 - viscosity 344
- Watt 5
- Weight 1
- Weir 79
- WEST A. S. 202, 203, 204
- Wet bulb temperature 307, 313
 - — — compared with adiabatic saturation temperature 309
 - — — effect of gas velocity 308
 - — — for determination of humidity
- WHITE R. R. 331
- Wien's Law 179
- WILLIAMS R. W. 197
- WILSON E. E. 212
- WINKELMANN N. A. 240
- Work done by fluid 17
 - — by pump 104
- YOUNGER K. R. 212



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